Variable Structure Adaptive Fuzzy Control for A Class of Nonlinear Time Delay Systems

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Abstract— **In this paper, the problem of variable structure adaptive fuzzy control for a class of nonlinear time delay systems is investigated. Sliding surface dependent of the time delay is constructed based on the Lyapunov-Krasovskii method. Two adaptive fuzzy logic systems are proposed to approximate two unknown continuous nonlinear functions containing the current state and the delayed state respectively. The corresponding reaching law is designed, which can drive the state trajectory of the closed-system onto the sliding surface within limited time.**

I. INTRODUCTION

Time-delay is often encountered in various engineering systems, such as electrical networks, turbojet engines, microwave oscillators, nuclear reactors, rolling mills, chemical processes, manual control, long transmission lines in pneumatic, and hydraulic systems, etc. Its existence is often a source of instability and poor performance. Therefore, the problem of stability analysis and robust control for dynamic time-delay systems has attracted considerable attention of a number of researchers over the past years [13, 14 and the references therein].

Fuzzy control has recently found extensive applications for a wide variety of industrial systems, and has attracted the attention of many control researchers due to its model free approach ([1] and [2]). Fuzzy control algorithms attempt to make use of information from human experts, which is generally represented by fuzzy terms, such as small, large, not very large or not very small, etc. More recently, an important adaptive fuzzy control system has been developed incorporated with the expert information systematically and the stability is guaranteed by theoretical analyses ([3] and [4]). An adaptive fuzzy system is a fuzzy logic system equipped with a training algorithm, in which the fuzzy logic system is constructed from a collection of fuzzy IF-THEN rules, and the training algorithm adjusts the parameters of the fuzzy logic system according to numerical input/output data.

Based on fuzzy logic systems which are capable of approximating, with arbitrary accuracy, any real continuous functions on a compact set, a globally stable adaptive controller is first synthesized from a collection of IF-THEN rules in [5]. In the existing literature on fuzzy control of nonlinear systems, fuzzy logic systems are used to approximate unknown nonlinear functions. Reference [6]

investigated H_∞ tracking problem of uncertain nonlinear SISO systems with external disturbance by using adaptive fuzzy controllers. Fuzzy indirect adaptive controller for a class of decentralized nonlinear systems is constructed in [7]. Paper [8] investigated robust adaptive fuzzy control for SISO nonlinear systems based on backstepping method. However, in the existing papers, few results on fuzzy control of time delay nonlinear systems are reported. Based on T-S fuzzy models, [15] was the first paper on the fuzzy control using Lyapunov-Krasovskii functional approach, while [16] used the Lyapunov-Razumikhin functional appraoch.

In this paper we consider the problem of robust variable structure adaptive fuzzy control for a class of nonlinear systems with time delay. As we know that a sliding mode control has attractive features such as fast response and good transient response. So we adopt the sliding mode control method in this paper. Fuzzy logic systems are also used as universal approximators of unknown nonlinear functions. The problem of designing both a linear sliding surface and reaching motion controller is investigated. A sliding surface is first designed by solving a Riccati inequality, which can guarantee that the sliding motion is exponentially stable dependent of the time delay. Then the reaching motion controller is designed, which can drive the state trajectory of the system onto the sliding surface within limited time.

The paper is organized as follows. The problem formulation is given in Section 2. A brief description of fuzzy logic systems is provided in Section 3. In Sections 4 and 5, the designs of the sliding surface and the reaching law are given respectively.

II. PROBLEM FORMULATION

Consider the following time delay system

$$\dot{x} = (A + \Delta A(t))x + (A_d + \Delta A_d(t))x(t-d) \quad (1)$$
$$+ B(u + F(x, x(t-d), \sigma(t)))$$

$$x(t) = \varphi(t), \ t \in \begin{bmatrix} -d & 0 \end{bmatrix}.$$

where $x \in R^n$ and $u \in R$ are the state and control input respectively, d is the delay parameter, A, B and A_d are matrices with appropriate dimensions. $\Delta A(t)$ and $\Delta A_d(t)$ are time-varying uncertainties. $F(x, x(t-d), \sigma(t))$ is an

unknown function containing uncertainties, where $\sigma(t)$ is a time varying disturbance parameter. In system (1), for simplicity we assume

