Proceeding of the 2004 American Control Conference
Boston, Massachusetts June 30 - July 2, 2004

A Robust Coordinated Control of the Doubly-Fed Induction Machine for Wind Turbines: a State-Space Based Approach

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Abstract— **A novel control methodology is proposed for the doubly-fed induction machines used for variable-speed wind-power plants. The machine speed and its reactive power are controlled in a coordinated manner using a linear state-space representation. The regulator is synthesized for the machine model linked to an infinite bus through an external equivalent reactance. The grid to which the machine is connected is thus taken into account in the control model by the infinite bus voltage and the external reactance values. The short-term voltage stability is improved on the wind farm side of the grid connection point especially in case of failure events in the external power system. The controller gains can be easily tuned in order to achieve a performance/robustness trade-off. This regulator is tested in comparison with the vector control approach by dynamic simulations using EUROSTAG software.**

I. INTRODUCTION

The wind energy is one of renewable energy sources which has been intensively developed the last two decades. The wind turbine technology has also continuously been improved; a great amount of new wind farms use variable speed generators with power electronics to optimally operate over a larger wind speed range. If a doubly-fed induction generator (DFIG) is used to operate a wind turbine at variable speed, in addition to the rotor speed control, a terminal voltage control can be achieved (see, *e.g.*, [13]). The more the wind power generation increases in a grid, the more this facility becomes important for the overall transmission grid security. The DFIG machine is one of the technologies which, thanks to the converters between the rotor winding and the grid, enables both speed (torque) and reactive power (voltage) control. If voltage-fed converters are used, the actuator variables are the real and imaginary (or direct and quadratic) components of the rotor excitation voltage so that the system to be controlled has two inputs and two outputs.

Most of the existing DFIG control schemes are based on the representation of the rotor currents in the stator flux reference frame introduced in [11]. Using such a reference frame under the hypothesis of stiff stator voltage a decoupling could be achieved in control: the machine speed could be regulated using only the quadratic component of the rotor current while the reactive power regulation is made with the direct component of the rotor current [1], [3], [7], [9], [15]. This control strategy will be called the *vector control* in what follows.

This paper proposes a new control of the DFIG machine which takes into account the interaction between the speed and the reactive power dynamics. The coordination of these two dynamics is achieved by using the whole linear state-space representation of the machine. A similar representation is used in [5] for the voltage control in the special context of the stand-alone DFIG operation (aircraft embedded generator). In the case treated here, the DFIG machine is connected to other generators via a grid and the effect of this external grid is taken into account in the control model which includes for this purpose an infinite bus connected to the DFIG machine through an equivalent short-circuit reactance. The robustness is improved especially in case of grid failure events which makes this controller appropriate for the grid connection of large wind farms for which transient stability is important (see, *e.g.*, [1]). Moreover, the gains of the linear regulator can be easily tuned in order to ensure closed-loop stability and to settle, if necessary, different dynamics of the mechanical and electrical variables.

The paper is organized as follows: in Section II it is shown how the control model for the machine and the grid is obtained. The coordinated speed/reactive power controller is synthesized and the structure of the resulting control law is analytically compared with the one of the vector control in Section III. Section IV presents a comparison of dynamic simulation results obtained with both controllers, while Section V is devoted to conclusions.

II. CONTROL MODEL

All the model variables are considered in the DFIG nominal power per unit.

A. Nonlinear machine model

The DFIG dynamic behavior is given by the Park equations (see, *e.g.*, [6], [8] and [15]). If the stator dynamic is neglected ($\dot{\Psi}_1 = 0$) and a reference frame attached to the stator flux is chosen, thoses equations written in a generator convention are:

