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Robust adaptive fuzzy controller for nonlinear systems based on approximation errors

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***Abstract*— A stable adaptive fuzzy control method is proposed for single input and single output nonlinear systems. This method needs not the assumption that the state variables need full observability. And the state variables are estimated by designing an observer. These unknown nonlinearities are approximated by the fuzzy systems. The key assumption is that the approximation errors satisfy certain bounding conditions. The Lyapunov synthesis approach is used to analyse the fuzzy system to obtain the corresponding parameters adaptive laws. The overall control system guarantees that the tracking error converges into a small neighborhood of zero and that all signals involved are uniformly bounded. A simulation results show the validity and efficiency of the proposed method.**

I. INTRODUCTION

Recently, fuzzy systems are successfully applied to many control problems because they need not accurate mathematical models of the system under control and can cooperate with human experts' knowledge. Furthermore, it is shown that fuzzy systems can approximate certain classes of functions to a given accuracy. Wang proved that the fuzzy systems are universal approximations and the output of the system can be represented by a linear combination of the so-called fuzzy basis functions [1, 2]. Based on this property, many researchers presented adaptive fuzzy control architectures for nonlinear systems [3-6]. However, since the fuzzy descriptions are imprecise and may be insufficient to achieve the desired accuracy, the approximation error introduced into the feedback loop makes it difficult to guarantee the stability of the closed-loop control system [3]. This problem was solved in [7] by means of sliding mode-like estimation of the approximations error bound, but nonsmooth control input is generated due to this estimation. In general, such discontinuous adaptive control schemes are avoided since it is well-known that they not only create problems of existence and uniqueness of solutions but also are known to display chattering phenomena and to excite high-frequency unmodeled dynamics [6]. Park tried to solve this problem by estimating these bounds on the plant dynamics using fuzzy inference in [8]. Chen et al. [9] proposed an adaptive fuzzy-based controller combined with a control technique. However, their controllers are intrinsically high-gain controllers since the small attenuation level results in a large gain on the additional robustifying control term. Park developed an indirect robust adaptive control algorithm against the approximation errors using fuzzy systems for single-input single-output (SISO) nonlinear dynamical systems with unknown nonlinearities [10]. In addition, he proposed a smooth control input with no chattering phenomena. However, adaptive fuzzy control algorithms given above are based on the assumption that the state vector of the system can be available for measurement. So this algorithm can not be applied into nonlinear systems whose state vectors can not be available.

The purpose of this paper is to propose an adaptive control algorithm based on the observer for single-input single-output (SISO) nonlinear dynamical systems whose state vector can not be available. The continuous control algorithm is applied to analyse approximation errors in the controller. The control algorithm not only avoids chattering phenomena but also improves performance of systems. The proposed controller guarantees that the tracking error converges into a small neighborhood of zero. Simulation example is given to testify the validity and efficiency of the proposed method.

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II. DESCRIPTION OF SYSTEMS AND CONTROL PROBLEMS

Consider the nth-order nonlinear system of the form

$$\begin{aligned} x^{(n)} &= f(x,\dot{x},\cdots x^{(n-1)}) + g(x,\dot{x},\cdots x^{(n-1)})u \\ y &= x \end{aligned} \tag{1}$$

where f and g are unknown continuous functions, $u \in R$, $y \in R$ are the input and output of the system, respectively, and $\underline{x} = [x_1, x_2, \cdots x_n]^T = [x, \dot{x}, \cdots, x^{(n-1)}]^T \in R^n$is the state vector of the system. The unknown nonlinear system (1) can be expressed as follows:

$$\begin{aligned} \dot{\underline{x}} &= A\underline{x} + B(f(\underline{x}) + g(\underline{x})u) \\ y &= C^T \underline{x} \end{aligned} \tag{2}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

