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# Feedback Control for a MEMS-Based High-Performance Operational Amplifier

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***Abstract*— A novel MEMS-based operational amplifier that uses extensive feedback control techniques to achieve robust performance is described. The input stage of the amplifier is a MEMS structure that provides a variable capacitance to transduce a low-frequency input voltage into a high-frequency AC current. This up-modulation of the input signal is exploited to reduce offsets and low-frequency noise, and the dielectric isolation of the MEMS structure provides high input impedance and low leakage currents. This paper details the closed-loop designs used to implement the signal up-modulation and to provide accurate gain to the input.**

## I. Introduction

Amplifiers that attempt to make precision DC measurements face two primary barriers. One difficulty is the inherent noise of the input stage transistors interfering with measurements. In particular, flicker noise has a frequency characteristic that presents a much larger noise floor for low-frequency measurements than for signals above the $1/f$ noise corner frequency. The other issue is the leakage current of the amplifier input stage, which limits the resolution of small input signals.

Our design approach addresses both issues by utilizing a MEMS-based input stage that acts as a capacitive transducer [1], [2]. The principle of operation is illustrated in Figure 1. A DC voltage is sensed by a sinusoidally varying capacitance, provided by the MEMS structure. This creates an AC current signal at the frequency of the capacitance modulation, which can then be amplified and demodulated back to DC. If the input signal is sufficiently bandlimited, and the frequency of the modulating capacitance is above the $1/f$ noise corner of the amplifier, then the input signal and the low-frequency noise of the amplifier do not share bandwidth, and the demodulated output results in a noise-reduced low-frequency measurement. Furthermore, as a fundamentally capacitive technique, coupled with dielectric trench isolation, the MEMS transducer facilitates very low leakage currents over a wide range of temperatures. Although the input bandwidth is constrained, this amplifier is well suited for low-frequency precision measurements, such as thermocouples, pH meters, and strain gauges.

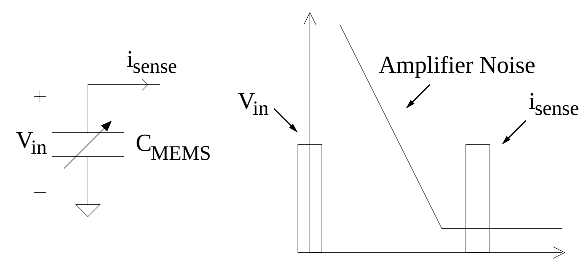


Fig. 1. Principle of operation. A DC voltage on a modulating capacitor creates an AC current in a frequency of less noise. The current signal is amplified and demodulated for a noise-reduced measurement of the input voltage.

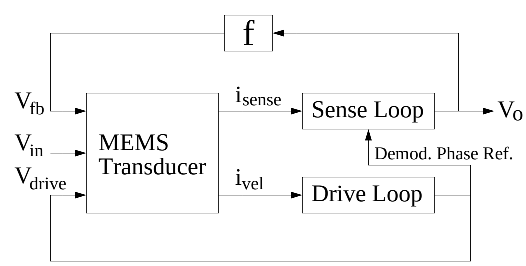


Fig. 2. System overview. The MEMS transducer converts the DC input voltage into an AC sense current, which is amplfied and demodulated by the sense loop. The output of the sense loop is fed back to the MEMS transducer to provide accurate gain to the input. The velocity current of the MEMS transducer provides information about the rate of capacitance modulation, and the drive loop controls the vibration of the MEMS transducer accordingly.

