

# Cartesian Feedback for RF Power Amplifier Linearization

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**Abstract**—We discuss two control problems that arise in connection with Cartesian feedback radio-frequency power amplifiers. New solutions to both problems are described, and the results of a working prototype are presented. The prototype, a integrated circuit (IC) fabricated in National Semiconductor’s 0.25 $\mu$ m CMOS process, represents the first known fully integrated implementation of the Cartesian feedback concept.

## I. INTRODUCTION

Designers of radio-frequency (RF) power amplifiers (PA’s) for modern wireless systems are faced with a difficult tradeoff. On one hand, the PA consumes the lion’s share of the power budget in most transceivers. It follows that in a cellular phone, for example, battery lifetime is largely determined by the power efficiency of the PA. On the other hand, it may be desirable to have high spectral efficiency—the ability to transmit data at the highest possible rate for a given channel bandwidth. The design conflict is that while spectral efficiency demands a highly linear PA, power efficiency is maximized when a PA is run as a constant-envelope, nonlinear element. The current state of the art is to design a moderately linear PA and employ some linearization technique. The amplifier operates as close to saturation as possible, maximizing its power efficiency, and the linearization system maximizes the spectral efficiency in this near-saturated region.

There are many different linearization techniques. Our work focuses on Cartesian feedback systems for two main reasons. First, because they employ analog feedback, the requirement for a detailed nonlinear model of the PA is greatly relaxed. This is an extremely compelling advantage, as RF PA’s are poorly understood and notoriously difficult to model. Second, Cartesian feedback systems automatically and elegantly compensate for process variations, temperature fluctuations, and aging. Nevertheless, historically the technique has suffered the shortcoming of relying on synchronous downconversion, which has been difficult to realize without manual trimming. This problem, combined with the recent trend toward fully monolithic systems, has caused Cartesian feedback to languish for years as little more than an academic curiosity.

We approach the synchronous downconversion, or phase alignment, problem from two directions. Detailed analysis of a Cartesian feedback system is performed, and it is shown to suggest a means of compensating the system

for robustness to phase misalignment. Alternatively, we describe and analyze a nonlinear, analog phase alignment regulator[1]. Test results for a fabricated IC, designed as a testbed for these ideas, are then presented and analyzed.

## II. CARTESIAN FEEDBACK

The idea of using Cartesian feedback to linearize power amplifiers has been discussed at least as early as the 1970’s[2], [3]. It is called *Cartesian* feedback because the feedback is based on the Cartesian coordinates of the baseband symbol,  $I$  and  $Q$ , as opposed to the polar coordinates. The a typical system is illustrated in figure 1. [2]. Fundamentally, the concept behind this system is negative

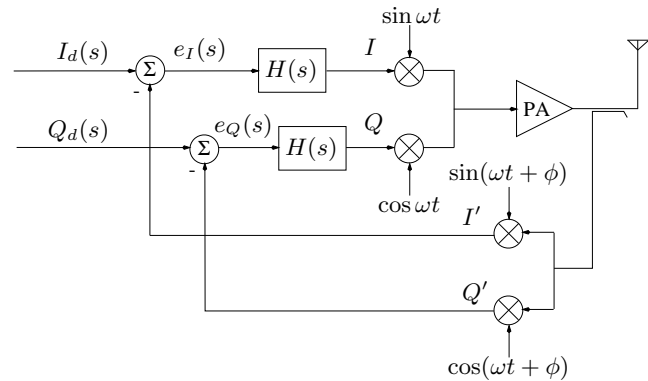


Fig. 1. Typical Cartesian feedback system. Ideally,  $\phi = 0$ .

feedback. A couple of factors complicate its expression in the context of an RF transmitter, however. The first is the extremely high frequency of many RF carriers, with modern standards calling for frequencies on the order of a few gigahertz. At this time, it is virtually impossible to build a high-gain, stable analog feedback loop with a crossover frequency in that range. The second factor is the recognition that in modulating an RF carrier, we are not shaping a voltage waveform in its entirety. Instead, we are shaping two independent characteristics of that carrier.