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$$u_{1d} = -r_1 i_{1d} - \omega_{ref}\psi_{1q} \quad (1)$$

$$u_{1q} = -r_1 i_{1q} + \omega_{ref}\psi_{1d} \quad (2)$$

$$u_{2d} = -r_2 i_{2d} + \dot{\Psi}_{2d} - (\omega_{ref} - \omega_R)\psi_{2q} \quad (3)$$

$$u_{2q} = -r_2 i_{2q} + \dot{\Psi}_{2q} + (\omega_{ref} - \omega_R)\psi_{2d} \quad (4)$$

where u is the voltage, i the current, Ψ the flux, r the resistance, ω_{ref} the grid frequency and ω_R the rotor speed; index 1 is used for the stator variables while index 2 denotes the rotor ones. The flux variables are linearly depending on currents:

$$\begin{pmatrix}\psi_{1d}\\ \psi_{2d}\\ \psi_{1q}\\ \psi_{2q}\end{pmatrix} = \begin{pmatrix}-L_m-l_1 & -L_m & 0 & 0\\ -L_m & -L_m-l_2 & 0 & 0\\ 0 & 0 & -L_m-l_1 & -L_m\\ 0 & 0 & -L_m & -L_m-l_2\end{pmatrix}\begin{pmatrix}i_{1d}\\ i_{2d}\\ i_{1q}\\ i_{2q}\end{pmatrix} \quad (5)$$

where l is the leakage inductance and L_m is the mutual inductance. If the rotor current i_2 and the stator flux Ψ_1 are eliminated using equations (5) and a reference frame attached to the initial load-flow (indices R and I denote the real and the imaginary part respectively) is chosen, one can write equations (1)-(4) in the form

$$U_{1R} = -r_1 I_{1R} + \omega_{ref} N I_{1I} - \omega_{ref}\frac{L_m}{L_2}\psi_{2I} \quad (1')$$

$$U_{1I} = -r_1 I_{1I} - \omega_{ref} N I_{1R} + \omega_{ref}\frac{L_m}{L_2}\psi_{2R} \quad (2')$$

$$\dot{\psi}_{2R} = U_{2R} - r_2\frac{L_m}{L_2}I_{1R} - \frac{r_2}{L_2}\psi_{2R} + (\omega_{ref}-\omega_R)\psi_{2I} \quad (3')$$

$$\dot{\psi}_{2I} = U_{2I} - r_2\frac{L_m}{L_2}I_{1I} - \frac{r_2}{L_2}\psi_{2I} - (\omega_{ref}-\omega_R)\psi_{2R} \quad (4')$$

where $L_1=L_m+l_1$, $L_2=L_m+l_2$ and $N=L_1-L_m^2/L_2$. Equations (3'), (4'), along with the machine motion equation

$$\frac{d\omega_R}{dt} = \frac{T_m}{2H} + \frac{L_m}{2HL_2}(\psi_{2I}I_{1R} - \psi_{2R}I_{1I}) \quad (6)$$

where H is the inertia constant and T_m is the mechanical torque provided by the turbine, completely describe the dynamic behavior of the DFIG.

B. Grid connection

The grid to which the DFIG machine is connected is represented here by an infinite bus of fixed voltage (V = 1pu) and by a reactance X_{sc} which can be varied in order to take into account different short-circuit powers of the grid (Fig. 1).

The algebraic equations for the grid connection written in generator convention are[1]:

$$\begin{cases} U_{1R} = V - X_{sc}I_{1I}\\ U_{1I} = X_{sc}I_{1R}\end{cases} \quad (7)$$

(the zero voltage reference angle is considered at the infinite bus).

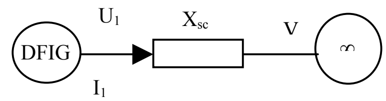

Fig. 1: DFIG connected to an infinite bus model

C. Nonlinear state-space model of the machine connected to the infinite bus

Using (1'), (2') and (7) one can compute I_{1R} and I_{1I} depending on Ψ_{2R}, Ψ_{2I} and the parameter X_{sc}. The resulting expressions can be fed into (3'), (4') and (6) in order to obtain a nonlinear state-space representation which corresponds to the following physical state-space variables

$$x \stackrel{\Delta}{=} [\Psi_{2R} \quad \Psi_{2I} \quad \omega_R]^T \quad (8)$$

and input variables U_{2R}, U_{2I}, T_m and ω_{ref}.