We assume that y_m is a given bounded reference signal, $\hat{\underline{x}}$ represents estimate of $\underline{x}$, $e_1 = y_m - y$ represents output tracking error. And

$$\underline{y}_m = \begin{bmatrix} y_m & \dot{y}_m & \dots & y_m^{(n-1)} \end{bmatrix}^T$$

$$\underline{e} = \underline{y}_m - \underline{x} = \begin{bmatrix} e & \dot{e} & \dots & e^{(n-1)} \end{bmatrix}^T$$

$$\hat{\underline{e}} = \underline{y}_m - \hat{\underline{x}} = \begin{bmatrix} \hat{e} & \dot{\hat{e}} & \dots & \hat{e}^{(n-1)} \end{bmatrix}^T$$

The purpose of this work is to design a feedback controller $u = u(\hat{x}, e_1|\underline{\theta})$ and adaptation laws of parameter vectors θ with appropriate dimensions in order that the following conditions are satisfied:

1. Variable of close-loop system are all bounded.
2. The tracking error $\underline{e}$ and $\underline{\hat{e}}$converge into a small neighborhood of zero.

III. Adaptive fuzzy controller design based on observer

If state vector of the system (1) can be available, according to method in [9], the control law is chosen as

$$u = u_c + u_r = \frac{1}{\hat{g}(\underline{x}|\theta_g)}(-\hat{f}(\underline{x}|\theta_f) + y_m^{(n)} + K^T e) + u_r \quad (3)$$

Suppose that $K = (k_n, \ldots, k_1)^T$ be such that all roots of $h(s) = s^n + k_1 s^{n-1} + \ldots + k_n$ are in the open left-half complex plane. The unknown functions $f(\cdot)$, $g(\cdot)$ are approximated by the fuzzy system $\hat{f}(\underline{x}|\theta_f)$ and $\hat{g}(\underline{x}|\theta_g)$ described as follows, respectively.

$$\begin{aligned} f(\underline{x}) &= \hat{f}(\underline{x}|\theta_f^*) + \delta_f(\underline{x}) \\ g(\underline{x}) &= \hat{g}(\underline{x}|\theta_g^*) + \delta_g(\underline{x}) \end{aligned} \quad (4)$$

$$\begin{aligned} \hat{f}(\underline{x}|\theta_f) &= \theta_f^T \xi_f(\underline{x}) \\ \hat{g}(\underline{x}|\theta_g) &= \theta_g^T \xi_g(\underline{x}) \end{aligned} \quad (5)$$

where θ_f and $\theta_g$are adjustable parameter vectors with appropriate dimensions, ξ_f and ξ_g are regressive vectors whose elements are fuzzy basis functions (FBF); $\delta_f(\underline{x})$ and $\delta_g(\underline{x})$are the approximation errors, θ_f^* and θ_g^* are some unknown optimal parameter vectors that are analytical quantities required only for analytical purposes. Typically, θ_f^* and θ_g^* are chosen as the values of θ_f and θ_g, respectively, that minimize the approximation errors, i.e.

$$\begin{aligned} \theta_f^* &= arg\min_{\theta_f}[\sup_{\underline{x}\in U_c} |\hat{f}(\underline{x}|\theta_f) - f(\underline{x})|] \\ \theta_g^* &= arg\min_{\theta_g}[\sup_{\underline{x}\in U_c} |\hat{g}(\underline{x}|\theta_g) - g(\underline{x})|] \end{aligned} \quad (6)$$

If the state vector of the system (1) is not available, the control law (3) can not be applied to control system (1). In this instance, choose the indirect adaptive control law as

$$u = \frac{1}{\hat{g}(\underline{\hat{x}}|\theta_g)}(-\hat{f}(\underline{\hat{x}}|\theta_f) + y_m^n + K^T e - u_s) + u_r \quad (7)$$

where $u_s$is feedback controller of approximation errors, u_r is an additional robustifying control term. Substituting(7) into (2), we obtain the closed-loop dynamics of the fuzzy control system as

$$\begin{aligned} \underline{\dot{e}} &= A\underline{e} - BK^T\underline{\hat{e}} + Bu_s + B[\hat{f}(\underline{x}|\theta_f) - f(\underline{x})) \\ &\quad + (\hat{g}(\underline{x}|\theta_g) - g(\underline{x}))u - \hat{g}(\underline{x}|\theta_g)u_r] \\ e_1 &= C^T\underline{e} \end{aligned} \quad (8)$$