Cartesian feedback’s way of dealing with the first factor is the inclusion of a frequency translation step in the feedback path, shown as a downconversion mixer in figure 1. The loop is then closed at baseband, rather than at the carrier frequency. The system consequence is to linearize only in a narrow band of the spectrum centered about the carrier,

rather than from DC to the carrier. This is an ingenious way to exploit the narrowband nature of most RF signals.

The second factor manifests as the “double loop” structure of the system. There are two degrees of freedom in shaping, or modulating, an otherwise free-running RF carrier, and at least two choices of coordinate systems that fully describe the modulation. For polar feedback the choice made is to consider an RF carrier as having an amplitude and a phase. The structure of a polar feedback system reflects this choice, having one control loop for the amplitude, and another for the phase. An equivalent choice of coordinates is the Cartesian components, in which we consider the modulated carrier as the sum

$$A(t) \sin(\omega_0 t + \phi(t)) = I(t) \sin \omega_0 t + Q(t) \cos \omega_0 t,$$

where

$$I(t) = A(t) \cos \phi(t)$$

and

$$Q(t) = A(t) \sin \phi(t).$$

It is seen that Cartesian feedback treats the two degrees of freedom in a symmetrical way, allowing the structure of the system to take the form of two identical loops. This is in direct contrast to polar feedback, where the two degrees of freedom must be treated very differently.

### III. CONSEQUENCES OF PHASE MISALIGNMENT IN CARTESIAN FEEDBACK SYSTEMS

Figure 1 shows a typical Cartesian feedback system. The system block  $H(s)$  represents the loop driver amplifiers, which provide the loop gain as well as the dynamics introduced by the compensation strategy. The loop drivers feed the baseband inputs of the upconversion mixer, which in turn drives the power amplifier. Some means of coupling the output of the power amplifier to the downconversion mixer is employed, and the output of this mixer is used to close the feedback system.

#### A. Impact of phase misalignment on stability

Ideally, a Cartesian feedback system functions as two identical, decoupled feedback loops: one for the  $I$  component, and one for the  $Q$  component. This corresponds to the case of  $\phi = 0$  in figure 1. In practice, however, this state of affairs must be actively enforced. Delay through the power amplifier, phase shifts of the RF carrier due to the reactive load of the antenna, and mismatched interconnect lengths between the local oscillator (LO) source and the two mixers all manifest as an effective nonzero  $\phi$ . Worse, the exact value of  $\phi$  varies with temperature, process variations, output power, and carrier frequency. A Cartesian feedback system in which  $\phi$  is nonzero is said to have phase misalignment. In this state the two feedback loops are coupled, and the stability of the system is compromised.

The impact of phase misalignment on system stability can be seen mathematically. We start by observing that

the demodulated symbol  $S'$  is rotated relative to  $S$  by an amount equal to the phase misalignment  $\phi$ . To see this, we write Cartesian components of the demodulated symbol

$$\begin{aligned} I' &= (I \sin \omega t + Q \cos \omega t) \sin(\omega t + \phi) \\ Q' &= (I \sin \omega t + Q \cos \omega t) \cos(\omega t + \phi), \end{aligned}$$

where  $\omega$  is the carrier frequency. Using trigonometric identities and assuming frequency components at  $2\omega$  are filtered out, we arrive at  $S'$

$$I' = \frac{1}{2}(I \cos \phi + Q \sin \phi) \quad (1)$$

$$Q' = \frac{1}{2}(-I \sin \phi + Q \cos \phi). \quad (2)$$

We see that for  $\phi \neq 0$ , an excitation on the  $I$  input of the modulator results in a signal on the  $Q'$  downconverter output (and similarly for  $Q$  and  $I'$ ). Accordingly, we say that the two loops are coupled.<sup>1</sup>

