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Some Control Problems for Uncertain Time-delay T-S Fuzzy Models Using LMI

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***Abstract*—In this paper, some robust fuzzy control problems are discussed for uncertain Takagi-Sugeno (T-S) fuzzy models with input as well as state delays. Both state feedback controllers and observer-based output feedback controllers are designed. Sufficient conditions for synthesis the robust asymptotically stability of the systems are established by LMIs forms; Furthermore, similar results are derived in the case of Lebesgue input disturbance in the T-S models. Sufficient conditions for the design of the state feedback H_∞ control are derived for robust H_∞ stabilization in the sense of Lyapunov asymptotic stability.**

I. INTRODUCTION

In recent years, there has been rapidly growing interests in fuzzy control of nonlinear systems, and there have also been some successful applications. However, many basic issues remain to be addressed. The most important issue for fuzzy control systems is how to achieve a systematic design with guaranteed stability and performance [1]-[4]. It is quite common to approximate a nonlinear plant by a T-S fuzzy linear model. Then control system design can be carried out based on the T-S fuzzy model via the so-called parallel distributed compensation (PDC) scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is nonlinear in general, is again a fuzzy blending of each individual linear controller. The PDC scheme is simple, nature and can be transformed into LMIs forms [5],[6].

Besides stability[3],[5],[7],[8], another important requirement for a control system is its robustness and this remains to be a central issue in the study of uncertain control systems and their controllers design. The parametric uncertainties are principal factors responsible for the degraded stability and performance of and uncertain nonlinear control system. Therefore, robust stability against parametric uncertainties in the T-S fuzzy-model-based control systems is an important problem [9],[10].

In traditional T-S fuzzy models, there is no delay in the control and state. However, time delays often occur in many dynamical systems such as biological systems, chemical systems, metallically processing systems and network systems. In the literature, the problem of stability and stabilization of time-delay systems has been dealt with two basic ways: the first one is dependent of the size of the time delays, in which the stability is guaranteed up to some maximum value for the time delays; the other is independent of the value of the time delays [6].

Since the pioneering work on the so-called H_∞ -optimal control theory [11], there has been a dramatic progress in H_∞ -control theory. Together with the robust stability, the problem of robust H_∞ disturbance attenuation for both linear and nonlinear systems has recently been studied. Cao[12] presented H_∞ controller design for fuzzy dynamic systems with state feedback. Chen et al[13] presented the design method of a observer-based fuzzy H_∞ controller via LMI approach. Gu et al[6] design robust H_∞ controller for fuzzy dynamic system with time-varying delayed state.

In this work, we address some robust control problems for T-S fuzzy models with both input and state delays. We design robust controllers for uncertain T-S model systems using different feedback configurations. H_∞ control synthesis schemes are developed for systems with disturbances. Furthermore, to achieve a systematic design with guaranteed robust asymptotic stability and H_∞ control performance, the analytical results are cast conveniently into a LMI framework that can be solved by the software package LMI-control toolbox.

The paper is organized as follows: Section 2 introduces the T-S fuzzy model and some preliminaries. In section 3 and 4, sufficient stabilization conditions using state-feedback control and observer-based output-feedback control are presented in LMI forms. In section 5, while a disturbance term is augmented to the system, a H_∞ control performance objective is presented for this disturbance and

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the H_∞ controllers are designed to guarantee the H_∞ performance. Sufficient conditions are also presented in LMI forms. The conclusions are given in section 6.

II. PRELIMINARIES AND SYSTEM DESCRIPTIONS

Firstly, consider the following uncertain time-delayed T-S model system that can be described by fuzzy IF-THEN rules:

Plant Rule i:

IF $x_1(t)$ is F_1^i and $x_2(t)$ is F_2^i , $\cdots$, and $x_n(t)$ is F_n^i , THEN

$$\begin{cases}\dot{x}(t)=(A_{i0}+\Delta A_{i0})x(t)+(B_{i0}+\Delta B_{i0})u(t)\\ \qquad +(A_{id}+\Delta A_{id})x(t-\tau_1)+(B_{id}+\Delta B_{id})u(t-\tau_2)\\ y(t)=C_i x(t) \qquad i=1,\cdots,q\end{cases} \tag{1}$$

where F_j^i is a fuzzy set, $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the control input vector, $y(t)\in R^l$ is the output vector, and $A_{i0},A_{id}\in R^{n\times n}$, $B_{i0},B_{id}\in R^{n\times m}$, and $C_i\in R^l$ are system matrix, input matrix and output matrix, respectively. $\Delta A_{i0},\Delta A_{id}$, $\Delta B_{i0},\Delta B_{id}$ are time-varying matrices with appropriate dimensions, which represent the parametric uncertainties in the plant model. q is the number of rules of the T-S fuzzy model, τ_1,τ_2 are the time –delay of the system state and control input, respectively.