Design the observer of(8) as

$$\begin{aligned} \underline{\dot{\hat{e}}} &= (A - BK^T)\underline{\hat{e}} + BK_0(e_1 - \hat{e}_1) \\ \hat{e}_1 &= C^T\underline{\hat{e}} \end{aligned} \quad (9)$$

Defining observation errors $\underline{\tilde{e}} = \underline{e} - \underline{\hat{e}}$, and using (8) and (9), we obtain

$$\begin{aligned} \underline{\dot{\tilde{e}}} &= (A - K_0C^T)\underline{\tilde{e}} + Bu_s + B[\hat{f}(\underline{x}|\theta_f) - f(\underline{x}) \\ &\quad + (\hat{g}(\underline{x}|\theta_g) - g(\underline{x}))u - \hat{g}(\underline{x}|\theta_g)u_r] \\ \underline{\tilde{e_1}} &= C^T\underline{e} \end{aligned} \quad (10)$$

Defining $\underline{\hat{x}} = \underline{y}_m - \underline{\hat{e}}$ as the input of fuzzy systems,(4)and(5) can be represented as fellows

$$\begin{aligned} f(\underline{x}) &= \hat{f}(\underline{\hat{x}}|\theta_f^*) + \delta_f(\underline{\hat{x}}) \\ g(\underline{x}) &= \hat{g}(\underline{\hat{x}}|\theta_g^*) + \delta_g(\underline{\hat{x}}) \end{aligned} \quad (11)$$

$$\begin{aligned} \hat{f}(\underline{\hat{x}}|\theta_f) &= \theta_f^T \xi_f(\underline{\hat{x}}) \\ \hat{g}(\underline{\hat{x}}|\theta_g) &= \theta_g^T \xi_g(\underline{\hat{x}}) \end{aligned} \quad (12)$$

The approximation errors $\delta_f(\underline{\hat{x}})$ and $\delta_g(\underline{\hat{x}})$ are critical quantities, representing the minimum possible deviation between the unknown functions f and g and the input/output functions of the fuzzy system $\hat{f}(\underline{x}|\theta_f)$ and $\hat{g}(\underline{x}|\theta_g)$. We make the following assumption on the approximation errors.

Assumption 1 On the compact region U_c, the following inequality are satisfied.

$$|\delta_f(\underline{\hat{x}})| \leq \varphi_f^* s_f(\underline{\hat{x}}), \forall \underline{\hat{x}} \in U_c \quad (13)$$

$$|\delta_g(\underline{\hat{x}})| \leq \varphi_g^* s_g(\underline{\hat{x}}), \forall \underline{\hat{x}} \in U_c \quad (14)$$

where $\psi_f^* \geq 0$ and $\psi_g^* \geq 0$ are unknown bounding parameters, s_f and s_g are known smooth bounding functions.

Assumption 2 If there exist symmetric and positive definite matrices P_1 and P_2, and some matrices K_0, Y_2 such that the following matrix inequalities are satisfied.

$$\begin{aligned} &P_2(A - 2BC^T) + (A - 2BC^T)^T P_2 - Y_2C^T \\ &-CY_2^T + 2CC^T + P_2BB^TP_2 < 0 \\ &(A - BK^T)^T P_1 + P_1(A - BK^T) - 2P_1K_0K_0^TP_1 < 0 \end{aligned}$$

Introducing the estimate error of parameter$\tilde{\theta}_f = \theta_f^* - \theta_f$ and $\tilde{\theta}_g = \theta_g^* - \theta_g$, from (11) and (12), Eq.(10) can be rewritten as

$$\begin{aligned} \underline{\dot{\tilde{e}}} &= (A - K_0C^T)\underline{\tilde{e}} + Bu_s + B[\tilde{\theta}_f^T \xi_f(\underline{\hat{x}}) \\ &\quad + \tilde{\theta}_g^T \xi_g(\underline{\hat{x}})u - \theta_f(\underline{\hat{x}}) - \theta_g(\underline{\hat{x}})u - \theta_g^T \xi_g(\underline{\hat{x}})u_r] \\ \underline{\tilde{e_1}} &= C^T\underline{\tilde{e}} \end{aligned} \quad (15)$$

For convenience of study, a lemma is given as follows.