D. Linear control model

Given values for the following variables

$$(P^*_{DFIG}, Q^*_{DFIG}, X^*_{sc}, \omega^*_{ref}) \quad (9)$$

where P_{DFIG} (respectively Q_{DFIG}) is the active (respectively reactive) power generated by the machine, an equilibrium operating point

$$(\Psi^*_{2R}, \Psi^*_{2I}, \omega^*_R, U^*_{2R}, U^*_{2I}, T^*_m, \omega^*_{ref}) \quad (10)$$

can be computed using equations (1'), (2'), (7) and (3'), (4') and (6) in which derivatives are made equal to zero. The linear approximation of equations (3'), (4') and (6) arround (10) can be written

$$\begin{cases}\Delta\dot{x} = A\Delta x + B\underbrace{\begin{bmatrix}\Delta U_{2R}\\ \Delta U_{2I}\end{bmatrix}}_{\Delta u} + E\underbrace{\begin{bmatrix}\Delta T_m\\ \Delta\omega_{ref}\end{bmatrix}}_{\Delta d}\\ \underbrace{\begin{bmatrix}\Delta\omega_R\\ \Delta Q\end{bmatrix}}_{y} = C\Delta x + D\Delta u + G\Delta d\end{cases} \quad (11)$$

where $\Delta\xi = \xi - \xi^*$ denote the deviation of each variable ξ from its equilibrium value ξ^* and matrices A, B, C, D, E and G depend on the values (9).

Remark:

In the following Section the output vector y will be regulated to reference values y_c. In (11) the second variable of the output vector y is the DFIG reactive power Q so that a reactive power control will be implemented. Another possible choice is $y = \begin{bmatrix}\Delta\omega_R\\ \Delta U_1\end{bmatrix}$ where U_1 is the

[1] To simplify this presentation, the current of the grid side converter is neglected. For a rigorous development, this current should be added to I_1 in (7) and in Fig.1.

modulus of the stator voltage and in this case a voltage control could be implemented insted of a reactive power one. The two choices lead almost to the same closed-loop performance. The reactive power control was chosen for this presentation to facilitate the comparison to most of the vector control schemes.

III. CONTROL SYNTHESIS AND ANALYSIS

A. Control objectives and control variables

The DFIG back-to-back voltage source converter applies to the rotor the voltage U_2 of desired magnitude and phase. U_{2R} and U_{2I} are thus the two actuator variables for the DFIG control problem.

A linear feedback of the state x given by (8) which allows one to robustly track constant references for maximum two chosen output varibles (y in (11)) will be presented in Section III.B.

For the purpose of this paper, the regulation will be limited to the DFIG machine variables; only references for the machine speed ω_R and reactive power Q will be thus tracked. The turbine mechanical torque T_m will be treated as a measured output exogenous signal and, for the moment, the control law will reject it along with ω_{ref} as a disturbance (signal d in (11)). The DFIG control law presented here will be extended by a pitch turbine blade control in forthcoming developments.

B. Two inputs/two outputs PI coordinated control law

Let $y_c = \begin{bmatrix} \omega_{R_c} \\ Q_c \end{bmatrix}$ be the vector of the set-points. For y_c to be assymptotically reached, the following *augmented system* has to be stabilized [4]:

$$\begin{cases} \dot{X} = \underbrace{\begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}}_{A_e} X + \underbrace{\begin{bmatrix} B \\ D \end{bmatrix}}_{B_e} U \\ y - y_c = \underbrace{\begin{bmatrix} 0 & I \end{bmatrix}}_{C_e} X \end{cases} \quad (12)$$

where $X \overset{\Delta}{=} \begin{bmatrix} \dot{x} \\ y - y_c \end{bmatrix}$ and $U \overset{\Delta}{=} \dot{u}$. As the pair (A, B) is assumed to be stabilizable, a sufficient condition for the augmented system (12) to be stabilizable is that zero is not an invariant zero of (A, B, C, D) [4]. Let

$$U = -KX \quad (13)$$

be a state-feedback which stabilizes the augmented system (12) and, as a consequence, ensures the asymptotic tracking of the set-points:

$$X \to 0 \;\Rightarrow\; y \to y_c \quad (14)$$

Set K=[k_p k_i]; (13) yields

$$\Delta u = -k_p \Delta x - k_i \int_0^t (y - y_c)dt + \alpha \quad (15)$$

which is a proportional-integral type control law (α is a constant which results from the values of the linearization point (10)).