$$B = [0, \cdots, 0, 1]^T$$

and other matrices can be decomposed into the following

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \qquad (2)$$

$$\Delta A(t) = \begin{pmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{pmatrix},$$

$$A_d = \begin{pmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{pmatrix},$$

$$\Delta A_d = \begin{pmatrix} \Delta A_{d11} & \Delta A_{d12} \\ \Delta A_{d21} & \Delta A_{d22} \end{pmatrix}$$

where $A_{11}, \Delta A_{11}, A_{d11}, \Delta A_{d11} \in R^{(n-1)\times(n-1)}$, $A_{22}, \Delta A_{22}, A_{d22}, \Delta A_{d22} \in R$. Thus system (1) can be written as

$$\dot{x}_1 = (A_{11} + \Delta A_{11})x_1 \qquad (3)$$
$$+ (A_{d11} + \Delta A_{d11}(t))x_1(t-d)$$
$$+ (A_{12} + \Delta A_{12})x_2 + (A_{d12} + \Delta A_{d12})x_2(t-d)$$
$$x_1(t) = \overline{\varphi}_1(t), \ t \in \begin{bmatrix} -d & 0 \end{bmatrix}$$

$$\dot{x}_2 = (A_{21} + \Delta A_{21})x_1 \qquad (4)$$
$$+ (A_{d21} + \Delta A_{d21}(t))x_1(t-d)$$
$$+ (A_{22} + \Delta A_{22})x_2 + (A_{d22} + \Delta A_{d22})x_2(t-d)$$
$$+ u + F(x, x(t-d), \sigma(t))$$
$$x_2(t) = \overline{\varphi}_2(t), \ t \in \begin{bmatrix} -d & 0 \end{bmatrix}$$

where $x_1 \in R^{n-1}, x_2 \in R$. We introduce the following assumptions imposed on the system (1):

Assumption 1 The uncertainties $\Delta A_{11}(t)$, $\Delta A_{12}(t)$, $\Delta A_{d11}(t)$ and $\Delta A_{d12}(t)$ satisfy

$$\begin{bmatrix} \Delta A_{11}(t) & \Delta A_{12}(t) \end{bmatrix} = D_1 F_1(t) \begin{bmatrix} E_{11} & E_{12} \end{bmatrix} \qquad (5)$$

$$\begin{bmatrix} \Delta A_{d11}(t) & \Delta A_{d12}(t) \end{bmatrix} = D_2 F_2(t) \begin{bmatrix} E_{21} & E_{22} \end{bmatrix}$$

where $\|F_1(t)\| \leq 1$, and $D_1, D_2, E_{11}, E_{12}, E_{21}$ and E_{22} are some known matrices with proper dimensions. In addition

$$\|\Delta A\| \leq \gamma_1, \|\Delta A_d\| \leq \gamma_2$$

Where positive scalars γ_1 and γ_2 are not required to be known.

Assumption 2 $F(x, x(t-d), \sigma(t))$ satisfies the following decomposition

$$F(x, x(t-d), \sigma(t))$$
$$= F_1(x) + F_2(x(t-d)) + \Delta F(x(t), x(t-d), \sigma(t))$$

where $F_1(x)$ and $F_2(x(t-d))$ are unknown continuous functions, $\Delta F(x(t), x(t-d), \sigma(t))$ is an uncertain function which is also unknown, but bounded

$$|\Delta F(x(t), x(t-d), \sigma(t))| \leq \alpha_0 + \alpha_1 \|x(t)\| + \alpha_2 \|x(t-d)\| \qquad (6)$$

where α_0, α_1 and α_2 are unknown positive scalars.

Problem: Design a sliding surface to guarantee the sliding motion exponentially stable with a given decay rate γ for system (1) and construct the reaching control law $u(t)$ to render the state trajectory of the closed-loop system convergent to the sliding surface in limited time.

III. DESCRIPTION OF FUZZY LOGIC SYSTEMS

A fuzzy system is a collection of fuzzy IF-THEN rules of the form:

$$R^{(j)} \ : \ IF \ x_1 \ is \ A_1^j \ and \ \cdots \ and \ x_n \ is \ A_n^j \qquad (7)$$
$$THEN \ y \ is \ B^j.$$

By using the strategy of singleton fuzzification, product inference and center-average defuzzification, the output of the fuzzy system

$$y(x) = \frac{\sum_{j=1}^l y^j \left(\prod_{i=1}^n \mu_{A_i^j}(x_i) \right)}{\sum_{j=1}^l \prod_{i=1}^n \mu_{A_i^j}(x_i)} \qquad (8)$$

where $\mu_{A_i^j}(x_i)$ is the membership function of linguistic variable x_i, and y^j is the point in R at which μ_{B^j} achieves its maximum value (assume that $\mu_{B^i}(y^i) = 1$).