Lemma 1 There exists $\kappa > 0$, such that the following inequalities holds for any $\varepsilon > 0$ and any $\eta > 0$.

$$0 \le |\eta| - \eta \tanh(\frac{\eta}{\varepsilon}) \le \kappa\varepsilon$$

In the following, the robustifying controller, the output feedback controller and the adaptive laws are chosen.

$$u_r = \frac{\beta}{\hat{g}(\hat{\underline{x}}|\theta_g}, u_s = -K_0^T P_1 \hat{\underline{e}}, \beta = -\frac{\varphi^T\omega}{1-\frac{\varphi_g s_g}{|\hat{g}(\hat{\underline{x}}|\theta_g}} \tag{16}$$

$$\begin{aligned}\dot{\theta}_f &= -\gamma_f[\tilde{\underline{e}}P_2B\xi_f + \sigma(\theta_f - \theta_f^0),\\ \dot{\theta}_g &= -\gamma_g[\tilde{\underline{e}}P_2B\xi_g + \sigma(\theta_g - \theta_g^0),\end{aligned} \tag{17}$$

$$\omega = \begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix} = \begin{bmatrix} s_f\tanh(\frac{zs_f}{\varepsilon}\\ s_g|u_c|\tanh(\frac{zs_g|u_c|}{\varepsilon}\end{bmatrix}$$

$$\dot{\varphi} = \gamma_\varphi \begin{bmatrix} z\omega_1 - \sigma(\varphi_f - \varphi_f^0)\\ z\omega_2 - \frac{z\beta s_g}{\|\hat{g}(\hat{x}|\theta_g)} - \sigma(\varphi_f - \varphi_f^0)\end{bmatrix} \tag{18}$$

θ_g^0, $\sigma > 0$ and $\varepsilon > 0$ are design constants.

Theorem 1 For nonlinear system (1), suppose the bounding assumption 1 and 2 hold globally. Then the robust adaptive laws (7) with the parameter adaptive (16), (17) and (18) guarantees that all the

a). signals and parameter estimates in on-line approximation based on control scheme are uniformly bounded.

b). given $\mu > \sqrt{\frac{2\rho}{\lambda_{\min(P_1)}}}$, there exists $T(\mu)$, for any $t \ge T$, such that $|\hat{\underline{e}}(t)| \le \mu$.

Proof: consider the Lyapunov function candidate

$$V(t) = \frac{1}{2}\hat{\underline{e}}^T P_1 \hat{\underline{e}} + \frac{1}{2}\tilde{\underline{e}}^T P_2 \tilde{\underline{e}} + \frac{1}{2\gamma_f}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\gamma_g}\tilde{\theta}_g^T\tilde{\theta}_g + \frac{1}{2\gamma_\varphi}\tilde{\varphi}^T\tilde{\varphi}$$

where $\tilde{\varphi} = \varphi - \varphi^*, \varphi^* = \begin{bmatrix}\varphi_f^* & \varphi_g^*\end{bmatrix}^T$. The time derivative of $V(t)$ is

