Benefits of over-actuation in motion systems

M.G.E. Schneiders, M.J.G. van de Molengraft and M. Steinbuch

***Abstract*— This paper studies the benefits of using additional actuators in the design of motion systems. The motion part and disturbance attenuation part require different placement of the actuators to work optimally, which is not possible in traditional designs. Allowing extra actuators in the design makes it possible to overcome this limitation. In this way the controllability of the internal dynamics of the structure can be both maximized for feedback and minimized for feedforward. By tuning a static relation between the actuators it is possible to minimize the excitation of resonances in the feedforward path. Feedback controller design is based on an existing vibration control strategy, which enables placement of the bandwidth beyond the lowest resonance frequencies. The idea is demonstrated by the example of a levitated beam.**

I. INTRODUCTION

A. Motion systems

The objective of a motion system is to perform a programmed motion task with the end-effector of the system. This task can be the tracking of a setpoint trajectory or the execution of a point-to-point movement. The type of motion systems intended in this paper are used in high-end industrial applications and with accuracies up to the sub-micron level (i.e. waferstages, wirebounders and pick-and-place robots). Specifications for this type of motion systems are becoming higher and higher nowadays: motion tasks have to be performed faster (shorter settling-times) and more accurate (smaller tracking errors and residual resonances after settling). The performance of a motion system is determined by its closed-loop dynamics. This includes amplifiers, actuators, mechanical structure, control system (including control strategy and implementation) and sensors. Normally, the mechanics are the limiting link in this chain with respect to total performance.

B. Traditional design

Traditionally motion control systems contain a number of actuators equal to the degree of free rigid-body modes. Since all parts of the system are supposed to behave as rigid bodies with inertia only, this is a plausible choice. Unfortunately, all mechanical structures have finite stiffness thus show flexibility. Induced forces will not only move the motion system in its rigid-body modes, but also resonance modes will be excited. Basically, there are two ways to deal with these resonances:

- **Feedforward**. Since the setpoint is known beforehand, ideally a force signal can be designed to drive the system conform the setpoint. Acceleration-feedforward is widely used and gives good results. Reduction of residual resonances can be obtained by using model-based feedforward designs and input-shaping techniques [1]. Adaptive approaches even give better results, i.e. iterative learning control [2]. In the intended class of high-end motion systems over 99 % of the tracking performance is achieved by feedforward control. However, designing complex feedforward signals has certain disadvantages: they are not generic and not robust to plant variations.
- **Feedback**. Unfortunately the performance cannot be obtained by feedforward control only: there will always be tracking errors due to imperfections in the feedforward signal. Furthermore, feedforward cannot correct for disturbances and noise in the system. Remaining vibrations and external disturbances should be dealt with by the feedback controller. This is actually a vibration regulation task; not a motion control task. By adding damping in the feedback controller, it is possible to dampen the excited resonances. In general, structural damping is low, so this property of the feedback controller is very welcome.

Traditionally, in motion systems the actuators are placed at such locations that excitation of resonances is avoided as much as possible; the objective is to move the total structure as a rigid body. For the feedforward indeed this is the best possible solution: it is better to avoid excitation at all than preventing residual resonances with a tuned feedforward. The same actuator is used for regulating residual resonances and other disturbances via the feedback path. For optimal damping of resonances, the actuators are best located at places were these resonances can be influenced as much as possible. However, these locations do not coincide with the optimal locations for feedforward control [3]. So it is hard to efficiently add damping to the significantly contributing modes in traditional designs. The only way to achieve higher performance is by designing a stiffer mechanical construction, which means increase of moving mass and heavier construction. Often this is unwanted, since this means more energy dissipation, more losses and more heat generation.

C. Over-actuated design

In this paper, the key to overcoming this limitation in design is to allow for more actuators than the number of rigid-

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body modes to be controlled. For motion systems this is regarded as overactuation (the number of actuators is larger than the number of rigid-body modes to be controlled). It will be shown that by over-actuation it is possible to both minimize modal controllability in the feedforward path and maximize the modal controllability in the feedback path at the same time. This idea is the main contribution of this paper.

First, based on knowledge of the most dominant modes, actuator placement is determined according to maximization of the controllability. Next, for the feedback design, standard concepts in vibration control can be used. Third step is the feedforward design. A static gain relation between the actuators is determined, such that the dominant modes are not excited by this parallel actuation. Now, only a rigid-body type of feedforward signal has to be determined to drive the system.