A system overview is illustrated in Figure 2. To function, the MEMS-based amplifier uses two co-dependent feedback loops. The ‘drive loop’ utilizes closed-loop control to vibrate the MEMS structure at a constant frequency to produce a modulating capacitance. The ‘sense loop’ senses

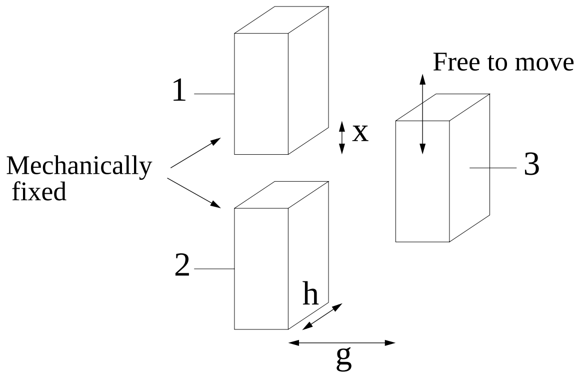


Fig. 3. Capacitor plates of the MEMS transducer. A modulating capacitor is achieved by varying the overlap area between two capacitor plates. The capacitor gap, height, and overlap distance are represented by $g$, $h$, and $x$, respectively. As plate 3 moves up and down, plates 1 and 2 see opposite phases of the same capacitance modulation.

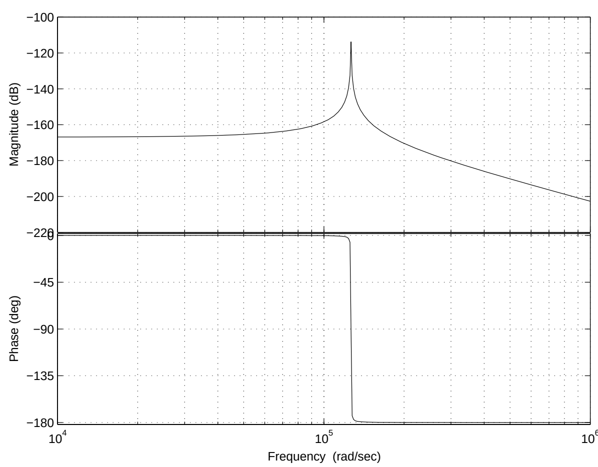


Fig. 4. Ideal force-to-displacement frequency response of the MEMS transducer. The resonant frequency is designed to be at 15kHz, and overlap area variation results in a mechanical system with a high quality factor. For maximum displacement, and thus maximum sense signal current, the transducer must be vibrated at its mechanical resonance.

the up-modulated signal from the MEMS structure, and provides gain and demodulation to this signal. Global feedback around the sense loop allows for accurate measurement of the input. The following sections detail the function of the MEMS structure as a capacitive transducer, and the design of the drive and sense loops.

## II. MEMS Transducer

The MEMS transducer consists of a series of fixed beams (ports) that interact with one common moving beam. For the two feedback loops, the MEMS transducer requires four ports. The sense loop must have a port for the input voltage, and a port for the feedback voltage. The up-modulated signal current is picked-off from the moving beam. The drive loop must have a port to apply electrostatic forces to move the beam, and a port to measure the velocity of the movement.

The MEMS structure creates a modulating capacitance by varying the overlap area between two capacitor plates, as illustrated in Figure 3. Plates 1 and 2 are mechanically fixed and form the input port, plate 3 is free to displace and is biased at a constant reference potential. The amount of overlap is represented by $x$. Because plates 1 and 2 see the same magnitude of capacitance modulation but with opposite phases, the induced signal current on plate 3 is proportional to the difference between the voltages on plates 1 and 2:

$$i_{sense} = (v_1 - v_2)\frac{dC_{\text{MEMS}}}{dt} = (v_{in+} - v_{in-})\frac{\epsilon_o h}{g}\frac{dx}{dt}.$$

Therefore, the MEMS transducer allows for differential inputs with common mode rejection.

Feedback around the sense loop can be accomplished by feeding the output of the sense loop back to the input port, as a classic operational amplifier feedback topology. Alternatively, for an instrumentation amplifier topology, the output may be connected to a feedback port, which is an additional set of fixed capacitor plates. The induced signal current on the moving beam is then

$$i_{sense} = ((v_{in+} - v_{in-}) - (v_{fb+} - v_{fb-}))\,\frac{\epsilon_o h}{g}\frac{dx}{dt}.$$

Such a feedback configuration allows for complete isolation of the input port, and the input common-mode voltage range is no longer constrained by the supply rails of the amplifier.