By introducing the concept of the fuzzy basic function vector $\zeta(x)$, (8) can be written as

$$y(x) = \theta^T \zeta(x)$$

where

$$\theta = (\theta_1, \cdots, \theta_l)^T, \zeta(x) = (\zeta_1(x), \cdots, \zeta_l(x))^T$$

and $\zeta^j(x)$ is defined as

$$\zeta_j(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^l \prod_{i=1}^n \mu_{A_i^j}(x_i)}$$

In this paper we will take two universal fuzzy systems $\widehat{F}_1(x, \Theta_1)$ and $\widehat{F}_2(x(t-d), \Theta_2)$ with $x, x(t-d) \in U_x$ for some compact set U_x, to approximate the uncertain terms $\widetilde{F}_1(x(t))$ and $\widetilde{F}_2(x(t-d))$ defined in (34), where Θ_1 and Θ_2 contain tunable parameters. We construct the fuzzy rule bases of $\widehat{F}_1(x, \Theta_1)$ and $\widehat{F}_2(x(t-d), \Theta_2)$ consisting of the following rules:

$$R_{F_1}^j \quad : \quad IF \ x_1 \ is \ A_1^j \ and \ \cdots \ and \ x_n \ is \ A_n^j$$
$$THEN \ \widehat{F}_1(x, \Theta_1) \ is \ F_1^j.$$

$$R_{F_2}^j \quad : \quad IF \ x_1(t-d) \ is \ \overline{A}_1^j \ and \ \cdots \ and \ x_n(t-d) \ is \ \overline{A}_n^j$$
$$THEN \ \widehat{F}_2(x(t-d), \Theta_2) \ is \ F_2^j.$$

and construct the following fuzzy systems

$$\widehat{F}_1(x, \Theta_1) = \Theta_1^T \xi_1(x)$$

and

$$\widehat{F}_2(x(t-d), \Theta_2) = \Theta_2^T \xi_2(x(t-d))$$

where $\xi_1(x)$ and $\xi_2(x(t-d))$ are fuzzy basis functions. According to the universal approximation theorem there exist optimal approximation parameters Θ_1^* and Θ_2^* [3], such that $\xi_1(x)^T \Theta_1^*$ and $\xi_2(x(t-d))^T \Theta_2^*$ can approximate $\widetilde{F}_1(x)$ and $\widetilde{F}_2(x(t-d))$ to any desired degree. The parameters Θ_1^* and Θ_2^* are defined as follows

$$\Theta_1^* = \arg \min_{\Theta_1 \in \Omega_{F_1}} \left(\sup_{x \in \Omega_x} \left| \widehat{F}_1(x, \Theta_1) - \widetilde{F}_1(x) \right| \right) \quad (9)$$

$$\Theta_2^* = \arg \min_{\Theta_2 \in \Omega_{F_2}} \left(\sup_{x(t-d) \in \Omega_x} \left| \begin{array}{c} \widehat{F}_2(x(t-d), \Theta_2) \\ -\widetilde{F}_2(x(t-d)) \end{array} \right| \right) \quad (10)$$

where Ω_{F_1}, Ω_{F_2} and Ω_x denote the sets of suitable bounds on Θ_1, Θ_2 and x, respectively. We assume that Θ_1, Θ_2 and x never reach the boundaries of Ω_{F_1}, Ω_{F_2} and Ω_x. And the minimum approximation error satisfies the following assumption

Assumption 3 The approximation error between $\widetilde{F}_1(x), \widetilde{F}_2(x(t-d))$ and $\xi_1(x)^T \Theta_1^*, \xi_2(x(t-d))^T \Theta_2^*$ satisfies the following inequality

$$\left| \begin{array}{c} \widetilde{F}_1(x) - \xi_1(x)^T \Theta_1^* + \widetilde{F}_2(x(t-d)) \\ -\xi_2(x(t-d))^T \Theta_2^* \end{array} \right| \quad (11)$$
$$\leq \quad \hbar_0 + \hbar_1 \|x\| + \hbar_2 \|x(t-d)\|$$

where $\hbar_0, \hbar_1$ and $\hbar_2$ are unknown scalars. These two parameters Θ_1^* and Θ_2^* will be learned by using the adaptive algorithms.