II. NEW APPROACH

A. Modelling

In general the dynamic behavior of a mechanical structure is given by the partial differential equation (1), known as the generalized wave equation [4]. These equations must hold in every point p in the domain P of the structure.

$$M(p)\frac{\partial^2 u(p,t)}{\partial t^2} + Lu(p,t) = f(p,t) \tag{1}$$

The displacement of a point p is notated as $u(p,t)$, while $M(p)$ represent the distributed mass. L is a linear differential self-adjoint operator, depending on the type of model, and $f(p,t)$ represents the external force distribution. Additionally, boundary conditions can be added. In theory, the system contains an infinite number of resonance modes, given by the set of eigenvalues $\lambda_r (= \omega_r^2)$ and corresponding eigenfunctions ϕ_r ($r \in \{1, 2, \ldots\}$). These modal parameters can be found by solving the associated eigenvalue problem. The expansion theorem [4] relates the displacement in any point p of the construction as a linear combination of the modal coordinates:

$$u(p,t) = \sum_{r=1}^{\infty} \phi_r(p)\eta_r(t) \tag{2}$$

The eigenfunctions ϕ_r are orthogonal and can be normalized to the distributed mass M. The modes related to zero eigenvalues are called the rigid-body modes. The system can be converted into an infinite set of ordinary differential equations (3). For a discrete set of n_i actuators with forces F_i at positions p_i the right side of (3) can be written as $\sum_{i=1}^{n_i} \phi_r(p_i)F_i$. From this set of decoupled equations it is possible to construct the transfer function from a force f_i at position p_i to the displacement u_o at position p_o as the sum of all individual modal contributions (4). The instantaneous potential energy for each mode is given by (5).

$$\ddot{\eta}_r(t) + \omega_r^2 \eta_r(t) = \int_P \phi_r(p) f(p,t) dP \qquad r \in \{1, 2, \ldots\} \tag{3}$$

$$H_{oi}(s) = \frac{U_o(s)}{F_i(s)} = \sum_{r=1}^{\infty} \frac{\phi_r(p_o)\phi_r(p_i)}{s^2 + w_r^2} \tag{4}$$

$$V_r = \frac{1}{2}\omega_r^2 \eta_r^2(t) \tag{5}$$

Often modal damping is added, which adds an individual level of damping to each resonance mode. In practice, damping is much more complex and will cause coupling between the different resonance modes. However, for weakly damped systems this effect can be discarded and the orthogonality properties of (3) can be used. In practice, only a limited number of modes will contribute significantly to the total system response. Some reasons for this are:

- Much more energy needs to be introduced into higher modes to reach the same level of excitation as for the lower modes.
- Forces induced by actuators are band-limited and higher modes will be hardly excited.
- Closed-loop controllers have low-pass properties in general and will not amplify higher frequencies. So, also in closed-loop a limited number of modes is dominant. This can be tested by calculating the Hankel Singular Values of the closed-loop system (i.e. [5]).

For the rest of the paper the modes are divided into the next sets:

Set	Description
R_1	suppressed modes (i.e. by a guidance)
R_2	free rigid-body modes (in motion direction)
R_3	regulated rigid-body modes (i.e. in levitated structures)
R_4	dominant resonance modes
R_5	residual resonance modes

Performance in a motion system is closely related to the level of excitation of the resonance modes of the system. It would be convenient to have a quantitative energy measure to what extend a certain resonance mode can be excited by an input force. Quantitative measures for the degree of controllability (and observability) for individual modes of flexible structures were introduced in [6]. These commonly used energetic measures are all based on controllability and observability gramians for different positions of actuators and sensors [7]. Fortunately, if the damping is small, these measures can be directly related to eigenfrequencies and eigenfunctions [8]. Considering a single mode, the controllability $\mathcal{C}_r$ is proportional to the square of the value of the eigenfunction at the position of the input force p_i:

$$\mathcal{C}_r(p_i) \sim \phi_r^2(p_i) \tag{6}$$

Low controllability indicates placement of the actuator near a nodal point of the mode (given by the eigenfunction), and control of that mode is hardly possible. High controllability means the mode can be easily influenced by the input and damping can be added efficiently by a feedback controller. Sensor placement (related to observability) is also very important for proper feedback control, but will not be treated in this paper. Maximum observability and controllability for the most important resonances is desired for good vibration control.