One method of stability analysis is to consider the error signals  $e_I(s)$  and  $e_Q(s)$  shown in Figure 1. Recall that for a single feedback loop, the error signal is written

$$e(s) = \frac{X(s)}{1 + L(s)},$$

where  $X(s)$  is the command input and  $L(s)$  is the loop transmission. In the present case, let the phase misalignment be  $\phi$ . Furthermore, we set  $Q_d = 0$  without loss of generality.<sup>2</sup> The error expressions, as a function of the single input  $I_d(s)$ , are written

$$\begin{aligned} e_I(s) &= I_d(s) - L(s)e_I(s) \cos \phi - L(s)e_Q(s) \sin \phi \\ e_Q(s) &= L(s)e_I(s) \sin \phi - e_Q(s)L(s) \cos \phi, \end{aligned}$$

where  $L(s)$  includes the dynamics of the loop compensation scheme  $H(s)$  and the (linearized) dynamics introduced by the modulator, power amplifier, and demodulator. From here, it is straightforward to show that

$$e_I(s) = \frac{X(s)}{1 + L(s) \cos \phi + \frac{[L(s) \sin \phi]^2}{1 + L(s) \cos \phi}}.$$

This reduction of the system to a single-input problem now yields considerable insight. We identify an effective loop transmission,  $L_{\text{eff}}(s, \phi)$ , as follows:

$$L_{\text{eff}}(s, \phi) = L(s) \cos \phi + \frac{[L(s) \sin \phi]^2}{1 + L(s) \cos \phi}. \quad (3)$$

For perfect alignment,  $\phi = 0$  and  $L_{\text{eff}}$  is simply  $L(s)$ . The worst alignment is  $\phi = \frac{\pi}{2}$ , for which  $L_{\text{eff}} = [L(s)]^2$  and so the loop dynamics are a cascade of the dynamics in the uncoupled case. Unless designed with this possibility in mind, most choices of  $H(s)$  yield unstable behavior in this second case. Equation 3 shows that traditional

<sup>1</sup>Technically,  $\phi = \pi$  is also an uncoupled case. However, there is now an inversion in both loops, resulting in positive feedback instead of the desired negative feedback.

<sup>2</sup>We do not lose generality as long as we stay with linear analysis.

measures of stability degrade continuously as  $\phi$  sweeps from 0 to  $\frac{\pi}{2}$ , a fact demonstrated experimentally by Briffa and Faulkner [4].

### B. Compensating the system for robustness to phase misalignment

Equation 3 offers a great deal of insight into what happens in a phase-misaligned Cartesian feedback system. Physically, the fully coupled ( $\phi = \frac{\pi}{2}$ ) case behaves as depicted in figure 2, where  $P(s)$  represents the dynamics that the upconversion mixer, power amplifier, and downconversion mixer contribute to the loop transmission. In the

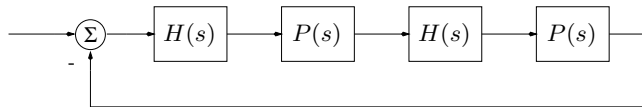


Fig. 2. Cartesian feedback under 90-degree misalignment.

literature, all efforts with regards to the phase alignment problem have focused, naturally, on ensuring phase alignment. But there is at least one other approach that deserves consideration: is it possible to choose  $H(s)$  such that it is stable for large phase misalignments?

The answer depends in part on what one means by “large.” Considering a misalignment of  $\phi = \pi$ , for instance, is discouraging. In this case  $L_{\text{eff}} = -L(s)$ , and there is simply no compensation strategy that is indifferent to the sign of the loop transmission. Cartesian feedback in fact does become a positive feedback system for misalignments in the open interval  $(\frac{\pi}{2}, \frac{3\pi}{2})$ , where the exact point of transition from negative to positive feedback depends on the details of  $L(s)$ . To avoid considering positive feedback cases, then, it is sensible to restrict the range of misalignments to the closed interval  $[-\frac{\pi}{2}, \frac{\pi}{2}]$ .

That stability margins degrade *continuously* with  $\phi$  suggests that finding a compensation strategy that works in the limiting cases of  $\phi = 0$  and  $\phi = \frac{\pi}{2}$  will solve the problem for the whole interval. Assuming the dynamics of the loop are dominated by  $H(s)$ , a compensation strategy that emerges is

$$H(s) = \frac{k}{s^x},$$

where  $0 < x < 1$ . Such “slow-rolloff” functions, while not truly realizable with a lumped-element network, can be approximated by alternating poles and zeros such that the average slope of  $H(s)$  is the appropriate dB-per-decade[5]. In the case of  $x = 0.5$ , for instance, stability as measured by phase margin would be excellent: 135 degrees in the aligned case, and 90 degrees in the  $\frac{\pi}{2}$  misaligned case.