Voltages applied to a drive port induce electrostatic forces that actuate the moving beam. Referring again to Figure 3, for $+E_o$ on plate 1 and $-E_o$ on plate 2, the attractive force acting on plate 3 is given by the gradient of the stored energy between the fixed and moving plates:

$$F = \frac{\partial}{\partial x}\left(\frac{1}{2}C_{\text{MEMS}}(v_1^2 + v_2^2)\right) = \frac{\epsilon_o h}{g}E_o^2.$$

The velocity of the moving beam is measured through another port. For a fixed bias voltage $V_{bias}$ on both plates 1 and 2, the current signal induced on these plates due to the displacement of plate 3 is

$$i_{vel,1} = -i_{vel,2} = V_{bias}\frac{dC_{\text{MEMS}}}{dt} = V_{bias}\frac{\epsilon_o h}{g}\frac{dx}{dt},$$

which is proportional to the velocity of the moving plate, as desired.

For an electrostatic force drive, the velocity of the moving plate can be modeled as a mass-spring-damper system [2]. The transfer function is

$$\frac{V}{F}(s) = s\frac{X}{F}(s) = \frac{s}{ms^2 + bs + k},$$

where $m$, $b$, and $k$ represent, respectively, the effective mass, damping, and restoring-force constants of the MEMS structure.[1] The transducer is expected to have a mechanical

[1]Correspondence regarding details of process parameters should be addressed to Tim Denison.

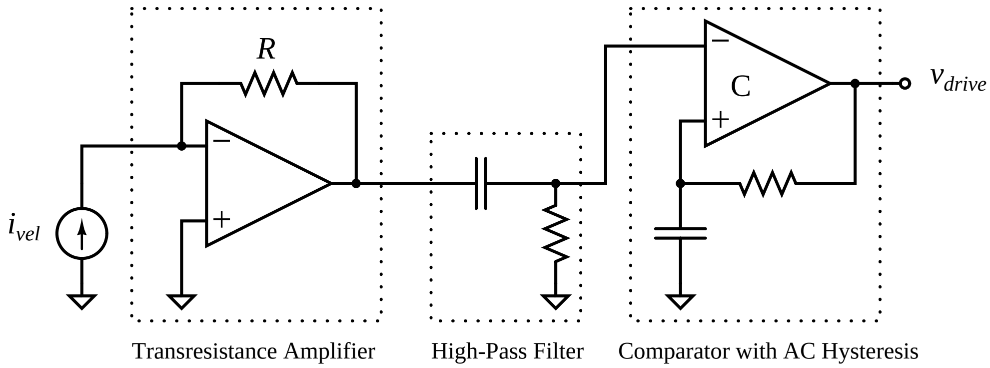


Fig. 5. Drive loop overview. The MEMS transducer generates a current proportional to the velocity of the moving beam, which is converted to a voltage by a transresistance amplifier. A high-pass filter suppresses DC offsets, and a comparator with AC hysteresis applies a square-wave voltage drive to the MEMS transducer.

resonant frequency at 15kHz, and a quality factor of 450. Figure 4 illustrates the anticipated frequency response.

Because the transduced sense current from the MEMS structure is proportional to its change in capacitance, a large displacement improves the signal-to-noise ratio of the amplifier. However, large voltages necessary to actuate large force drives can capacitively couple into the sense node through parasitics, interfering with the sensing of small signals. Because the displacement is at a maximum when the driving force is at the resonant frequency of the system, it is desirable to vibrate the transducer at its mechanical resonance.

## III. Resonant Drive Loop

The purpose of the drive loop is to vibrate the MEMS transducer at its mechanical resonant frequency, yielding the maximum sense current from the beam for a given force drive. However, the frequency of the resonant peak illustrated in Figure 4 can shift with process variations and temperature, so a fixed-frequency drive can result in a significant decrease in displacement, severely degrading amplifier sensitivity. Therefore, a robust approach is needed to ensure that the moving structure will consistently vibrate at its mechanical resonance. This robustness is achieved through closed-loop control.