The parameter time-varying uncertainties $\Delta A_{i0},\Delta A_{id}$, $\Delta B_{i0},\Delta B_{id}$ considered here are norm-bounded, in the form:

$$[\Delta A_{i0},\Delta B_{i0}]=D_{i0}F_{i0}(t)[E_{1i}^0,E_{2i}^0] \tag{2a}$$

$$[\Delta A_{id},\Delta B_{id}]=D_{id}F_{id}(t)[E_{1i}^d,E_{2i}^d] \tag{2b}$$

where $D_{i0},E_{1i}^0,E_{2i}^0,D_{id},E_{1i}^d,E_{2i}^d$ are all known real constant matrices of appropriate dimension. $F_{i0}(t),F_{id}(t)$ are unknown matrix function with Lebesgue-measurable elements, which are defined as followings:

$$F_{i0}(t),F_{id}(t)\in\Omega:=\left\{F_{i0}(t),F_{id}(t)\left|\begin{matrix}F_{i0}^{\mathrm{T}}(t)F_{i0}(t)\le \mathrm{I}\\ F_{id}^{\mathrm{T}}(t)F_{id}(t)\le \mathrm{I}\end{matrix}\right.\right\} \tag{3}$$

where the elements of $F_{i0}(t),F_{id}(t)$ are Lebesgue measurable, i.e., every uncertain term is norm-bounded.

Let $\mu_i(x(t))$ be the normalized membership function of the inferred fuzzy set $h_i(x(t))$, i.e.,

$$\mu_i(x(t))=h_i(x(t))\Big/\sum_{i=1}^q h_i(x(t)) \tag{4}$$

$$h_i(x(t))=\prod_{j=1}^n F_j^i(x_j(t)) \tag{5}$$

where $x(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^{\mathrm{T}}$, $F_j^i(x_j(t))$ is the grade of membership of $x_j(t)$ in F_j^i , which has following properties:

$$\begin{aligned}&h_i(x(t))\ge 0 \qquad i=1,\cdots,q\\ &\sum_{i=1}^q h_i(x(t))>0\end{aligned} \tag{6}$$

for all t , Then we can obtain the following conditions:

$$\mu_i(x(t))\ge 0,\ and\ \sum_{i=1}^q \mu_i(x(t))=1 \quad i=1,\cdots,q \tag{7}$$

By using a center average defuzzifer, product inference, and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model:

$$\begin{aligned}\dot{x}(t)=&\sum_{i=1}^q \mu_i(x(t))\{(A_{i0}+\Delta A_{i0})x(t)+(B_{i0}+\Delta B_{i0})u(t)\\ &+(A_{id}+\Delta A_{id})x(t-\tau_1)+(B_{id}+\Delta B_{id})u(t-\tau_2)\}\\ y(t)=&\sum_{i=1}^q \mu_i(x(t))C_i x(t)\end{aligned} \tag{8}$$

III. ROBUST STATE FEEDBACK CONTROL OF T-S FUZZY MODEL

Consider the T-S fuzzy model system with state delay and control delay in the form of (8), to achieve the robust stability, we design the following state feedback controllers:

Controller Rule i:

IF $x_1(t)$ is F_1^i, and $\cdots$, and $x_n(t)$ is F_1^i,

THEN $\qquad u(t)=-K_i x(t) \qquad (9)$

where $K_i\in R^{m\times n}$ is the constant feedback control gain to be determined.