$$\begin{aligned}\dot{V} &= \frac{1}{2}\hat{\underline{e}}^T[(A-BK^T)^T P_1 + P_1(A-BK^T)]\hat{\underline{e}} + \hat{\underline{e}}^T P_1 K_0 C^T \tilde{\underline{e}}\\ &+ \tilde{\underline{e}}P_2Bu_s - \tilde{\underline{e}}P_2B\delta_f(\hat{x}) + \frac{1}{2}\tilde{\underline{e}}^T[(A-K_0C^T)^T P_2\\ &+ P_2(A-K_0C^T)]\tilde{e} + \tilde{\underline{e}}^T P_2 B\theta_f^T\xi(\hat{x}) + \tilde{\underline{e}}^T P_2 B\theta_g^T\xi(\hat{x})u\\ &- \tilde{\underline{e}}^T P_2 B\delta_g(\hat{x})u - \tilde{\underline{e}}^T P_2 B\hat{g}(\hat{x}|\theta_g)\xi(\hat{x})u_r + \frac{1}{\gamma_f}\tilde{\theta}_f^T\dot{\theta}_f\\ &+ \frac{1}{\gamma_g}\tilde{\theta}_g^T\dot{\theta}_g + \frac{1}{\gamma_\varphi}\tilde{\theta}_\varphi^T\dot{\theta}_\varphi\\ &\le -\frac{1}{2}\hat{\underline{e}}^T Q_1\hat{\underline{e}} - \frac{1}{2}\tilde{\underline{e}}^T Q_2\tilde{\underline{e}} + \tilde{\theta}_f^T(\frac{1}{\gamma_f}\dot{\theta}_g + \tilde{\underline{e}}^T P_2 B\xi(\hat{x}))\\ &+ \tilde{\theta}_g^T(\frac{1}{\gamma_g}\dot{\theta}_g + \tilde{\underline{e}}^T P_2 B\xi(\hat{x})u) + A_v\end{aligned} \tag{19}$$

where

$$\begin{aligned}-Q_1 &= P_2(A-2BC^T) + (A-2BC^T)^T P_2 - Y_2C^T - CY_2^T\\ &\quad + 2CC^T + P_2BB^TP_2 - 2P_1K_0K_0^TP_1\\ -Q_2 &= (A-BK^T)^T P_1 + P_1(A-BK^T) - 2(C-P_2B)(C-P_2B)^T\end{aligned} \tag{20}$$

$$A_v = -\tilde{\underline{e}}^T P_2 B\delta_f(\hat{x}) - \tilde{\underline{e}}^T P_2 B\delta_g(\hat{x})u - \tilde{\underline{e}}^T P_2 B\beta + \frac{1}{\gamma_\varphi}\tilde{\varphi}^T\dot{\varphi} \tag{21}$$

Defining $z = -\tilde{\underline{e}}^T P_2 B$, combining (13) and (14) with (16) gives

$$\begin{aligned}A_v &= \frac{1}{\gamma_\varphi}\tilde{\varphi}^T\dot{\varphi} + z\delta_f(\hat{x}) + z\delta_g(\hat{x})u + z\beta\\ &= \frac{1}{\gamma_\varphi}\tilde{\varphi}^T\dot{\varphi} + z\delta_f(\hat{x}) + z\delta_g(\hat{x})u_c + z\beta z + \delta_g(\hat{x})\frac{\beta}{\hat{g}}\\ &\le \frac{1}{\gamma_\varphi}\tilde{\varphi}^T\dot{\varphi} + |z|(\varphi_f^* s_f + \varphi_g^* s_g|u_c|) + z\beta\\ &+ |z\beta|\frac{\varphi_g^* s_g}{|\hat{g}|}\end{aligned}$$

where

$$\varphi^* = \begin{bmatrix}\varphi_f^* & \varphi_g^*\end{bmatrix}^T, \varphi^* = \begin{bmatrix}s_f & s_g|u_c|\end{bmatrix}^T \tag{22}$$

By $z\beta \le 0$,it is obtained that

$$\begin{aligned}A_v &\le \frac{1}{\gamma_\varphi}\tilde{\varphi}^T\dot{\varphi} + |z|{\varphi^*}^T s - z\varphi^T\omega + z\beta\frac{\varphi_g s_g}{|\hat{g}|}\\ &+ |z\beta|\frac{\varphi_g^* s_g}{|\hat{g}|}\\ &\le \tilde{\varphi}^T(\frac{1}{\gamma_\varphi}\dot{\varphi} - z\omega) + {\varphi^*}^T(|z|s - z\omega) + z\beta\frac{\tilde{\varphi}_g s_g}{|\hat{g}|}\end{aligned}$$

By using (18) and Lemma 1, it follows that

$$A_v \le -\sigma\tilde{\varphi}^T(\varphi - \varphi^0) + \kappa\varepsilon|\varphi^*|_1 \tag{23}$$