IV. SLIDING SURFACE DESIGN

Without loss of generality, we suppose that the sliding surface is

$$S = \widetilde{C}x = \begin{bmatrix} -C & 1 \end{bmatrix} \begin{bmatrix} x_1^T & x_2 \end{bmatrix}^T = (-Cx_1 + x_2) = 0 \quad (12)$$

where $C \in R^{n-1}$. Hence, substituting $x_2 = Cx_1$ into (3) gives the sliding motion

$$\dot{x}_1 = (A_{11} + \Delta A_{11})x_1 \quad (13)$$
$$+ (A_{d11} + \Delta A_{d11}(t))x_1(t-d)$$
$$+ (A_{12} + \Delta A_{12})Cx_1$$
$$+ (A_{d12} + \Delta A_{d12})Cx_1(t-d)$$
$$x_1(t) = \overline{\varphi}_1(t), \quad t \in \begin{bmatrix} -d & 0 \end{bmatrix}$$

If system (13) is exponentially stable dependent of the delay, the sliding motion (13) will be exponentially stable dependent of the delay.

Theorem 1 Consider system (13), if assumption 1 is met and the following inequality is satisfied

$$P(A_{11} + \gamma I) + (A_{11} + \gamma I)^T P \quad (14)$$
$$+ \varepsilon^{-1} PD_1 D_1^T P + \varepsilon E_{11}^T E_{11}$$
$$+ \varepsilon^{-1} e^{2\gamma d} PA_{d12} A_{d12}^T P + e^{2\gamma d} PA_{d11} X A_{d11}^T P + X^{-1}$$
$$+ \varepsilon^{-1} e^{2\gamma d} PD_2 D_2^T P + \varepsilon E_{21}^T E_{21}$$
$$- \left[\varepsilon \left(1 + E_{12}^T E_{12} + E_{22}^T E_{22} \right) \right]^{-1}$$
$$\left(PA_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)$$
$$\left(PA_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)^T$$
$$< \quad 0$$

where P and X are positive matrices, ε is a positive scalar, then the following sliding surface

$$S = \left[\varepsilon \left(1 + E_{12}^T E_{12} + E_{22}^T E_{22} \right) \right]^{-1} \quad (15)$$
$$\left(PA_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)^T x_1$$
$$+ x_2$$

will render the sliding motion exponentially stable dependent of the delay with a given decay rate γ.

Proof: For system (13), we choose the following coordinate transformation

$$z(t) = e^{\gamma t} x_1(t)$$

we can obtain

$$\dot{z} = \gamma e^{\gamma t} x_1 + e^{\gamma t} \dot{x}_1 \quad (16)$$
$$= (A_{11} + A_{12}C + \gamma I + \Delta A_{11} + \Delta A_{12}C)z$$
$$+ e^{\gamma d}(A_{d11} + A_{d12}C + \Delta A_{d11} + \Delta A_{d12}C)z(t-d)$$

As we know, if system (16) is robust asymptotically stable, system (13) will be exponential stable with a given attenuance γ dependent of the delay.

For system (16), let us define the following Lyapunov-Krasovskii function

$$V(z) = z^T Pz + \int_{t-d}^t \varepsilon z^T(\upsilon)$$
$$\left[\begin{array}{c} (E_{21} + E_{22}C)^T (E_{21} + E_{22}C) \\ + X^{-1} + C^T C \end{array} \right] z(\upsilon) d\upsilon$$

Then the time derivative along the state trajectory of the system (16) is

$$\dot{V} = 2z^T P \left(\begin{array}{c} A_{11} + A_{12}C + \gamma I \\ + \Delta A_{11} + \Delta A_{12}C \end{array} \right) z \qquad (17)$$
$$+ 2e^{\gamma d} z^T P \left(\begin{array}{c} A_{d11} + A_{d12}C \\ + \Delta A_{d11} + \Delta A_{d12}C \end{array} \right) z(t-d)$$
$$+ \varepsilon z^T(t) \left[\begin{array}{c} (E_{21} + E_{22}C)^T (E_{21} + E_{22}C) \\ + X^{-1} + C^T C \end{array} \right] z(t)$$
$$- \varepsilon z^T(t-d) \left[\begin{array}{c} (E_{21} + E_{22}C)^T (E_{21} + E_{22}C) \\ + X^{-1} + C^T C \end{array} \right]$$
$$z(t-d)$$