The defuzzified output of (9) is represented as follows:

$$u(t)=-\sum_{i=1}^q \mu_i(x(t))K_i x(t) \tag{10}$$

The closed-loop system of (10) and (8) is

$$\begin{aligned}\dot{x}(t)=&\sum_{i=1}^q\sum_{j=1}^q \mu_i(x(t))\mu_j(x(t))\{[(A_{i0}+\Delta A_{i0})\\ &-(B_{i0}+\Delta B_{i0})K_j]x(t)+(A_{id}+\Delta A_{id})x(t-\tau_1)\\ &-(B_{id}+\Delta B_{id})K_j x(t-\tau_2)\}\end{aligned} \tag{11}$$

Moreover, we can get

$$\begin{aligned}\dot{x}(t)=&\sum_{i=1}^q \mu_i^2(x(t))\{[(A_{i0}+\Delta A_{i0})-(B_{i0}+\Delta B_{i0})K_i]x(t)\\ &+(A_{id}+\Delta A_{id})x(t-\tau_1)-(B_{id}+\Delta B_{id})K_i x(t-\tau_2)\}\\ &+2\sum_{i<j}^q \mu_i(x(t))\mu_j(x(t))\{[\frac{A_{i0}+\Delta A_{i0}+A_{j0}+\Delta A_{j0}}{2}\\ &\frac{-(B_{i0}+\Delta B_{i0})K_j-(B_{j0}+\Delta B_{j0})K_i}{2}]x(t)\\ &[\frac{A_{id}+\Delta A_{id}+A_{jd}+\Delta A_{jd}}{2}]x(t-\tau_1)\end{aligned}$$

$$\frac{-(B_{id}+\Delta B_{id})K_j-(B_{jd}+\Delta B_{jd})K_i}{2}]x(t-\tau_2) \quad (12)$$

For the above global closed-loop T-S uncertain fuzzy time-delayed system, the main result is summarized in the following theorem:

Theorem 1 If there exist a symmetric and positive definite matrix P, some matrices K_i, and some scalars $\varepsilon_1^i,\varepsilon_2^i,r_1^i,r_2^i,\alpha_1^i,\varepsilon_{ij}(i,j=1,\cdots,q)$, such that the following LMIs are satisfied, then T-S fuzzy time-delayed system (8) is asymptotically stabilizable via the T-S fuzzy model-based state-feedback controller (10), (Note: (13a) and (13b) are in the appendix)

$$\begin{aligned}&-r_1^{-1}PP+r_2^{-1}[K_i^T(E_{2i}^d)^T E_{2i}^d K_i\\&-K_j^T(E_{2i}^d)^T E_{2i}^d K_j-K_i^T(E_{2j}^d)^T E_{2j}^d K_i]<0\end{aligned} \quad (13c)$$

where

$$\begin{aligned}\Phi_{ii}&=QA_{i0}^T+A_{i0}Q-M_i^T B_{i0}^T-B_{i0}M_i+\alpha_1^i D_{i0}D_{i0}^T\\&+\varepsilon_1^i A_{id}A_{id}^T+\varepsilon_2^i D_{id}D_{id}^T+r_2^i D_{id}D_{id}^T+2(r_1^i)^{-1}I\end{aligned} \quad (13d)$$

$$\sigma_{ij}=-\varepsilon_2^i[(E_{1i}^d)^T E_{1i}^d+(E_{1j}^d)^T E_{1j}^d]^{-1} \quad (13e)$$

$$\begin{aligned}\Phi_{ij}&=Q(A_{i0}+A_{j0})^T+(A_{i0}+A_{j0})Q-M_j^T B_{i0}^T\\&-M_i^T B_{j0}^T-B_{i0}M_j-B_{j0}M_i+\{\varepsilon_1^i(A_{id}A_{id}^T\\&+A_{jd}A_{jd}^T)+(\varepsilon_2^i+r_2^i)(D_{id}D_{id}^T+D_{jd}D_{jd}^T)\\&+2(r_1^i)^{-1}I+\varepsilon_{ij}\begin{bmatrix}D_{i0} & D_{j0}\end{bmatrix}\begin{bmatrix}D_{i0} & D_{j0}\end{bmatrix}^T\}\end{aligned} \quad (13f)$$

and $Q=P^{-1},M_i=K_iP^{-1}=K_iQ$, where * denotes the transposed elements in the symmetric positions.