Root locus analysis confirms that slow-rolloff compensation is a viable approach to designing for large misalignments. Figure 3 shows the root loci for the dominant-pole and  $\frac{k}{\sqrt{s}}$  compensation strategies. It can be seen that even in the case of the dominant-pole, 90-degree misalignment

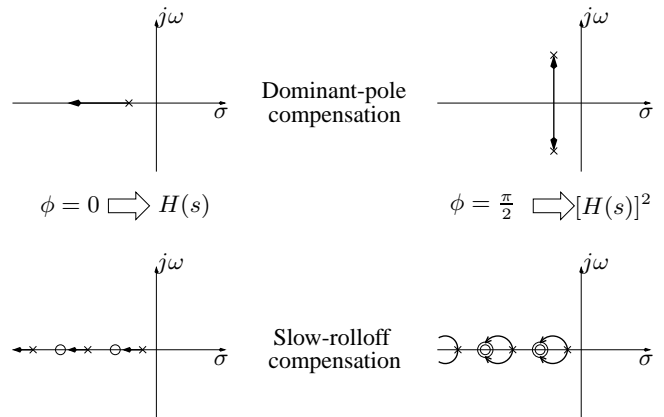


Fig. 3. Root locus plots for dominant-pole and slow-rolloff ( $\frac{k}{\sqrt{s}}$ ) compensation.

doesn’t necessarily lead to right-half-plane poles. At best, however, the result is a lightly damped, complex pair of poles. At worst, high-frequency poles not shown here (or not modeled) push this complex pair into the right-half plane. By contrast, the slow-rolloff compensation is seen to lead to heavily-damped complex pole pairs, and one expects a corresponding reduction in overshoot and ringing in response to an input step. One also expects the low-frequency, zero-pole doublets of the root loci to manifest themselves as slow-settling “tails” in the step response [5].

Experiments carried out on the final IC in accordance with this compensation discussion validate these expectations. As seen in section V-B, the slow-rolloff technique stabilizes the system for all misalignments up to 90 degrees. In addition to shedding light on compensation strategies for Cartesian feedback systems, the importance of these experiments is that they confirm the understanding developed in section III-A.

## IV. A NONLINEAR REGULATOR FOR MAINTAINING PHASE ALIGNMENT

Occasionally continuous regulation of the phase alignment is not needed, and it suffices to introduce a manually adjustable delay between the LO source, and, say, the demodulator. This approach is only feasible, however, if the system is not subject to variations in temperature, carrier frequency, process parameters, or, in some cases, output power. For cases in which the alignment must be regulated, various methods have been proposed in the literature; see, for example [6].

We present our control concept as a compact, truly continuous solution to the problem of LO phase alignment. It is truly continuous because it does not, for example, rely on the appearance of a specific symbol or pattern in the outgoing data stream. It is compact because it is easily implemented without digital signal processing, as presented here. This is a particularly compelling advantage, as the signals in a Cartesian feedback system are necessarily in

analog form. And we emphasize that, because the concept is based on the processing of baseband symbols, its realization is independent of carrier frequency.

#### A. Nonlinear dynamical system

Figure 4 represents a baseband symbol at the inputs of the modulator and at the outputs of the demodulator of a Cartesian feedback system. Mathematically the vectors

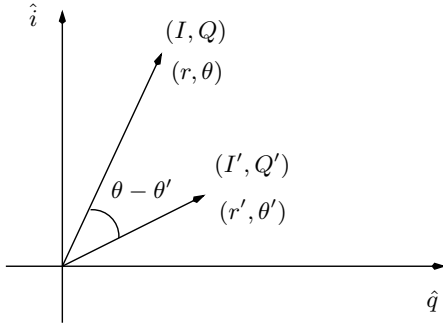


Fig. 4. Rotation of the baseband symbol due to phase misalignment.

are described in both Cartesian and polar coordinates, with primed coordinates denoting the demodulated power amplifier output and unprimed coordinates denoting the modulator input. In addition to undergoing a distortion in magnitude, the demodulated symbol is rotated by an amount exactly equal to the phase misalignment (see equations 1 and 2).