Note

$$2z^T P (\Delta A_{11} + \Delta A_{12}C) z \qquad (18)$$
$$= 2z^T P D_1 F_1(t) (E_{11} + E_{12}C) z$$
$$\leq \varepsilon_1^{-1} z^T P D_1 D_1^T P z$$
$$+ \varepsilon_1 z^T (E_{11} + E_{12}C)^T (E_{11} + E_{12}C) z$$

$$2e^{\gamma d} z^T P (A_{d12}C + \Delta A_{d11} + \Delta A_{d12}C) z(t-d) \qquad (19)$$
$$\leq \varepsilon_2^{-1} e^{2\gamma d} z^T P A_{d12} A_{d12}^T P z$$
$$+ \varepsilon_2 z(t-d)^T C^T C z(t-d) + \varepsilon_3^{-1} e^{2\gamma d} z^T P D_2 D_2^T P z$$
$$+ \varepsilon_3 z(t-d)^T (E_{21} + E_{22}C)^T (E_{21} + E_{22}C) z(t-d)$$

$$2e^{\gamma d} z^T P A_{d11} z(t-d) \qquad (20)$$
$$\leq e^{2\gamma d} z^T P A_{d11} X A_{d11}^T P z + z(t-d)^T X^{-1} z(t-d)$$

where X is a positive matrix, $\varepsilon_1, \varepsilon_2$ and ε_3 are positive scalars. For simplicity we choose $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon$, then by substituting (18),(19) and (20) into (17), we get

$$\dot{V} \leq 2z^T P (A_{11} + A_{12}C + \gamma I) z + \varepsilon^{-1} z^T P D_1 D_1^T P z$$
$$+ \varepsilon z^T (E_{11} + E_{12}C)^T (E_{11} + E_{12}C) z$$
$$+ \varepsilon^{-1} e^{2\gamma d} z^T P A_{d12} A_{d12}^T P z + \varepsilon z^T C^T C z$$
$$+ e^{2\gamma d} z^T P A_{d11} X A_{d11}^T P z + z^T X^{-1} z$$
$$+ \varepsilon^{-1} e^{2\gamma d} z^T P D_2 D_2^T z$$
$$+ \varepsilon z^T (E_{21} + E_{22}C)^T (E_{21} + E_{22}C) z$$
$$= 2z^T P (A_{11} + \gamma I) z + \varepsilon^{-1} z^T P D_1 D_1^T P z$$

$$+ \varepsilon z^T E_{11}^T E_{11} z + \varepsilon^{-1} e^{2\gamma d} z^T P A_{d12} A_{d12}^T P z$$
$$+ e^{2\gamma d} z^T P A_{d11} X A_{d11}^T P z + z^T X^{-1} z$$
$$+ \varepsilon^{-1} e^{2\gamma d} z^T P D_2 D_2^T z + \varepsilon z^T E_{21}^T E_{21} z$$
$$+ \varepsilon z^T C^T \left(1 + E_{12}^T E_{12} + E_{22}^T E_{22} \right) C z$$
$$+ 2z^T \left(P A_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right) C z$$

So if we let

$$C = - \left[\varepsilon \left(1 + E_{12}^T E_{12} + E_{22}^T E_{22} \right) \right]^{-1}$$
$$\left(P A_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)^T$$

then

$$\dot{V} \leq 2z^T P (A_{11} + \gamma I) z \qquad (21)$$
$$+ \varepsilon^{-1} z^T P D_1 D_1^T P z + \varepsilon z^T E_{11}^T E_{11} z$$
$$+ \varepsilon^{-1} e^{2\gamma d} z^T P A_{d12} A_{d12}^T P z$$
$$+ e^{2\gamma d} z^T P A_{d11} X A_{d11}^T P z + z^T X^{-1} z$$
$$+ \varepsilon^{-1} e^{2\gamma d} z^T P D_2 D_2^T P z + \varepsilon z^T E_{21}^T E_{21} z$$
$$- z^T \left(P A_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)$$
$$\left[\varepsilon \left(1 + E_{12}^T E_{12} + E_{22}^T E_{22} \right) \right]^{-1}$$
$$\left(P A_{12} + \varepsilon E_{11}^T E_{12} + \varepsilon E_{21}^T E_{22} \right)^T z$$

If inequality (14) is satisfied, (21) holds. So we complete the proof of theorem 1.