IV. Robust Observer-based Output Feedback Control of T-S Uncertain Time-delayed Fuzzy Model

Consider uncertain time-delayed fuzzy model system (8), and design the observer-based output feedback controller as follows:

$$\begin{cases}\dot{\hat{x}}(t)=\sum_{i=1}^{q}\mu_i(t)[A_{i0}\hat{x}(t)+B_{i0}u(t)]+\sum_{i=1}^{q}\mu_i(t)[A_{id}\hat{x}(t-\tau_1)\\ \quad +B_{id}u(t-\tau_2)]+\sum_{i=1}^{q}\mu_i(t)L_i[y_i(t)-C_i\hat{x}(t)]\\ \hat{y}(t)=\sum_{i=1}^{q}\mu_i(t)C_i\hat{x}(t) \qquad i=1,\cdots,q\end{cases} \quad (14)$$

Define the observation errors as:

$$e(t)=x(t)-\hat{x}(t) \quad (15)$$

Design a fuzzy output feedback controller based on the observer for robust stabilization of system (8) in the form

$$u(t)=-\sum_{i=1}^{q}\mu_i(x(t))K_i\hat{x}(t) \quad (16)$$

By substituting (16) into (8), and with (15), we get the closed-loop error system equations:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))[(A_{i0}+\Delta A_{i0})\\&-(B_{i0}+\Delta B_{i0})K_j]u(t)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))(B_{i0}+\Delta B_{i0})K_je(t)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))(A_{id}+\Delta A_{id})x(t-\tau_1)\\&-\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))(B_{id}+\Delta B_{id})K_jx(t-\tau_2)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))(B_{id}+\Delta B_{id})K_je(t-\tau_2)\end{aligned} \quad (17a)$$

$$\begin{aligned}\dot{\hat{x}}(t)&=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))[A_{i0}-B_{i0}K_j]\hat{x}(t)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))A_{id}\hat{x}(t-\tau_1)\\&-\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))B_{id}K_j\hat{x}(t-\tau_2)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))L_iC_ie(t)\end{aligned} \quad (17b)$$

$$\begin{aligned}\dot{e}(t)&=\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))[A_{i0}+B_{i0}K_j-L_iC_i]e(t)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))[\Delta A_{i0}+\Delta B_{i0}K_j]x(t)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))A_{id}e(t-\tau_1)\\&-\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))B_{id}K_je(t-\tau_2)\\&+\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))\Delta A_{id}x(t-\tau_1)\\&-\sum_{i=1}^{q}\sum_{j=1}^{q}\mu_i(x(t))\mu_j(x(t))\Delta B_{id}K_jx(t-\tau_2)\end{aligned} \quad (17c)$$

The main result for the global asymptotic stability of T-S fuzzy model, with parametric uncertainties and unavailable state variables, are summarized in the following theorem.

Theorem 2 If there exist symmetric and positive definite matrices P_1 and P_2, some matrices K_i and L_i, and scalars $\varepsilon_i^m,\alpha_i^m,\varepsilon_{jk}(i=1,\cdots,8;m=1,\cdots,q;j=1,\cdots,q;k=1,\cdots,q)$, such that the following LMIs are satisfied, then the T-S fuzzy system (8) is asymptotically stabilizable via the T-S fuzzy-model-based output-feedback controller (14) and (16). (Note: (18a) and (18b) are in the appendix)

$$\begin{bmatrix}T_{ii} & P_2\\ P_2 & -\Theta_{ii}^{-1}\end{bmatrix}<0 \quad (18c)$$

$$\begin{bmatrix} \Xi_{ij} & * & * & * & * & * \\ E_{2i}^{0}K_j & -[(\varepsilon_2^m)^{-1}+\varepsilon_{ij}^{-1}]^{-1} & & & & \\ E_{2j}^{0}K_i & & -[(\varepsilon_2^m)^{-1}+\varepsilon_{ij}^{-1}]^{-1} & & & \\ E_{2i}^{d}K_j & & & -[(\alpha_3^m)^{-1}+(\varepsilon_8^m)^{-1}]^{-1} & & \\ E_{2j}^{d}K_i & & & & -[(\alpha_3^m)^{-1}+(\varepsilon_8^m)^{-1}]^{-1} & \\ P_2 & & & & & -\Theta_{ij}^{-1} \end{bmatrix} < 0 \tag{18d}$$