Because the applied force drive and the beam velocity are in phase at resonance, it is possible to couple the transducer with a nonlinear element to form a self-exciting oscillator that maintains constant-amplitude oscillations at the resonant frequency of the MEMS structure. An overview of the drive loop electronics and a block diagram for this feedback loop are shown in Figures 5 and 6, respectively. The beam velocity is converted to a current by the modulating capacitance of the transducer, and this current is converted to a voltage by a transresistance amplifier. A high-pass filter suppresses DC offsets, and a comparator applies a square-wave electrostatic drive to the MEMS structure. In addition, because the drive frequency and the frequency of up-modulation are the same, the comparator output can

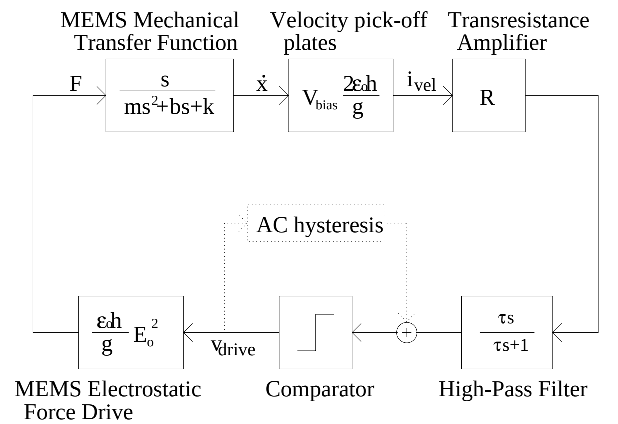


Fig. 6. Drive loop block diagram. The voltage drive of the comparator creates electrostatic forces that move the MEMS structure. The velocity of the movement induces a current signal, which is amplified and high-pass filtered, and in turn affects the comparator voltage drive.

also serve as a phase reference for demodulation in the sense loop.

For a comparator without hysteresis, its describing function is

$$G_D(E) = \frac{4}{\pi}\frac{E_o}{E},$$

where $E$ is the amplitude of the input of the comparator, and $E_o$ is the amplitude of the square-wave output of the comparator [3]. Intuitively, the high-Q bandpass nature of the transducer attenuates the harmonics of the square wave drive to yield the desired sinusoidal displacement of the beam, in line with the describing function approximation.

The open-loop transfer function of the drive loop is given by

$$L(s) = \left(\frac{V}{F}(s)\right)\left(V_{bias}\frac{2\epsilon_0 h}{g}\right)(R)\left(\frac{4}{\pi}\frac{E_o}{E}\right)\left(E_o^2\frac{\epsilon_0 h}{g}\right).$$

If the transresistance amplifier, high-pass filter, and comparator contribute no phase shift, the net loop transfer function phase is zero at the resonant frequency of the

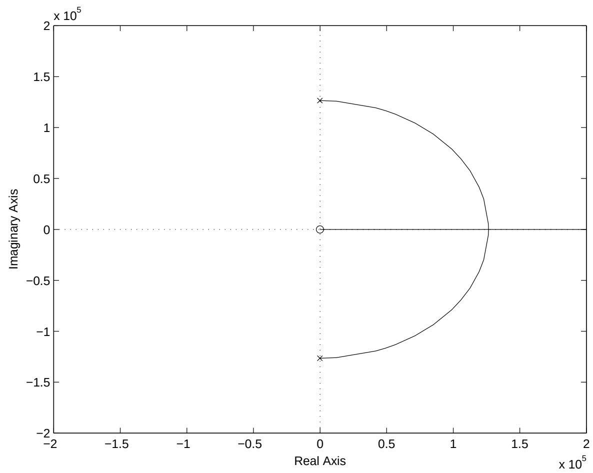


Fig. 7. Root locus for the drive loop. The dynamics of the drive loop control yield constant-amplitude oscillations at the resonant frequency of the MEMS transducer.