In this section, we have designed the sliding surface dependent of the time delay. Compared to the papers [11] and [12] on variable structure control for time delay system, our results have two merits: 1) Under condition (14), the sliding surface is exponentially stable with decay rate γ. 2) The condition (14) is dependent of the time delay, so it is less conservative than the results independent of the time delay when the time delay is sufficiently small.

V. DESIGN OF THE REACHING LAW

In this section, we will design the controller which can drive the trajectory of system (1) onto the sliding surface in limited time.

For system (1), consider the following controller

$$u = u_1 + u_2 + u_3 \qquad (22)$$

where

$$u_1 = -\Theta_1^T \xi_1(x) - \Theta_2^T \xi_2(x(t-d)) \qquad (23)$$

$$u_2 = -\vartheta_0 sign\left(\tilde{C}x\right) - \vartheta_1 \|x\| sign\left(\tilde{C}x\right) \qquad (24)$$
$$- \vartheta_2 \|x(t-d)\| sign\left(\tilde{C}x\right)$$

$$u_3 = -\beta \tilde{C}x \qquad (25)$$

where β is a positive scalar, $\Theta_1, \Theta_2, \vartheta_0, \vartheta_1$ and ϑ_2 are adaptive parameters, whose adaptive laws are as follows

$$\dot{\Theta}_1 = \Gamma_1 \tilde{C}x \xi_1(x) \qquad (26)$$

$$\dot{\Theta}_2 = \Gamma_2 \tilde{C}x \xi_2(x(t-d)) \qquad (27)$$

$$\dot{\vartheta}_0 = k_0 \left| \tilde{C}x \right| \qquad (28)$$

$$\dot{\vartheta}_1 = k_1 \left| \widetilde{C}x \right| \|x\| \tag{29}$$

$$\dot{\vartheta}_2 = k_2 \left| \widetilde{C}x \right| \|x(t-d)\| \tag{30}$$

where Γ_1 and Γ_2 are positive matrices with proper dimensions, k_1, k_2 and k_3 are positive scalars.

In the above controller, u_1 contains two fuzzy logic systems used to approximate the unknown nonlinear functions, u_2 is an adaptive controller used to compensate for the time-varying uncertainties and the approximation error, and u_3 is used to further guarantee that the sliding function $S(\cdot)$ asymptotically converges to zero.

Theorem 2 Given system (1) with the sliding surface (15), the above reaching law (23) will drive the trajectory of the closed-loop system onto the sliding surface in limited time.

Proof: For system (1), define the following Lyapunov function

$$
\begin{aligned}
V(x) &= \frac{1}{2}[S^2 + \overline{\Theta}_1^T \Gamma_1^{-1} \overline{\Theta}_1 + \overline{\Theta}_2^T \Gamma_2^{-1} \overline{\Theta}_2 \\
&\quad + k_0^{-1}\overline{\vartheta}_0^2 + k_1^{-1}\overline{\vartheta}_1^2 + k_2^{-1}\overline{\vartheta}_2^2]
\end{aligned}
\tag{31}
$$

where $\overline{\Theta}_1 = \Theta_1^* - \Theta_1, \overline{\Theta}_2 = \Theta_2^* - \Theta_2, \overline{\vartheta}_0 = \vartheta_0^* - \vartheta_0, \overline{\vartheta}_1 = \vartheta_1^* - \vartheta_1$ and $\overline{\vartheta}_2 = \vartheta_2^* - \vartheta_2$.