A start to the design of a phase alignment regulator is to observe that the signals  $I$ ,  $Q$ ,  $I'$ , and  $Q'$ , taken together, represent enough information to determine the phase misalignment. Further, they are easily accessible within the system. We seek to combine these variables such that, over a suitable range, the derived signal is monotonic in the phase misalignment.

One such combining of the variables is the sum of products  $IQ' - QI'$ . Recognizing that  $I = r \cos \theta$  and  $Q = r \sin \theta$ , and using trigonometric identities, we write the key relation

$$IQ' - QI' = rr' \sin(\theta - \theta'). \quad (4)$$

We see that using two multipliers and a subtractor, operations easily realizable in circuit form, one can derive a control signal that is indeed monotonic in the phase misalignment over the range  $-\frac{\pi}{2} < \theta - \theta' < \frac{\pi}{2}$ .

Figure 5 details a nonlinear dynamical controller built around equation 4. Using the notation  $\Delta\theta = \theta - \theta'$ , an implementation can be understood as mechanizing the equation

$$\frac{d\theta}{dt} = -\kappa[r(t)]^2 G \sin(\Delta\theta), \quad (5)$$

where  $\kappa$  is a constant of proportionality and gain  $G$  is associated with the integrator.

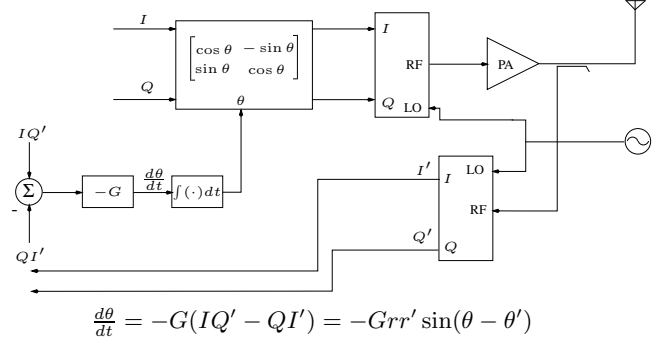


Fig. 5. Phase alignment concept.

Equation 5 presupposes the ability to correct the phase shift by changing  $\theta$ . The original prototype described in [7] realizes the required rotation by directly phase shifting the modulator LO. However, substantial power savings result from doing symbol rotation at baseband as shown in figure 5. Regardless, rotation should be performed in the forward path of the Cartesian feedback system, where the unavoidable artifacts of imperfect rotation are suppressed.

#### B. Stability concerns

Our control solution for the phase alignment problem is the simplest of nonlinear dynamical systems. It is seen from equation 5 to have two equilibrium points. The first, for which the symbols are aligned, is *stable*. The second, for which the symbols are misaligned by  $\pi$  radians, is *unstable*. For the ideal system represented by equation 5, this is the extent of a rigorous stability analysis.

The real-world situation can be complicated by dynamics associated with the phase shifter (and, possibly, the subtractor). If we provisionally consider a modulation scheme in which the magnitude of transmitted symbols is held constant,<sup>3</sup>  $r(t)$  in equation 5 loses its time dependence. Linearizing for small phase misalignments, and including the dynamics of the phase shifter as  $P(s)$ , we can represent the system as shown in figure 6. Drawing the system this

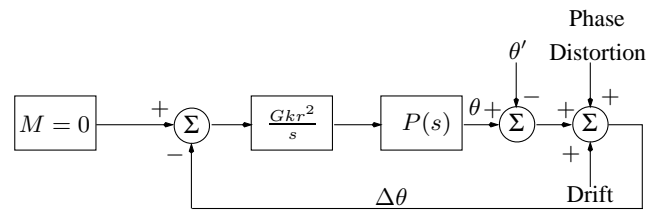


Fig. 6. Linearized phase regulation system. 'M' is the desired misalignment, which is nominally zero.

way requires some manipulation. The output of the phase shifter is not really  $\theta$ , but rather an additive *part* of  $\theta$  that