The control law (15) could be supplemented by a feedforward term of the disturbance d

$$\Delta u = -k_p \Delta x - k_i \int_0^t (y - y_c)dt - k_f \Delta d + \alpha \quad (15')$$

where $k_f = B^{\#}E$ with $B^{\#}$ the pseudo-inverse of B.

The technique for determining the stabilizing matrix gain K is based on linear quadratic (LQ) synthesis in order to get suitable (and, if necessary, different) dynamics for rotor speed and reactive power. The gains obtained in this way are of lower magnitude than the ones of vector control which makes this approach more robust against unmodelled dynamics and measurement noise.

To implement the control law (15)-(15') measurements of the state x are needed. To overcome the problem of rotor flux measurement, x in (15)-(15') could be replaced by its optimal estimation calculated using rotor current measurements (classical Kalman filtering) (see, *e.g.*, [2]).

C. A comparison with the vector control

The coordinated control law (15) can be detailed:

$$\begin{aligned} \Delta U_{2R} &= -k_{p_{11}} \Delta\Psi_{2R} - k_{p_{12}} \Delta\Psi_{2I} - k_{p_{13}} \Delta\omega_R - \\ &\quad - k_{i_{11}} \int_0^t (\omega_R - \omega_{R_c})dt - k_{i_{12}} \int_0^t (Q - Q_c)dt + \alpha_1 \\ \Delta U_{2I} &= -k_{p_{21}} \Delta\Psi_{2R} - k_{p_{22}} \Delta\Psi_{2I} - k_{p_{23}} \Delta\omega_R - \\ &\quad - k_{i_{21}} \int_0^t (\omega_R - \omega_{R_c})dt - k_{i_{22}} \int_0^t (Q - Q_c)dt + \alpha_2 \end{aligned} \quad (16)$$

Thanks to the cross-gain k_{i12} (k_{i21}) the command U_{2R} (U_{2I}) is computed not only for rotor speed (reactive power) but also for reactive power (rotor speed) regulation. Also, the state components Ψ_{2R}, Ψ_{2I} and ω_R are used to compute both real and imaginary command components. This leads to a coordinated two inputs/two outputs control law which takes into account the machine and grid characteristics via the state modelization (8) used for the synthesis.

In all vector control schemes, under some approximations, the rotor speed control is treated independently of the reactive power one. Indeed, if it is assumed that the terminal voltage is constant, thus the stator flux is constant. It follows that the electrical torque becomes proportional to i_{2q} and the rotor speed control can be synthesized in a vector control type approach only using the rotor curent component of axis q

$$i_{2q_{ref}} = -\gamma_{P1}(\omega_R - \omega_{Rc}) - \gamma_{I1}\int_0^t (\omega_R - \omega_{Rc})dt \quad (17)$$

while the direct axis rotor curent component is used exclusevely for the reactive power control:

$$i_{2d_{ref}} = -\gamma_{P2}(Q - Q_c) - \gamma_{I2}\int_0^t (Q - Q_c)dt \quad (18)$$

This assumption simplifies the control design which instead of a two inputs/two outputs control problem is reduced to two problems of one input/one output, but it has consequences when the stator voltage varies as a result of a fault [9]. In this case, in [7] is proposed to increase the proportional gains γ_{P1} and γ_{P2} in order to achieve a better response in case of grid short-circuit but this could amplify the measurement noise and reduce the robustness against unmodelled dynamics. Moreover, as shown in [14], the gains cannot be increased too much because one could thus excite the low frequency mechanical mode. The coordinated control law (15) which takes into account the whole machine model and the grid connection seems to be a good trade-off performance/robustness.

IV. SIMULATION RESULTS

A. Test grid

The proposed control methodology was tested in comparison with the vector control on the case showed in Fig. 2. This represents a typical wind farm connection to a grid represented in a simplified manner (the rest of the grid is modelized by an infinite bus connected to bus N2). The wind farm with DFIG induction generators is represented by an equivalent machine (with its converters) at bus N7. It supplies along with the synchronous generator at bus N1 the load of bus N2.