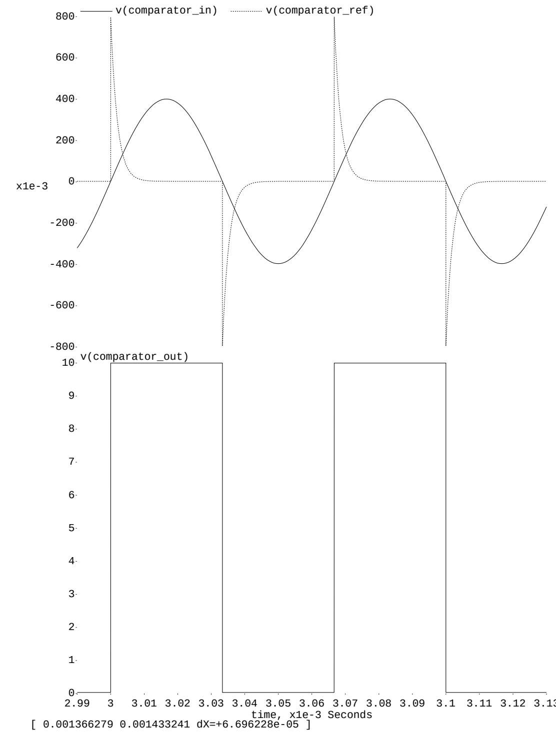


Fig. 8. Effect of comparator AC hysteresis. A comparator transition briefly changes the comparator switching threshold, preventing the comparator from triggering more than once during a fixed amount of time. This 'blanking' interval is set to be smaller than the oscillation period, so the dynamics of the resonant feedback loop are unaffected.

transducer, when the velocity of the beam is linearly related to the drive force. Thus, this frequency is the self-oscillation frequency of the drive loop, and the amplitude of oscillations into the comparator will be such that the magnitude of the loop transfer function is unity at this frequency:

$$E = \left(\frac{1}{\beta}\right)\left(V_{bias}\frac{2\epsilon_0 h}{g}\right)(R)\left(\frac{4E_o}{\pi}\right)\left(E_o^2\frac{\epsilon_0 h}{g}\right).$$

This analysis indicates that poles on the $j\omega$-axis are possible. Observation of the root locus of the open-loop drive transfer function, illustrated in Figure 7, indicates that the system poles return to the $j\omega$-axis under perturbations, yielding a stable limit cycle.

However, in reality, the transresistance amplifier and the comparator, although designed for wide bandwidths, contribute small negative phase shifts at the resonant frequency of the MEMS transducer. The high-pass filter can be designed to provide positive phase shift for cancellation, but precise cancellation of phase shifts is not practical, and a phase error between one to five degrees is expected. Thus, the oscillation frequency of the drive loop is not at the resonant peak of the transducer. However, due to the high-Q nature of the mechanical transfer function of the MEMS, a 20° phase error from resonance results in less than 5% loss in sensitivy of the amplifier, so a small phase error is not a concern.

In practice, the presence of parasitic paths coupling force and velocity nodes in the drive loop can result in unwanted high-frequency oscillations. This problem is addressed by using a small amount of AC hysteresis around the comparator, as shown in Figure 5. This creates a 'blanking' interval such that once the comparator changes state, it cannot make another transition for a fixed amount of time. The effect of the AC hysteresis is illustrated in Figure 8. The time constant of the AC hysteresis is set to be faster than the period of the resonant frequency, and thus the AC hysteresis does not contribute phase at the oscillation frequency of the feedback loop, and the describing function of the comparator remains unchanged.

Another design concern is the presence of higher order resonant modal frequencies of the MEMS structure, which can also potentially form stable limit cycles. However, the nature of the force drive cannot support many of these mechanical modes, so these modes can never be excited. In addition, the other resonant modes are at frequencies much higher than the fundamental resonance, so the modes that can plausibly be realized by the force drive are prevented by the blanking interval of the AC hysteresis [2].

## IV. Sense Loop

An overview of the sense loop is shown in Figure 9. A differential input voltage is transduced into an AC current by the MEMS structure, and detected by a low-noise preamplifier. The following stages provide AC coupling, demodulation, integration, and buffering. The capacitor $C_{par}$ represents the input capacitance of the preamplifier.