<sup>3</sup>Unlikely when using Cartesian feedback, of course. Temporarily making this assumption, however, yields insight that is broadly relevant to the stability analysis.

gets combined with the polar angle of the symbol being transmitted. However, in the absence of phase distortion and drift, the symbol-by-symbol changes of the polar angle  $\theta$  are tracked by identical changes in  $\theta'$ . These symbol-rate changes are thus invisible to an alignment system, and it is appropriate to label the output of  $P(s)$  as  $\theta$ . We can then include the effects of phase distortion and phase alignment drift as the additive disturbances of figure 6.

One can ensure stability by choosing  $G$  such that, for the largest symbol magnitude, loop crossover occurs before non-dominant poles become an issue. Fortunately, the drift disturbance will normally occur on time scales associated with temperature drift and aging [2]. Suppression of the phase distortion is the domain of the Cartesian feedback itself. It follows that for many systems, little of the design effort need be focused on fast phase alignment.

## V. EXPERIMENTAL RESULTS

What the new phase alignment regulator enables is the building of highly integrated Cartesian feedback systems. This is a compelling design goal, as it may allow this linearization technique to be used for modern, handheld wireless devices. As a demonstration, a fully monolithic prototype, fabricated in National Semiconductor's  $0.25\mu\text{m}$  CMOS technology, was designed and tested. To our knowledge, this is the first successful integration of a power amplifier, phase alignment system, and Cartesian feedback linearization circuitry all on the same die.

### A. Phase alignment system

Figure 7 is a trace capture of the type of experiment used to characterize the performance of the phase alignment system. The Cartesian feedback loop is open, a 500mV

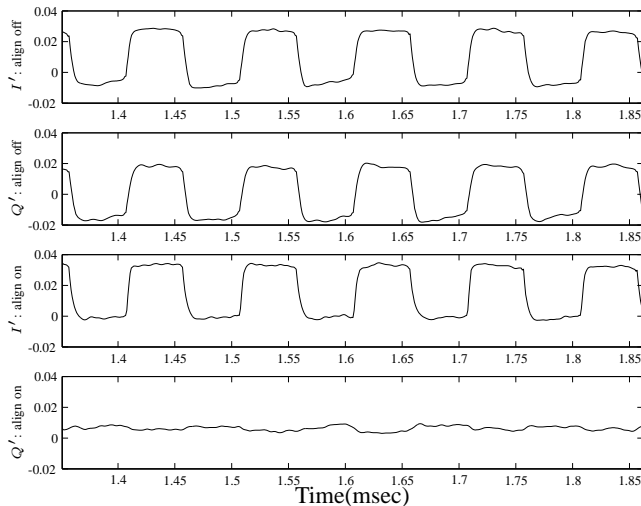


Fig. 7. Trace capture of a phase alignment experiment. The Cartesian feedback loop is open.

amplitude, 10kHz square wave drives the  $I$  channel, and

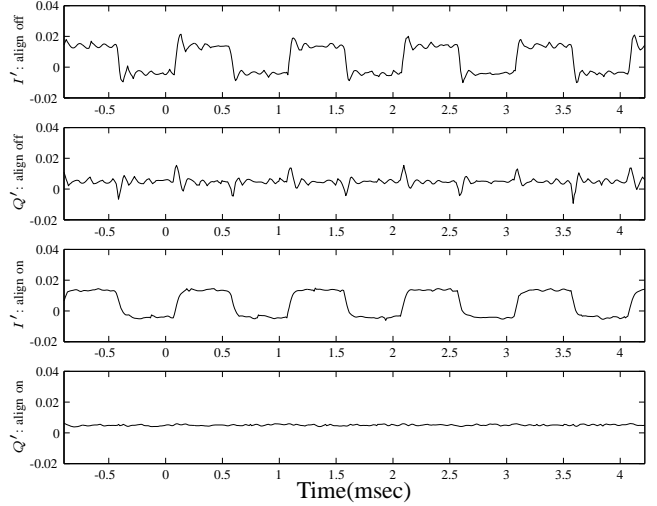


Fig. 8. Illustration of phase alignment stabilizing the closed-loop CFB system.

the  $Q$  channel is grounded.<sup>4</sup> The top two traces show that, initially, the misalignment is manually set to 45 degrees. The bottom two traces show the result of turning on the phase alignment system (releasing it from the a “reset” mode). Over the full,  $\pm 90$  degree range that the symbol rotator permitted, the regulator kept the phase misalignment below 9 degrees.