B. Actuator placement

First, actuator placement based on the needs for the feedback system is considered: high controllability $\mathcal{C}_r$ of all significantly contributing modes $r \in \{R_3, R_4\}$. Since extra actuators mean extra costs, the number will be limited, and n_i will be smaller than the number of significantly contributing modes $r \in \{R_3, R_4\}$. An optimization criterium is proposed to optimize actuator placement (7). A comparable criteria is proposed in [8] and it is believed to provide a good balance between the importance of all modes. The function $J_1(\underline{p})$ will decrease rapidly if the controllability of any mode is lost. The individual controllability of the modes can be weighted by weighting functions w_r before multiplying them into the total optimization function $J_1(\underline{p})$. The weighting functions can be based on prior knowledge of the energy distribution in the feedback path and the significance of each the mode in the total performance of the system.

The mechanical design will put constraints on the placement of actuators, so possible locations are limited to certain regions $[a_i, b_i]$ (9). Orientation of the actuators may even be fixed beforehand: maximal driving of the free rigid-body modes $r \in R_2$ without moving the suppressed modes in R_1. Actuator positions are obtained by solving the nonlinear optimization problem (8) under the constraints (9).

$$J_1(\underline{p}) = \quad \prod_{r\in\{R_3,R_4\}} \quad w_r(\sum_{i=1}^{n_i} \phi_r(p_i))^2 \qquad (7)$$

$$\underline{p} \qquad = [p_1 \dots p_{n_i}]$$

$$\underline{p}^* = \arg\max J_1(\underline{p}) \qquad (8)$$

$$a_i \leq p_i \leq b_i \quad i = 1, \dots, n_i \qquad (9)$$

C. Feedback design

In the design of the feedback controller, the actuator positions $\underline{p}^*$ follow from (8) and disturbance attenuation can be obtained by common vibration control techniques. The problem of controlling flexible structures is an area of active research (i.e [9],[10],[11],[12]). Much of this work considers the use of distributed sensors and actuators (mostly piezo-electric materials), in combination with robust control techniques. Other approaches focus on the decoupling of the control problem in modal space, developed in the early 80's [13]. It enables SISO control design for a limited set of modal contributions. This approach is more appropriate for this application, since it gives much insight in the design process and is closely related to control design in traditional motion systems (loop-shaping).

D. Feedforward design

Since more actuators than strictly needed are included there are many possibilities to design the feedforward controller: feedforward design is not straightforward anymore. The benefits of multiple actuators in input-shaping design has been proven [14], but the disadvantages of traditional input-shaping are still present. This new approach focusses on minimal controllability of $r \in R_4$ without designing the feedforward signal itself. A static gain relation K_{ff} (see Fig. 1) between the actuators is created that avoids excitation (and minimizes controllability) of the significantly contributing modes as much as possible. A cost function representing the modal excitations as function of the static actuator ratios k_i is proposed (10). The individual modal contributions can be tuned by choosing the weights v_r (10), e.g. based on the spectral energy of the setpoint. To make optimization of (11) possible, the gains are lower and upper bounded (12). Since the regulated rigid-body modes in R_3 may not be influenced by the feedforward path at all, constraint (13) is added. The only modes that have to be "excited" (moved) are the rigid-body modes $r \in R_2$. To ensure a sufficiently high level of controllability for these modes, and to enable a feedforward signal within the saturation limits of the actuators, constraint (14) is added.

$$J_2(\underline{k}) = \quad \sum_{r\in R_4} \quad v_r \sum_{i=1}^{r_i} (\phi_r(p_i^*)k_i)^2 \qquad (10)$$

$$\underline{k} \qquad = [k_1 \dots k_{n_i}]$$

$$\underline{k}^* = \arg\min J_2(\underline{k}) \qquad (11)$$

$$-1 \leq k_i \leq 1 \quad k_i = 1, 2, \dots, n_i \qquad (12)$$

$$\sum_{i=1}^{n_i} \phi_r(p_i)k_i = 0 \qquad \forall r \in R_3 \qquad (13)$$

$$\sum_{i=1}^{n_i} \phi_r(p_i)k_i \geq \alpha_r \qquad \forall r \in R_2 \qquad (14)$$

Now modes in R_4 do not contribute significantly in the total transfer (4) any more, and only the rigid-body modes (in R_2) with $\omega_r = 0$ are significantly present. The system $K_{ff}P$ as seen by the feedforward controller C_{ff} now resembles almost a perfect inertia up to a certain frequency. The feedforward controller C_{ff} can now be designed as a relatively simple mass-feedforward for each rigid-body mode to be controlled (see Fig. 1). Different setpoints are allowed as long as their frequency content is similar; the designed K_{ff} will still be optimal.