$$\{-(\varepsilon_5^m)^{-1}I+[(\varepsilon_6^m)^{-1}+(\alpha_4^m)^{-1}][M_i^{\mathrm{T}}(E_{2i}^d)^{\mathrm{T}}E_{2i}^dM_i - M_j^{\mathrm{T}}(E_{2i}^d)^{\mathrm{T}}E_{2i}^dM_j - M_i^{\mathrm{T}}(E_{2j}^d)^{\mathrm{T}}E_{2j}^dM_i]\}<0 \tag{18e}$$

where

$$Q = P_1^{-1} \tag{18f}$$

$$N_i = P_2 L_i \tag{18g}$$

$$M_i = K_i Q \tag{18h}$$

$$\Phi_{ii}^1 = QA_{i0}^{\mathrm{T}} + A_{i0}Q - M_i^{\mathrm{T}}B_{i0}^{\mathrm{T}} - B_{i0}M_i + 2(\varepsilon_5^m)^{-1}\,\mathrm{I} + (\varepsilon_2^m + \varepsilon_{ii})D_{i0}D_{i0}^{\mathrm{T}} + \varepsilon_3^m A_{id}A_{id}^{\mathrm{T}} + (\varepsilon_4^m + \varepsilon_6^m + \varepsilon_8^m)P_{id}P_{id}^{\mathrm{T}} \tag{18i}$$

$$\Omega_{ii} = 2(\varepsilon_3^m)^{-1}\,\mathrm{I} + [(\alpha_4^m)^{-1} + (\alpha_5^m)^{-1}][(E_{1i}^d)^{\mathrm{T}}E_{1i}^d + (E_{1j}^d)^{\mathrm{T}}E_{1j}^d]^{-1} \tag{18j}$$

$$T_{ii} = A_{i0}^{\mathrm{T}}P_2 + P_2A_{i0} - C_i^{\mathrm{T}}N_i^{\mathrm{T}} - N_iC_i + 2(\alpha_2^m)^{-1}I + [(\varepsilon_2^m)^{-1} + (\varepsilon_{ii})^{-1}][K_i^{\mathrm{T}}(E_{2i}^0)^{\mathrm{T}}(E_{2i}^0)K_i] + [(\varepsilon_1^m)^{-1} + 2(\varepsilon_7^m)^{-1}]P_1P_1 + [(\alpha_3^m)^{-1} + (\alpha_8^m)^{-1}][K_j^{\mathrm{T}}(E_{2i}^d)^{\mathrm{T}}(E_{2i}^d)K_j + K_i^{\mathrm{T}}(E_{2j}^d)^{\mathrm{T}}(E_{2j}^d)K_i] \tag{18k}$$

$$\Theta_{ii} = (\alpha_1^m + \varepsilon_{ii})D_{i0}D_{i0}^{\mathrm{T}} + \alpha_2^m A_{id}A_{id}^{\mathrm{T}} + (\varepsilon_3^m + \varepsilon_4^m + \varepsilon_5^m)D_{id}D_{id}^{\mathrm{T}} \tag{18l}$$

$$\Psi_{ij} = Q(A_{i0} + A_{j0})^{\mathrm{T}} + (A_{i0} + A_{j0})Q - M_j^{\mathrm{T}}B_{i0}^{\mathrm{T}} - M_i^{\mathrm{T}}B_{j0}^{\mathrm{T}} - B_{i0}M_j - B_{j0}M_i + \varepsilon_2^m(D_{i0}D_{i0}^{\mathrm{T}} + D_{j0}D_{j0}^{\mathrm{T}}) + \varepsilon_3^m(A_{id}A_{id}^{\mathrm{T}} + A_{jd}A_{jd}^{\mathrm{T}}) + 2(\varepsilon_5^m)^{-1}I + (\varepsilon_4^m + \varepsilon_6^m + \varepsilon_8^m)(D_{id}D_{id}^{\mathrm{T}} + D_{jd}D_{jd}^{\mathrm{T}}) \tag{18m}$$

$$\Omega_{ij} = 2(\varepsilon_3^m)^{-1}I + [(\varepsilon_4^m)^{-1} + (\varepsilon_5^m)^{-1}][(E_{1i}^d)^{\mathrm{T}}E_{1i}^d + (E_{1j}^d)^{\mathrm{T}}E_{1j}^d] \tag{18n}$$