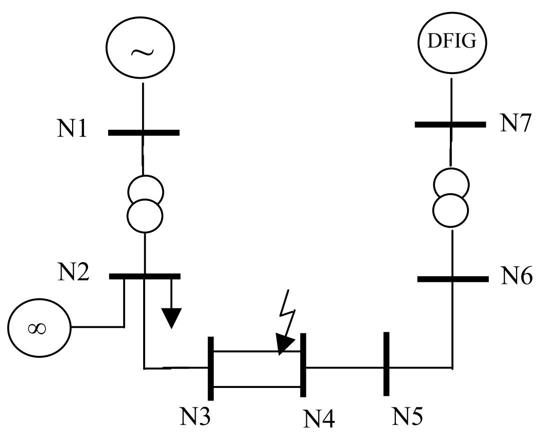

Fig. 2: DFIG connection to a test grid

The DFIG dynamic parameters given in the machine system base S_{base} = 12MVA are: r_1 = 0.00489pu, r_2 = 0.0055pu, l_1 = 0.092pu, l_2 = 0.0992pu, L_m = 3.95pu and H = 3.5s. The voltage levels are as follows: N7 at 0.69kV, N1 at 1kV and the rest of the grid is at 20kV.

B. Test scenario

The initial DFIG rotor speed is ω_{R0} = 1.05pu. The behavior of both closed loops is simulated from time t = 0s to t = 20s using the following script: at time t = 2s, the rotor speed reference is increased by a step of 20% magnitude, *i.e.*, the DFIG has to speed up to 1.26pu. Next, at t = 12s, a direct permanent short-circuit is simulated on one of the two identical lines N3-N4, close to the end N4. This fault is cleared in two phases: firstly, at t = 12.15s the N4 end of the line is opened and, next, at t = 12.7s the other end (N3) of the line is opened so that the line is totally tripped-out. The mechanical torque is constant during all the simulations.

The response is studied for two values of the N4-N5 line reactance. The first case corresponds to the most encountered situation when the DFIG machine is connected to a low equivalent impedance grid (X_{sc} = 10% in Fig. 1 in this case), while in the second one the machine is connected to a grid with a strong equivalent impedance (X_{sc} = 30%) which leads to a severe short-circuit behavior.

C. Simulation results

The scenario described above is tested using EUROSTAG time simulation software for stability studies [12].

Short-term stability

Fig. 3 shows the response of the proposed control (solid lines) and of the vector control (dashed lines) in case X_{sc} = 10%.

As it is shown in the first curve display, the step rotor speed response is rather similar in both cases. However, the perturbation in stator voltage response (second curve display in Fig. 3) due to the step on the speed reference is slightly less important in the case of the optimal state feedback thanks to the coordination between the speed and reactive power (voltage) regulation introduced by the cross-terms in (16). A more important difference can be found between the responses of the two regulators in case of the short-circuit. Indeed, the rotor speed response is improved using the state-feedback in the sense that the overshoot is diminished. Since in this approach the control effort is optimized, the command level (magnitude of U_{2R} and U_{2I}) is less important than the one of the vector control (last two curve displays in Fig. 3). Also, the stator voltage drop is less significatif in this case (v_{1min}=0.53pu) in comparison with the case where the vector control is used and the voltage drops to v_{1min}=0.33pu (second curve display in Fig. 3). This value is, however, not critical as is the case when the DFIG is connected to a high impedance grid. The simulation results of this case are ploted in Fig. 4 where it can be seen on the second curve display that while the short-circuit voltage drop obtained using the proposed approach is almost the same as previous (v_{1min}=0.57pu), the stator voltage drops below usual minimum protection voltage limits when using the vector control. Indeed, v_{1min}=0.15pu in this case, and the machine should normally be disconnected from the grid [9].