The overall gain of the input voltage signal depends on the magnitude of the capacitance modulation, in addition to other potentially variable gain elements in the signal path. However, while the modulating capacitances of the MEMS transducer are capable of matching to 0.1%, their absolute values can vary as much as 20%. Furthermore, the amount

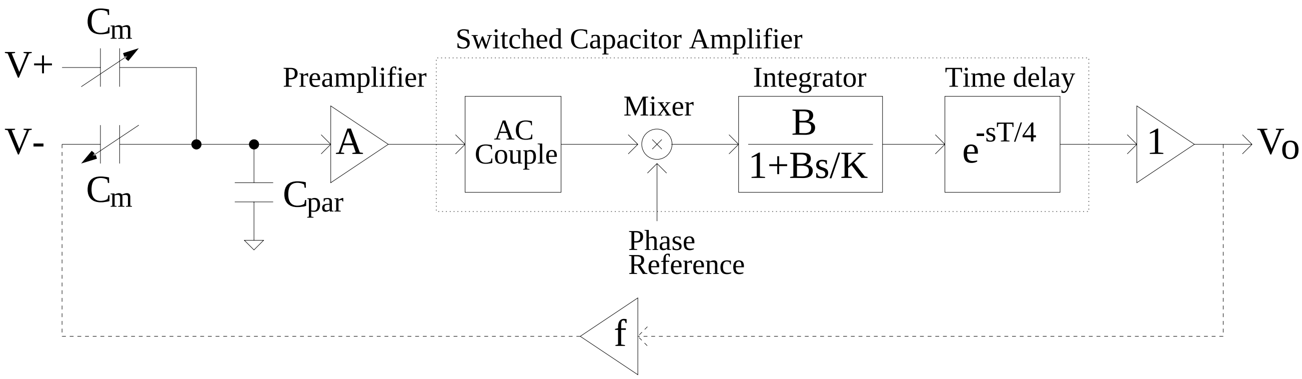


Fig. 9. Sense loop overview. The modulating capacitance of the input port of the MEMS transducer converts the differential input voltage into a signal current. A low-noise preamplifier provides gain, and a switched capacitor amplifier provides AC coupling, demodulation, and integration, and also introduces a time-delay. The capacitor $C_{par}$ represents the parasitic capacitance at the input of the preamplifier. Global feedback around the sense loop buys immunity to changes in the absolute magnitude of the modulating capacitance and other variable forward path gains.

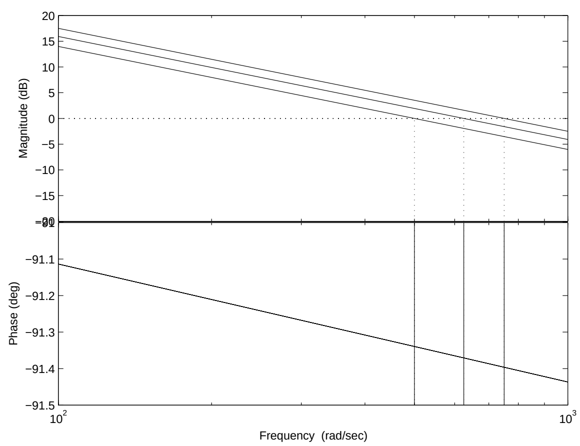


Fig. 10. Open-loop frequency response of the sense loop with unity gain feedback. For a $\pm 20\%$ variation in the open-loop gain, the crossover frequency varies by $\pm$20Hz, with negligible change in phase margin.