III. EXAMPLE

A. Modelling

The proposed idea will be illustrated with the example of a flexible beam. For this purpose an idealized positioning

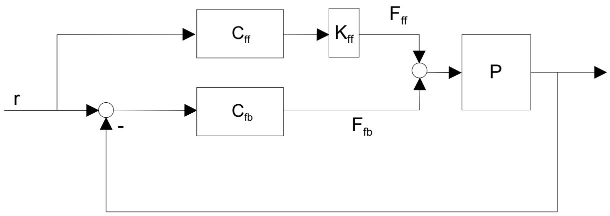

Fig. 1. Control scheme for integrated feedforward/feedback design. Optimal feedforward is reached if $K_{ff} = \underline{k}^*$.

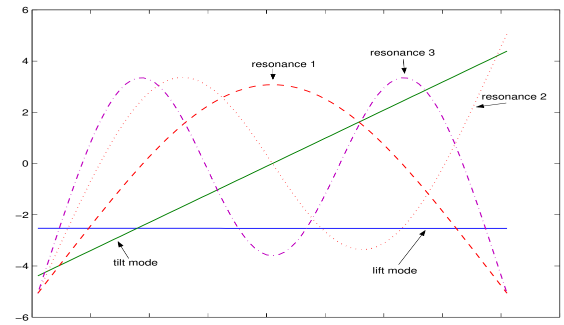

Fig. 3. The first 5 eigenfunction of the beam $(\phi_1, \ldots, \phi_5)$

system is defined. The positioning task is to move the entire beam over a certain trajectory $r(t)$ in z-direction Fig. 2. This motion must be performed as fast as possible with as little as possible residual vibrations. So, in this case the performance criterium is defined over a line and not in a single point. Gravity is not included in the model.

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + EI \frac{\partial^4 w(x,t)}{\partial x^4} = f(x,t) \tag{15}$$

For small deflections this system can be modelled as a Bernoulli-Euler beam, which fits within the class of distributed parameter systems in (1). For uniform cross-section of the beam the equations of motion for the system are given by (15). Here $w(x,t)$ is the deflection of the beam in z-direction along the position x. ρ is the density of the used material, A the area of the cross-section, E the Young's modulus of the material and I the second moment of area of the cross-section. $f(x,t)$ represents the external distributed force applied to the beam. For

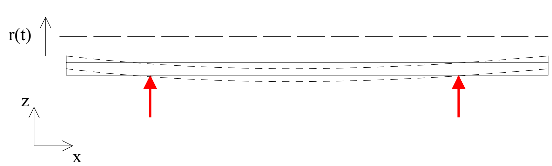

Fig. 2. Bernoulli-Euler beam

simulation purposes a lumped parameter model is created and implemented in Matlab/Simulink. The corresponding mass and stiffness matrices of the element-formulation can be derived from numerous finite element references (i.e. [15] or [16]).

B. Traditional design

The system possesses two rigid-body modes; a tilt mode and the desired lift mode. Traditionally, two actuators would be used to drive the system. In the normal case, also two position measurements would be used for feedback. In this case collocated control is proposed for each actuator/sensor pair. Bandwidths are tuned below the first resonance (around 15 Hz), which is quite common in traditional designs. The lowest resonance mode (mode 3) will be limiting for the performance. Only this mode is taken into consideration for this moment to make the problem clear. Two extreme situations for the placement of actuators are:

- **Situation A.** Actuators are placed for maximal controllability of mode 3 ($p_i = [0, 1]$). The middle position of the beam shows the largest controllability, but this configuration will not be considered since the tilt mode cannot be controlled. Placement at both ends of the beam seems the best suboptimal solution according to (8). A simple 2-mass-spring system (see Fig. 4) can represent the transfer from F_1 to x_1 (or from F_2 to x_2). Motion of the system will excite the internal resonance heavily, but it is easy to add damping by the feedback controller (see sensitivity plot in Fig. 8). However, for good tracking performance, a modal-based feedforward is necessary.

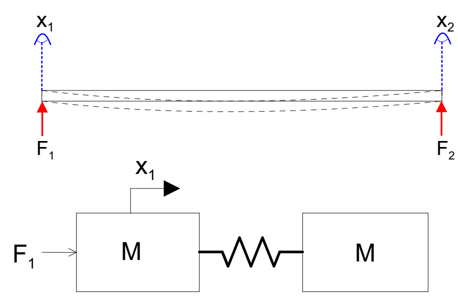

Fig. 4. Actuator-placement at the both ends of the beam. The equivalent 2-mass-spring system show equal masses for this situation (for uniform cross-sections only).