Figure 8 serves to illustrate the impact of phase misalignment on the stability margins of the closed-loop CFB system. Dominant-pole compensation is used in the CFB loop, and for the upper two traces the misalignment is manually set to 74 degrees. Overshoot and ringing is evident on these waveforms, and further misalignment causes outright oscillation. For the bottom two traces the phase alignment system is turned on, and one sees the classic first-order step responses that are expected when using dominant-pole compensation.

### B. Compensation experiments

The prototype was designed such that the loop transmission could be varied. Among possible choices of loop compensation, the slow-rolloff network is of particular interest as a demonstration of the ideas developed in section III. Our slow-rolloff network realizes three poles and two zeros, and is shown in figure 9. The component values were chosen such that the dominant-pole compensation and the slow-rolloff compensation result in identical unity gain frequencies.

Figure 10 provides a dramatic comparison of the system under dominant-pole versus slow-rolloff compensation. For this experiment, the phase misalignment of the system is manually set to 90 degrees. The top two traces show

<sup>4</sup>The voltage droop on what is normally the flat part of the square waves is due to the fact that, at the board-level, the inputs have been AC-coupled.

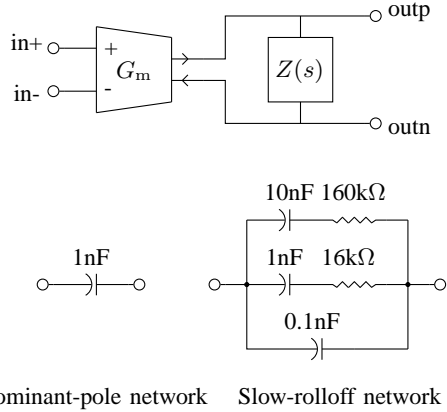


Fig. 9. Adjustable compensation scheme.

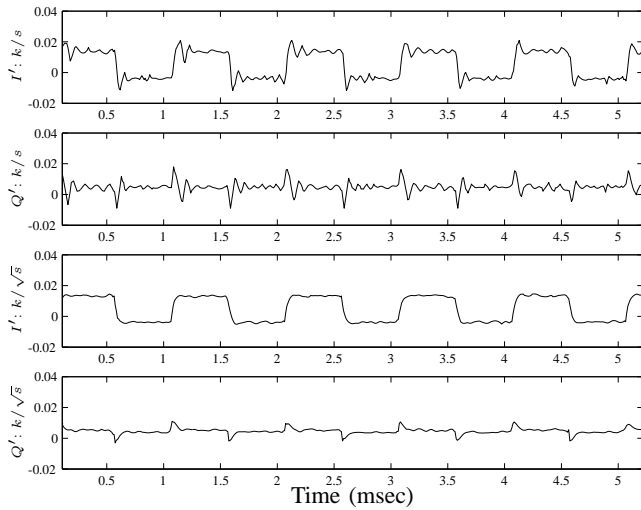


Fig. 10. Step response comparison between dominant-pole and slow-rolloff compensated systems for 90-degree misalignment.

the dominant-pole compensated system under 90-degree

misalignment. Substantial overshoot and ringing is visible, indicative of a lightly damped, complex pole pair. The bottom two traces show the system under slow-rolloff compensation. The step response is remarkably similar to that of a single pole system.

## VI. CONCLUSION

The problem of phase alignment has stood as the primary barrier to the widespread use Cartesian feedback. In this paper we describe a new analysis, and use the resulting insight to design a system that is tolerant to  $\pm 90$  degrees of misalignment. A new phase alignment regulator is also reviewed. Taken together, these results considerably lower the barrier to implementing Cartesian feedback in modern wireless transceivers.

## VII. ACKNOWLEDGEMENTS

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