Substituting (23) into (19) yields

$$\begin{aligned}\dot{V} &\le -\frac{1}{2}\hat{\underline{e}}^T Q_1\hat{\underline{e}} - \frac{1}{2}\tilde{\underline{e}}^T Q_2\tilde{\underline{e}} - \sigma\tilde{\theta}_f^T(\theta_f - \theta_f^0)\\ &- \sigma\tilde{\theta}_g^T(\theta_g - \theta_g^0) - \sigma\tilde{\varphi}^T(\varphi - \varphi^0) + \kappa\varepsilon|\varphi^*|_1\\ &\le -\frac{1}{2}\hat{\underline{e}}^T Q_1\hat{\underline{e}} - \frac{1}{2}\tilde{\underline{e}}^T Q_2\tilde{\underline{e}} - \frac{\sigma}{2}(|\tilde{\theta}_f|^2| + |\tilde{\theta}_g|^2 + |\tilde{\varphi}|^2)\\ &+ \frac{\sigma}{2}|\theta_f^* - \theta_f^0|^2 + \frac{\sigma}{2}|\theta_g^* - \theta_g^0|^2 + \frac{\sigma}{2}|\varphi^* - \varphi^0|^2 + \kappa\varepsilon|\varphi^*|_1\end{aligned}$$

Solving the LMI in Assumption 2, matrices P_1, P_2 and K_0 can be obtained. From (21) we get Q_1 and Q_2. By (19), it is followed that

$$\dot{V} \le -\alpha V + \lambda \tag{24}$$

where

$$\begin{aligned}\alpha &:= \min\frac{\lambda_{min}(Q_1)}{\lambda_{max}(P_1)}, \frac{\lambda_{min}(Q_2)}{\lambda_{max}(P_2)}, \sigma\gamma_f, \sigma\gamma_g, \sigma\gamma_\varphi\\ \lambda &:= \frac{\sigma}{2}|\theta_f^* - \theta_f^0|^2 + \frac{\sigma}{2}|\theta_g^* - \theta_g^0|^2 + \frac{\sigma}{2}|\varphi^* - \varphi^0|^2 + \kappa\varepsilon|\varphi^*|_1\end{aligned} \tag{25}$$

Selecting $\rho = \frac{\lambda}{\alpha} > 0$, then (24) satisfies

$$0 \le V(t) \le \rho + (V(0) - \rho)\exp(-\alpha t) \tag{26}$$

Therefore, $\underline{x}$ and θ_f and φ are uniformly bounded. Furthermore, using (19) and (26), we obtain that, given any $\mu > \sqrt{\frac{2\rho}{\lambda_{min}(P_1)}}$, there exists $T(\mu)$ such that for all $t \ge T$, the error $\hat{\underline{e}}(t)$ satisfies $|\hat{\underline{e}}(t)| \le \mu$.

Remark: It is shown that $z\omega_i \ge 0$ from Lemma 1. To guarantee $z\beta \le 0$, the following inequality must be satisfied

$$\varphi_g < \frac{g_M}{\alpha_g}$$

where

$$\alpha_g = \sup_{\hat{\underline{x}} \in U_c} s_g(\hat{\underline{x}}), g_M = \min_{\hat{\underline{x}} \in U_c} \hat{g}(\hat{\underline{x}}|\theta_g).$$

IV. Simulation Example

To illustrate the control procedure and performance, we apply the method proposed in this paper to control the inverted pendulum to track a sine-wave trajectory. The dynamic equations of the system are described as follows [11].