- **Situation B.** Actuators are placed in the nodal points of the first parasitic mode ($\phi_3(p_1) = \phi_3(p_2) = 0$). The motion task, introduced by either feedback or feedforward, will (almost) not excite the mode (see Fig. 7). Exact cancellation will be difficult to achieve in practice. Furthermore, excitation of mode 3 due to disturbances or noise cannot be damped out by the controller, which can be seen from the sensitivity in Fig. 8. Disturbance attenuation of the system will be bad (see also Fig. 5).

For both situations the ratio between the two actuators needed for any feedforward signal is rather trivial. The first mode must be maximally driven, while the tilt mode ($\in R_3$)

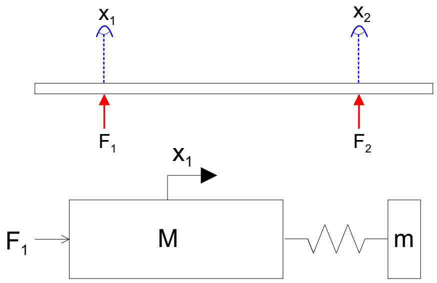

Fig. 5. Actuator-placement in nodal points (almost). Equivalent 2-mass-spring system show that the resonance is hardly controllable

has to be avoided. This means $k_1 = k_2$. No more freedom of suppressing resonances is possible in this strictly-actuated system.

C. Over-actuated design

In the over-actuated design we allow for two more actuators. A constraint is added that prohibits placement in the middle region $x \in \langle 0.35, 0.65 \rangle$, which is often the case in practical situations (i.e. a gantry system). Optimizing (8) results in a placement as in Fig. 6; two actuators at both beam ends (**situation A** in the traditional design) and the two extra actuators placed at the borders of the constraint: $p_i = [0 \;\; 0.35 \;\; 0.65 \;\; 1]$. Since the eigenfunctions (see Fig. 3) are ranged in the same order, equal modal coordinates η_r bring about equal levels of physical excitation. Claiming equal controllability measures for each mode for the same level of physical excitation, weighting functions are chosen as $w_r = \frac{1}{\omega_r^2}$, according to (5). Now we have the freedom to

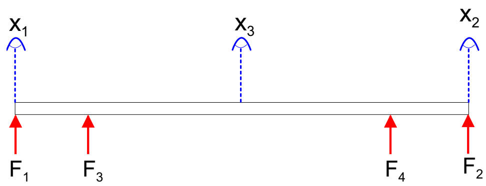

Fig. 6. Overactuated design

tune the static gain relation K_{ff} for the feedforward controller. Since the resonances are well spaced, suppressing both mode 3 and 4 completely (together with the tilt mode $\in R_3$), gives the best overall performance. For this situation the optimal gain relation is calculated (16) according to (11).

$$\underline{k}^* = [0.41 \quad 1 \quad 1 \quad 0.41]^T \tag{16}$$

To show how also feedback performance can be enhanced in the over-actuated case, an independent modal-space controller for the lowest 3 modes is designed. For this purpose, another sensor is added in the middle of the beam. Controllability at this place is high for the lowest three modes.

$$C_{fb} = K_a C(s) K_s \tag{17}$$

Independent modal-space control relies on decoupled control design for a limited set of modes [13]. The decoupling into modal space K_s and back from modal space K_a is based on inversion of the expansion theorem (2). Consequently, the actual (dynamic) controller $C(s)$ is diagonal (17).

In this case, the 2 rigid-body modes and the first resonance are now individually controlled, whereas the second resonance mode is not controlled at all (this idea is similar to the approach in [17]). The approach may not be optimal, but it prevents stability problems due to the higher modes (spillover). With this design, the bandwidth can be placed beyond the first resonance mode at 43 Hz (see Fig. 8), without decreasing overall performance. The tracking performance for the 3 situations is compared by applying a third-order polynomial setpoint to the systems (see Fig. 9). For the overactuated case, mode 3 and mode 4 are not excited (see Fig. 7) and the resulting tracking error is almost completely caused by mode 5. After settling almost no residual resonances are present.