$$\Xi_{ij} = A_{i0}^{\mathrm{T}}P_2 + P_2A_{i0} + A_{j0}^{\mathrm{T}}P_2 + P_2A_{j0} - C_i^{\mathrm{T}}N_i^{\mathrm{T}} - N_iC_i - C_j^{\mathrm{T}}N_j^{\mathrm{T}} - N_jC_j + 2(\alpha_2^m)^{-1}\,\mathrm{I} + [2(\varepsilon_1^m)^{-1} + 2(\varepsilon_7^m)^{-1}]P_1P_1 \tag{18o}$$

$$\Theta_{ij} = (\alpha_1^m + \varepsilon_{ii})(D_{i0}D_{i0}^{\mathrm{T}} + D_{j0}D_{j0}^{\mathrm{T}}) + \alpha_2^m(A_{id}A_{id}^{\mathrm{T}} + A_{jd}A_{jd}^{\mathrm{T}}) + (\varepsilon_3^m + \varepsilon_4^m + \varepsilon_5^m)(D_{id}D_{id}^{\mathrm{T}} + D_{jd}D_{jd}^{\mathrm{T}}) \tag{18p}$$

where * denotes the transposed elements in the symmetric positions.

V. ROBUST H_∞ STATE FEEDBACK CONTROL OF T-S FUZZY MODEL

Consider the following T-S fuzzy model with uncertainties and disturbances, which has been defuzzified to be a global model:

$$\dot{x}(t) = \sum_{i=1}^{q}\mu_i(x(t))\{(A_{i0} + \Delta A_{i0})x(t) + (B_{i0} + \Delta B_{i0})u(t) + (A_{id} + \Delta A_{id})x(t-\tau_1) + (B_{id} + \Delta B_{id})u(t-\tau_2) + D_iw(t)\} \tag{19a}$$

$$z(t) = \sum_{i=1}^{q}\mu_i(x(t))E_ix(t) \tag{19b}$$

$$y(t) = \sum_{i=1}^{q}\mu_i(x(t))C_ix(t) \tag{19c}$$

where $w(t) \in R^q$ is the input disturbance which belongs to $L_2[0,\infty)$, $z(t) \in R^l$ is the controlled output vector, D_i, E_i are known matrices with appropriate dimensions, the other terms in the system is the same as the ones in system (1) and (8).

The objective is to design a state feedback controller in the form of (10) such that for all of $T > 0$ and $w(t) \in L_2[0,T), w(t) \neq 0$, the equilibrium of the disturbance-free fuzzy system (19) is asymptotically stable, and the response $z(t)$ of the system under the zero initial condition $x(0) = 0$ satisfies

$$\|z(t)\|_2 < \gamma\|w(t)\|_2 \tag{20}$$

where $\gamma > 0$ is the prescribed level of disturbance attenuation, the above system is also said to be robust stable with $\gamma-$ disturbance attenuation for all admissible uncertainties[12].

Theorem 3 If there exist a symmetric and positive definite matrix P, some matrices K_i ,and scalars $\varepsilon_1^i, \varepsilon_2^i, r_1^i, r_2^i, \alpha_1^i, \varepsilon_{ij}$, $(i, j = 1, \cdots, q)$ such that the following LMIs are satisfied, then the T-S fuzzy system (19) with uncertainties and time-delays is robust stable with $\gamma-$ disturbance attenuation via the state-feedback controller (10), i.e., for all $T > 0$, and $w(t) \in L_2[0,T), w(t) \neq 0$, (20) holds.

$$\left[\begin{array}{c|c} (13a) & \begin{matrix} QE_i^{\mathrm{T}} \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline \begin{matrix} E_iQ & 0 & \cdots & 0 \end{matrix} & -\mathrm{I} \end{array}\right] \tag{21a}$$

$$\left[\begin{array}{c|c} (13b) & \begin{matrix} QE_i^{\mathrm{T}} \\ 0 \\ \vdots \\ 0 \end{matrix} \\ \hline \begin{matrix} E_iQ & 0 & \cdots & 0 \end{matrix} & -\mathrm{I} \end{array}\right] \tag{21b}$$

where (13a) and (13b) are the left part of the LMI (13a) and (13b), respectively, except that Φ_{ii}, Φ_{ij} have minor changes as follows:

$$\begin{aligned}\Phi_{ii} &= QA_{i0}^{\mathrm{T}} + A_{i0}Q - M_i^{\mathrm{T}}B_{i0}^{\mathrm{T}} - B_{i0}M_i \\ &\quad + \alpha_1^i D_{i0}D_{i0}^{\mathrm{T}} + \varepsilon_1^i A_{id}A_{id}^{\mathrm{T}} + \varepsilon_2^i D_{id}D_{id}^{\mathrm{T}} \\ &\quad + r_2^i D_{id}D_{id}^{\mathrm{T}} + 2(r_1^i)^{-1}I + \gamma^{-2}D_iD_i^{\mathrm{T}}\end{aligned} \tag{21c}$$

$$\begin{aligned}\Phi_{ij} &= Q(A_{i0}+A_{j0})^{\mathrm{T}} + (A_{i0}+A_{j0})Q \\ &\quad -M_j^{\mathrm{T}}B_{i0}^{\mathrm{T}} - M_i^{\mathrm{T}}B_{j0}^{\mathrm{T}} - B_{i0}M_j - B_{j0}M_i \\ &\quad +\{\varepsilon_1^i(A_{id}A_{id}^{\mathrm{T}} + A_{jd}A_{jd}^{\mathrm{T}}) + (\varepsilon_2^i + r_2^i) \\ &\quad (D_{id}D_{id}^{\mathrm{T}} + D_{jd}D_{jd}^{\mathrm{T}}) + D_iD_i^{\mathrm{T}} \\ &\quad +2(r_1^i)^{-1}I + \varepsilon_{ij}\begin{bmatrix} D_{i0} & D_{j0}\end{bmatrix}\begin{bmatrix} D_{i0} & D_{j0}\end{bmatrix}^{\mathrm{T}}\}\end{aligned} \tag{21d}$$

For system (19), if the states are not available but the output can be measurable, a observer-based output feedback control methodology will be adopted to achieve the robust H_∞ performance, which is similar to the state feedback control methodology. Here it is omitted.

For the limited space, simulations are omitted here.

VI. CONCLUSION

In this paper, based on Lyapunov stability theory and the LMI method, we have addressed some robust control problems in T-S fuzzy systems with both input and state delays, uncertainties and norm disturbances. Firstly, sufficient conditions for the existences of the state feedback controllers and observer-based output feedback controllers are established for systems without disturbances; Secondly, feedback H_∞ robust controllers are designed for systems with disturbances, and sufficient conditions for systems to achieve certain H_∞ control performances are formulated in the LMIs forms.

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APPENDIX

$$\begin{bmatrix} \Phi_{ii} & (E_{1i}^0Q - E_{2i}^0M_i)^{\mathrm{T}} & B_{id}M_i & M_j^{\mathrm{T}}(E_{2i}^d)^{\mathrm{T}} & M_i^{\mathrm{T}}(E_{2j}^d)^{\mathrm{T}} & Q & Q \\ E_{1i}^0Q - E_{2i}^0M_i & -\alpha_1^i\mathrm{I} & 0 & 0 & 0 & 0 & 0 \\ M_i^{\mathrm{T}}B_{id}^{\mathrm{T}} & 0 & -(r_1^i)^{-1}\mathrm{I} & 0 & 0 & 0 & 0 \\ E_{2i}^dM_j & 0 & 0 & -(r_2^i)\mathrm{I} & 0 & 0 & 0 \\ E_{2j}^dM_i & 0 & 0 & 0 & -(r_2^i)\mathrm{I} & 0 & 0 \\ Q & 0 & 0 & 0 & 0 & -\frac{1}{2}\varepsilon_1^i\mathrm{I} & 0 \\ Q & 0 & 0 & 0 & 0 & 0 & \sigma_{ij} \end{bmatrix} < 0 \tag{13a}$$