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{g\sin(x_1) - mlx_2^2\cos(x_1)/(m_c+m)}{l(4/3 - ml\cos^2(x_1)/(m_c+m)} \\
&\quad + \frac{\cos(x_1)/(m_c+m)}{l(4/3 - ml\cos^2(x_1)/(m_c+m)}
\end{aligned} \tag{27}$$

where $x_1 = \theta$ represents the angle of the pendulum, x_2 represents angular velocity, $g = 9.8m/s^2$ is the acceleration due to gravity, $m_c$is the mass of cart, m is the mass of pole, l is the half-length of pole, u is the applied force (control). Choose $m_c = 1kg$,$m = 0.1kg$, $m = 0.1kg$. Obviously, (27) is on the form of (1), thus our fuzzy controller applies to this system. Select the membership function

$$\begin{aligned}
&\mu_{F_i^1}(x_i) = \frac{1}{1+\exp[5(x_i+0.6)]}, \mu_{F_i^2}(x_i) = \exp[-(x_i+0.4)^2], \\
&\mu_{F_i^3}(x_i) = \exp[-(x_i+0.2)^2], \mu_{F_i^4}(x_i) = \exp[-(x_i)^2], \\
&\mu_{F_i^5}(x_i) = \exp[-(x_i-0.2)^2], \mu_{F_i^6}(x_i) = \exp[-(x_i-0.4)^2], \\
&\mu_{F_i^7}(x_i) = \frac{1}{1+\exp[-5(x_i+0.6)]}
\end{aligned}$$

choose

$$\begin{aligned}
&y_m = (pi/30)\sin(t), K = [18, 80], \gamma_f = 1000, \\
&\gamma_g = 10, \gamma_\varphi = 1, \sigma = 0.1, s_f = 100, \\
&s_g = 0.8, \theta_f^0 = \theta_g^0 = 1.
\end{aligned}$$

The simulation results are shown in Fig.1 to Fig.3

From the Fig1 and Fig2, it can be seen that the system output tracks the desired output well by the proposed controller. From the Fig3, the observe error converges to zero, which testifies the validity of design of the observer (9).

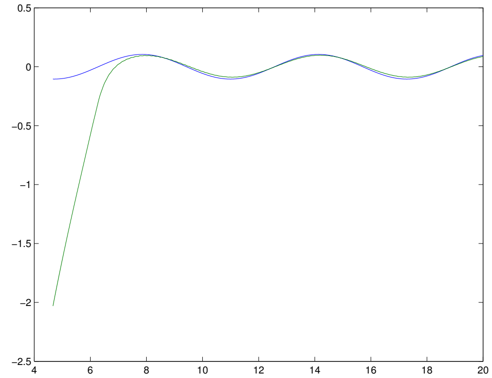

Fig. 1. system output y and desired output y_m

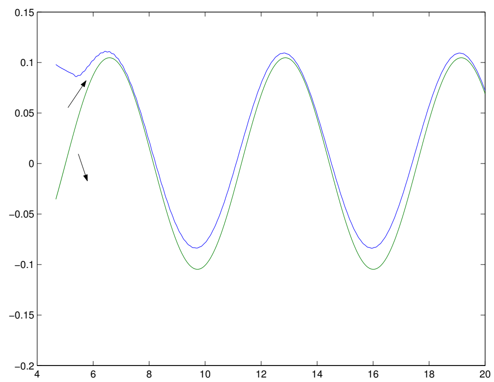

Fig. 2. the trajectory of $\dot{y}$ and $\dot{y_m}$

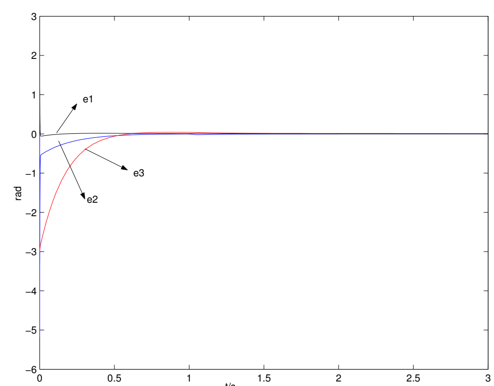

Fig. 3. the trajectory of observe error $\hat{e}$

V. Conclusion

A stable adaptive fuzzy control method is proposed for single input and single output nonlinear systems. This method needs not the assumption that the state variables are measure. And the state variables are estimated by designing observer. The Lyapunov synthesis approach is used to analyse the fuzzy system to obtain the corresponding parameters adaptive laws. Simulation example is given to show that the proposed method is validated and efficient.

VI. ACKNOWLEDGMENTS

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