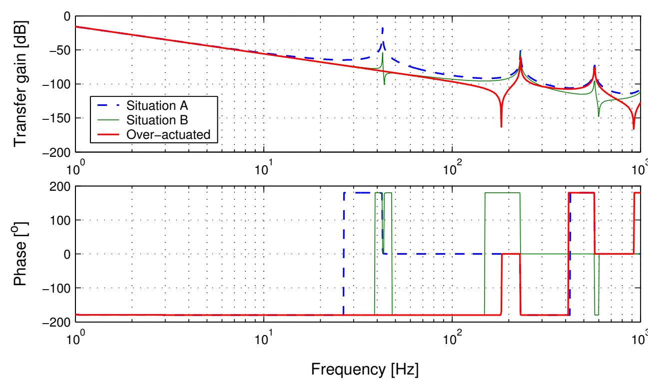

Fig. 7. Plant transfers from the feedforward signal F_{ff} (see Fig. 1) to the deflection at the middle of the beam. Note that mode 4 (second resonance mode at 117 Hz) is not excited in all cases.

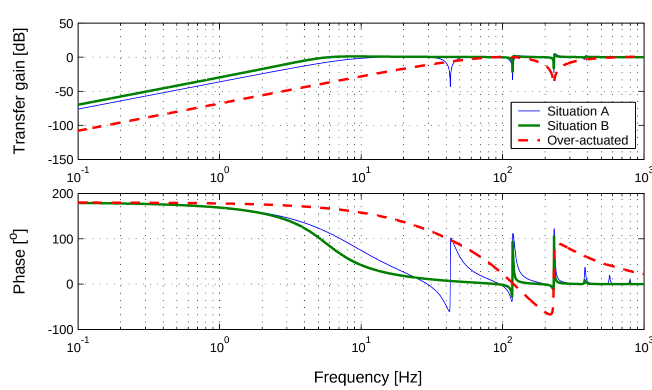

Fig. 8. Sensitivity transfers for situation A and situation B. For the over-actuated case, only the sensitivity transfer from the lift mode is given. It is clear that the over-actuated case enables better disturbance attenuation.

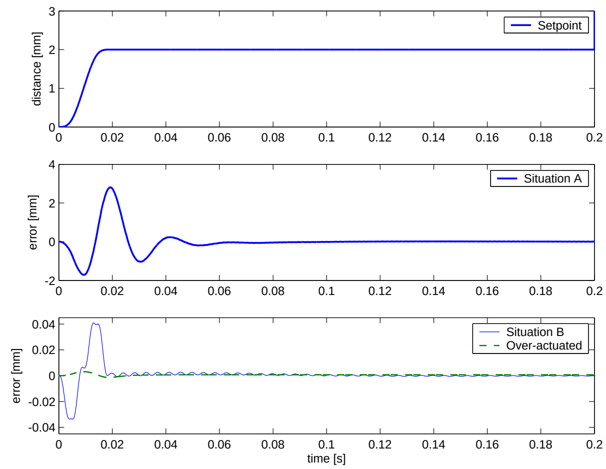

Fig. 9. Tracking errors for the 3 situations using acceleration-feedforward only.

IV. CONCLUSIONS

A. Discussion approach

The design approach presented in this paper makes it possible to decouple the controllability of resonances modes for the feedback and the feedforward part. This is only possible by allowing extra actuators. The largest benefit of this approach is the possibility to design motion systems with higher accuracy without increasing mechanical stiffness and moving mass. Piezo-actuators are widely used nowadays and they can be easily integrated in the design since they are flat and lightweight. Further integration of control design and mechanical design is likely to open new doors to enable over-actuation.

The feedforward gain K_{ff} has to be tuned only once for optimal design, which is another benefit of this approach. Varying setpoints are allowed without retuning, as long as the frequency content is more or less the same. Furthermore, the disturbance attenuation will be much better, since *more* modes show *higher* controllability in comparison with the traditional design.

Of course their are certain drawbacks. The motion system design must allow for the placement of extra actuators, which is not always possible. Often ideal actuator placement is not possible and the benefits of this approach could then be limited. In practice, a lot of compounded motion systems have a number of moving parts and show time-variant behavior. If corresponding eigenfunctions vary too much in time, the performance and the applicability of this new approach is not guaranteed. However, it seems possible to extend the method by introducing a position or time dependent gain relation K_{ff}.

B. Future work

This paper only presents the main benefits of over-actuation and a first design optimization approach in which feedback and feedforward are separately (actually in series) optimized. If disturbances are included in the design, it is possible to impose certain performance criteria on the feedback controller. Then it is possible to compare the performance needed from the feedback controller with the performance asked from the feedforward path. The final goal is to develop a design approach in which the contributions of the feedback controller and the feedforward path together are optimized. Feedback design has to be incorporated then, so also sensor placement (observability) will have to be included.

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