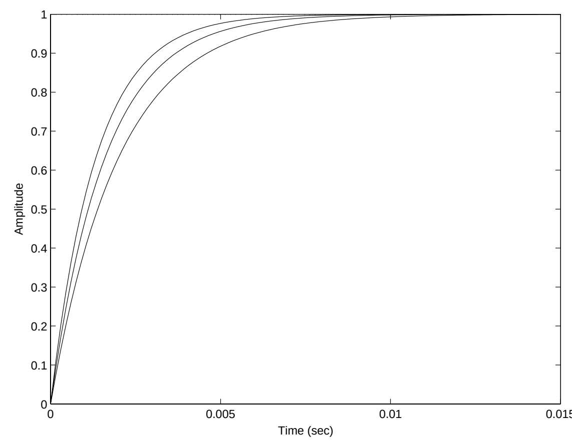


Fig. 11. Closed-loop step response of the sense loop with unity gain feedback. For a $\pm 20\%$ variation in the open-loop gain, the rise time of the step response varies by $\pm 20\%$, but the closed-loop gain varies by less than 0.1ppm.

of capacitance modulation also depends on the viscosity of air between the capacitive plates, which changes with temperature. To provide consistent and predictable gain to the input signal over temperature and process variations, a global feedback solution maintains a robust closed-loop gain over variations in the forward-path gain. As discussed earlier, the MEMS-based amplifier may be configured as either an operational or instrumentation amplifier.

The target closed-loop bandwidth of the sense loop is 100Hz. However, because of the up-modulation of the input signal, the sense loop is not a linear time-invariant system, so traditional feedback analysis cannot be applied. Nevertheless, previous work has confirmed that the output step response is well-predicted by the effective loop transfer function seen by the DC input voltage [2].

The input voltage first sees a capacitive divider between the modulating capacitance $C_m$ and the input parasitic capacitance $C_{par}$ as it is up-modulated. The preamplifier is designed for sufficiently wide bandwidth so that its phase contribution at the modulation frequency is negligible. A switched capacitor amplifier simultaneously provides AC coupling, demodulation, and integration.

The integrator transfer function, $H(s)$, looks like an ideal integrator over most frequencies:

$$H(s) \approx \frac{K}{s}.$$

However, although the switched capacitor topology is capable of realizing a very high DC gain, the DC gain is nonetheless finite, and the integrator pole is shifted slightly into the left-half plane. With a DC gain of $B$, the integrator transfer function is more precisely

$$H(s) = \frac{B}{1 + \frac{B}{K}s}.$$

In addition, the switched capacitors also introduce an inherent quarter-cycle time delay [2]. Therefore, for unity feedback, the effective open-loop transfer function of the sense loop is

$$L(s) = \left(\frac{C_m}{C_{par}}\right)(A)\left(\frac{B}{1+\frac{B}{K}s}\right)\left(e^{-sT/4}\right)(f).$$

Figures 10 and 11 illustrate the predicted frequency and step responses of the sense loop for unity gain feedback. For a $\pm$20% variation in $C_m$, the crossover frequency varies by $\pm$20%, which has a negligible effect on phase margin. The closed-loop gain changes by less than 0.1ppm. With feedback, variations in the open-loop gain are traded-off for variations in the closed-loop bandwidth, and the accuracy of the closed-loop gain depends instead on the matching of the feedback resistors (if used) and the modulating capacitance values.

## V. Conclusions

A novel amplifier utilizing a MEMS transducer input stage addresses the measurement limitations of existing amplifier topologies. Capacitive sensing of the signal, made possible by the MEMS, circumvents low-frequency noise through up-modulation, and facilitates very low leakage currents due to isolation trenches.

Feedback control of the capacitance modulation frequency is made possible by the linear relationship between the drive force and the transducer beam velocity at its desired frequency of vibration. While an open-loop solution using a fixed-frequency oscillator to drive the beam would severely compromise sensitivity and performance as the resonant peak shifts with temperature and process variations, the closed-loop design is guaranteed to consistently vibrate the beam at its resonance, maintaining the maximum signal-to-noise ratio of the amplifier.

The open-loop gain of the sense loop is sensitive to the absolute value of the modulating capacitance of the MEMS, which also changes significantly with temperature and process variations. Accurate amplification of the input voltage is made possible by global feedback, which maintains a relatively constant gain despite variations in the modulating capacitance. The MEMS transducer has a feedback port which may be used, allowing for complete isolation of the input port.


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