$$\begin{bmatrix}
\Phi_{ij} & * & * & * & * & * & * & * & * \\
E_{1i}^{0}Q - E_{2i}^{0}M_i & -\varepsilon_{ij}\,\mathrm{I} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{1i}^{0}Q - E_{2j}^{0}M_i & 0 & -\varepsilon_{ij}\,\mathrm{I} & 0 & 0 & 0 & 0 & 0 & 0 \\
E_{2i}^{d}M_j & 0 & 0 & -(r_2^i)\,\mathrm{I} & 0 & 0 & 0 & 0 & 0 \\
E_{2j}^{d}M_i & 0 & 0 & 0 & -(r_2^i)\,\mathrm{I} & 0 & 0 & 0 & 0 \\
M_j^{\mathrm{T}}B_{id}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -(r_1^i)^{-1}\,\mathrm{I} & 0 & 0 & 0 \\
M_i^{\mathrm{T}}B_{jd}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & -(r_1^i)^{-1}\,\mathrm{I} & 0 & 0 \\
Q & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}\varepsilon_1^i\,\mathrm{I} & 0 \\
Q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{ij}
\end{bmatrix} < 0 \tag{13b}$$

$$\begin{bmatrix}
\Phi_{ii}^{1} & * & * & * & * & * & * \\
(E_{1i}^{0}Q - E_{2i}^{0}M_i) & -[\varepsilon_{ii}^{-1} + (\alpha_1^m)^{-1}]^{-1}\,\mathrm{I} & 0 & 0 & 0 & 0 & 0 \\
Q & 0 & -(\Omega_{ii}^{-1})\,\mathrm{I} & 0 & 0 & 0 & 0 \\
E_{2i}^{d}M_j & 0 & 0 & -[(\varepsilon_6^m)^{-1} + (\alpha_4^m)^{-1}]^{-1}\,\mathrm{I} & 0 & 0 & 0 \\
E_{2j}^{d}M_i & 0 & 0 & 0 & -[(\varepsilon_6^m)^{-1} + (\alpha_4^m)^{-1}]^{-1}\,\mathrm{I} & 0 & 0 \\
M_i^{\mathrm{T}}B_{i0}^{\mathrm{T}} & 0 & 0 & 0 & 0 & -(\varepsilon_1^m)^{-1}\,\mathrm{I} & 0 \\
M_i^{\mathrm{T}}B_{id}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & -(\varepsilon_5^m + \varepsilon_7^m)^{-1}\,\mathrm{I}
\end{bmatrix} < 0 \tag{18a}$$

$$\begin{bmatrix}
\Psi_{ij} & * & * & * & * & * & * & * & * & * & * & * \\
(E_{1i}^{0}Q - E_{2i}^{0}M_j) & -[\varepsilon_{ij}^{-1} + (\alpha_1^m)^{-1}]^{-1}\,\mathrm{I} & & & & & & & & & & \\
(E_{1j}^{0}Q - E_{2j}^{0}M_i) & & -[\varepsilon_{ij}^{-1} + (\alpha_1^m)^{-1}]^{-1}\,\mathrm{I} & & & & & & & & & \\
E_{2i}^{d}M_j & & & -[(\varepsilon_6^m)^{-1} + (\alpha_4^m)^{-1}]^{-1}\,\mathrm{I} & & & & & & & & \\
E_{2j}^{d}M_i & & & & -[(\varepsilon_6^m)^{-1} + (\alpha_4^m)^{-1}]^{-1}\,\mathrm{I} & & & & & & & \\
D_{i0} & & & & & -\varepsilon_{ij}^{-1}\,\mathrm{I} & & & & & & \\
D_{j0} & & & & & & -\varepsilon_{ij}^{-1}\,\mathrm{I} & & & & & \\
Q & & & & & & & -\Omega_{ij}^{-1}\,\mathrm{I} & & & & \\
M_j^{\mathrm{T}}B_{i0}^{\mathrm{T}} & & & & & & & & -(\varepsilon_1^m)^{-1}\,\mathrm{I} & & & \\
M_i^{\mathrm{T}}B_{j0}^{\mathrm{T}} & & & & & & & & & -(\varepsilon_1^m)^{-1}\,\mathrm{I} & & \\
M_j^{\mathrm{T}}B_{id}^{\mathrm{T}} & & & & & & & & & & -(\varepsilon_5^m + \varepsilon_7^m)^{-1}\,\mathrm{I} & \\
M_i^{\mathrm{T}}B_{jd}^{\mathrm{T}} & & & & & & & & & & & -(\varepsilon_5^m + \varepsilon_7^m)^{-1}\,\mathrm{I}
\end{bmatrix} < 0 \tag{18b}$$