The time derivative along the state trajectory of the system (1) is

$$
\begin{aligned}
\dot{V} &= S\dot{S} + \overline{\Theta}_1^T \Gamma_1^{-1} \dot{\overline{\Theta}}_1 + \overline{\Theta}_2^T \Gamma_2^{-1} \dot{\overline{\Theta}}_2 \\
&\quad + k_0^{-1}\overline{\vartheta}_0\dot{\overline{\vartheta}}_0 + k_1^{-1}\overline{\vartheta}_1\dot{\overline{\vartheta}}_1 + k_2^{-1}\overline{\vartheta}_2\dot{\overline{\vartheta}}_2
\end{aligned}
\tag{32}
$$

As we know

$$
\begin{aligned}
\dot{S}S &= \widetilde{C}x\dot{\widetilde{C}x} \\
&= \widetilde{C}x\widetilde{C}[(A+\Delta A(t))x + (A_d+\Delta A_d(t))x(t-d) \\
&\quad + B(u+F(x,x(t-d),\sigma(t)))] \\
&= \widetilde{C}x\Big[\widetilde{C}((A+\Delta A(t))x + (A_d+\Delta A_d(t))x(t-d)) \\
&\quad + u + F(x,x(t-d),\sigma(t))\Big]
\end{aligned}
$$

Substituting the controller (22) into the above equality, we can obtain

$$
\begin{aligned}
&\dot{S}S \\
&= \widetilde{C}x[\widetilde{C}((A+\Delta A(t))x + (A_d+\Delta A_d(t))x(t-d)) \\
&\quad + u_1 + u_2 + u_3 + F(x,x(t-d),\sigma(t))]
\end{aligned}
\tag{33}
$$

The unknown nonlinear continuous functions are defined as follows

$$
\begin{aligned}
\widetilde{F}_1(x) &= \widetilde{C}Ax + F_1(x) \\
\widetilde{F}_2(x(t-d)) &= \widetilde{C}A_d x(t-d) + F_2(x(t-d))
\end{aligned}
\tag{34}
$$

From the knowledge of section 3, we use two fuzzy logic systems $\widehat{F}_1(x,\Theta_1) = \Theta_1^T \xi_1(x)$ and $\widehat{F}_2(x(t-d),\Theta_2) = \Theta_2^T \xi_2(x(t-d))$, to approximate $\widetilde{F}_1(x)$ and $\widetilde{F}_2(x(t-d))$, substituting (23) into (33) we further get

$$
\begin{aligned}
\dot{S}S &= \widetilde{C}x[\widetilde{C}\Delta A(t)x + \widetilde{C}\Delta A_d(t)x(t-d) \\
&\quad + \widetilde{F}_1(x) - \Theta_1^{*T}\xi_1(x) \\
&\quad + \widetilde{F}_2(x(t-d)) - \Theta_2^{*T}\xi_2(x(t-d)) \\
&\quad + (\Theta_1^* - \Theta_1)^T\xi_1(x) \\
&\quad + (\Theta_2^* - \Theta_2)^T\xi_2(x(t-d)) \\
&\quad + u_2 + u_3 + \Delta F(x,x(t-d),\sigma(t))]
\end{aligned}
\tag{35}
$$

From assumptions 2 and 3, it is easy to obtain that there exist unknown positive scalars $\vartheta_0^*, \vartheta_1^*$ and ϑ_2^* satisfying the following inequality

$$
\begin{aligned}
&\widetilde{C}x\big(\Delta F(x,x(t-d),\sigma(t)) \\
&\quad + \widetilde{C}\Delta A_d(t)x(t-d) + \widetilde{F}_1(x) \\
&\quad - \Theta_1^{*T}\xi_1(x) + \widetilde{C}\Delta A(t)x \\
&\quad + \widetilde{F}_2(x(t-d)) - \Theta_2^{*T}\xi_2(x(t-d))\big) \\
&\leq \left|\widetilde{C}x\right|[\vartheta_0^* + \vartheta_1^*\|x\| + \vartheta_2^*\|x(t-d)\|]
\end{aligned}
\tag{36}
$$

Substitute (24), (25) and (36) into (35), we obtain

$$
\begin{aligned}
\dot{S}S &\leq \left|\widetilde{C}x\right|\begin{bmatrix} (\vartheta_0^* - \vartheta_0) + (\vartheta_1^* - \vartheta_1)\|x\| \\ + (\vartheta_2^* - \vartheta_2)\|x(t-d)\| \end{bmatrix} \\
&\quad + \begin{bmatrix} (\Theta_1^* - \Theta_1)^T\xi_1(x) \\ + (\Theta_2^* - \Theta_2)^T\xi_2(x(t-d)) \end{bmatrix}\widetilde{C}x \\
&\quad - \beta S^2
\end{aligned}
\tag{37}
$$

So we can get the following inequality by substituting (37) and (26-30) into (32)

$$
\begin{aligned}
\dot{V} &= S\dot{S} + \overline{\Theta}_1^T \Gamma_1^{-1}\dot{\overline{\Theta}}_1 + \overline{\Theta}_2^T \Gamma_2^{-1}\dot{\overline{\Theta}}_2 \\
&\quad + k_0^{-1}\overline{\vartheta}_1\dot{\overline{\vartheta}}_1 + k_1^{-1}\overline{\vartheta}_2\dot{\overline{\vartheta}}_2 + k_2^{-1}\overline{\vartheta}_3\dot{\overline{\vartheta}}_3 \\
&\leq -\beta S^2
\end{aligned}
\tag{38}
$$

From (38), it is easy to see that the reaching law (22) guarantees that the trajectory of system (1) is driven onto the sliding surface in limited time.

In the control process, we assume that Θ_1, Θ_2 never reach the boundary of $\Omega_{F_1}, \Omega_{F_2}$ defined as follows

$$
\begin{aligned}
\Omega_{F_1} &= \{\Theta_1 \,|\, \|\Theta_1\| \leq M_1\} \\
\Omega_{F_2} &= \{\Theta_2 \,|\, \|\Theta_2\| \leq M_2\}
\end{aligned}
\tag{39}
$$

In order to guarantee the boundedness of the adaptive parameters Θ_1, Θ_2, we introduce the projection operator [5] to restrict them in the closed sets $\Omega_{F_1}, \Omega_{F_2}$. The parameter adaptive law (26) and (27) can be changed into

$$\dot{\Theta}_1 = \begin{cases} \Gamma_1 \widetilde{C} x \xi_1(x) & \text{If } \|\Theta_1\| < M_1 \\ \quad \text{or } (\|\Theta_1\| = M_1 \text{ and } \widetilde{C} x \Theta_1^T \xi_1(x) \le 0) \\ P_r\left[\Gamma_1 \widetilde{C} x \xi_1(x)\right] & \text{If } \|\Theta_1\| = M_1 \\ \quad \text{and } \widetilde{C} x \Theta_1^T \xi_1(x) > 0) \end{cases} \qquad (40)$$

and

$$\dot{\Theta}_2 = \begin{cases} \Gamma_2 \widetilde{C} x \xi_2(x(t-d)) & \text{If } \|\Theta_2\| < M_2 \\ \quad \text{or } (\|\Theta_2\| = M_2 \text{ and } \widetilde{C} x \Theta_2^T \xi_2(x(t-d)) \le 0) \\ P_r\left[\Gamma_2 \widetilde{C} x \xi_2(x(t-d))\right] & \text{If } \|\Theta_2\| = M_2 \\ \quad \text{and } \widetilde{C} x \Theta_2^T \xi_2(x(t-d)) > 0) \end{cases}$$
$$(41)$$

where P_r is determined by the following equations

$$P_r\left[\Gamma_1 \widetilde{C} x \xi_1(x)\right] = \Gamma_1 \widetilde{C} x \xi_1(x) - \Gamma_1 \frac{\widetilde{C} x \xi_1^T(x) \Theta_1}{\|\Theta_1\|^2} \Theta_1$$

$$P_r\left[\Gamma_2 \widetilde{C} x \xi_2(x(t-d))\right]$$
$$= \Gamma_2 \widetilde{C} x \xi_2(x(t-d)) - \Gamma_2 \frac{\widetilde{C} x \xi_2^T(x(t-d)) \Theta_2}{\|\Theta_2\|^2} \Theta_2$$

We can find that the theorem can also be easily proved by utilizing the adaptive laws (40) and (41) instead of (26) and (27). Here the proof is omitted.

VI. CONCLUSION

This paper investigates the problem of variable structure adaptive fuzzy control for a class of nonlinear systems with time delays. The sliding surface and the corresponding reaching law are designed. Different from the existing literature, the sliding surface in this paper is dependent on the time delay, and the sliding motion is exponentially stable. During the design of reaching law, fuzzy logic systems are adopted to approximate unknown functions. The constructed reaching law can drive the state trajectory of the system onto the sliding surface in limited time.

Acknowledgements

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