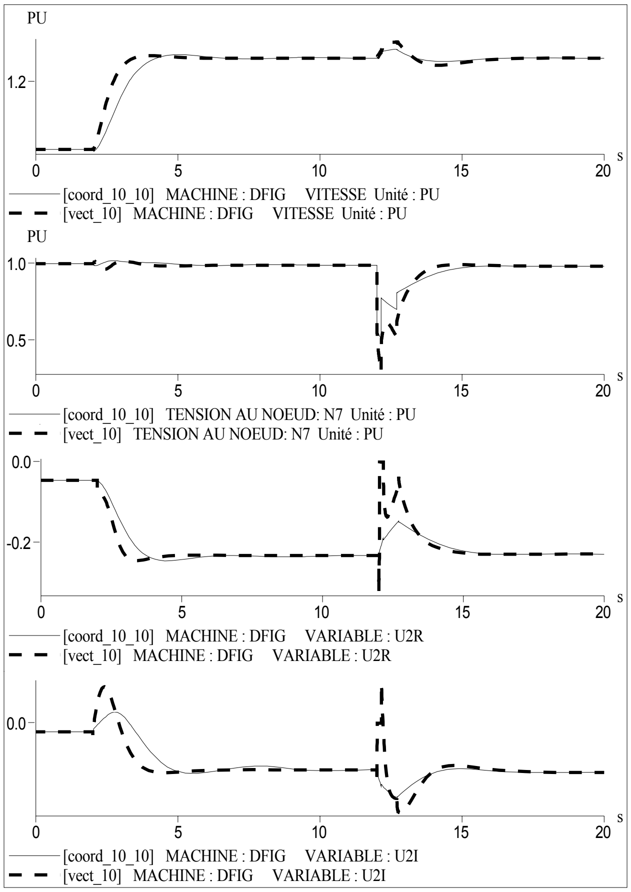

Fig. 3: Response in case of grid connection with X_{sc}=10%

The short-circuit behavior of the proposed control is improved by two factors: first, the controller gains can be adjusted in function of the grid to which the machine is connected since the control law is synthesized using the machine connected to the infinite bus model in Fig. 1. Second, the controller uses all the machine state, *i.e.*, the electrical variables are used along with the mechanical ones in a coordinated manner. This reduces the perturbation induced by the rotor speed regulation onto the voltage dynamics and vice-versa and leads to an improved overall dynamics. Indeed, in Fig. 4, it can be seen that the overshoot on the voltage dynamics due to the step on the speed reference (second curve display in Fig. 4) and the overshoot of the speed response in case of short-circuit (first curve display in Fig. 4) are of less magnitude that the ones obtained when the vector control is used.

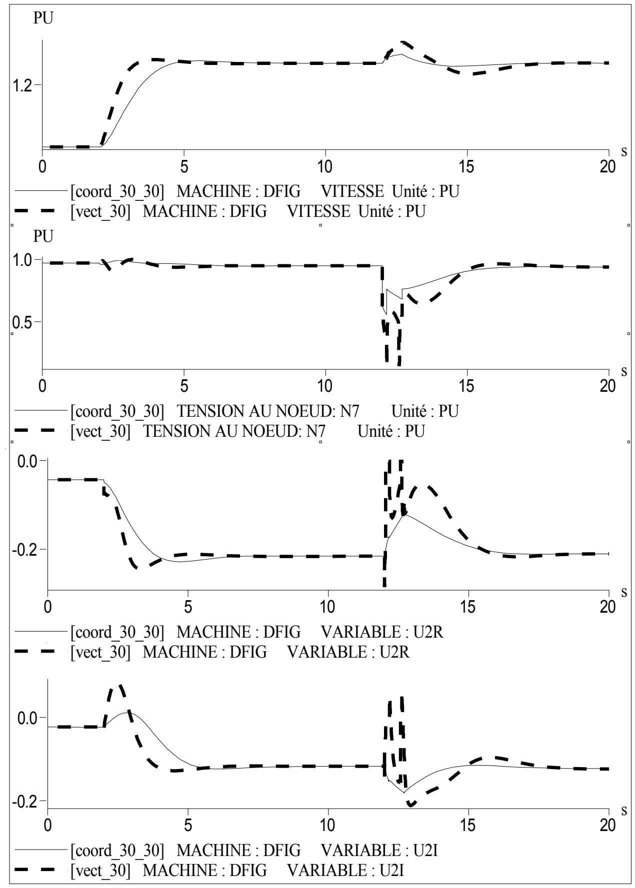

Fig. 4: Response in case of grid connection with X_{sc}=30%

Robustness

As it is shown in Section II, one could tune the regulator gains in function of the grid to which the machine is connected by varying X_{sc} in the grid connection model (7) used in the regulator synthesis. However, since the control law (15) is built in accordance to the Internal Model Principle [4], it is robust for parametric uncertainties. Indeed, the augmented system (12) contains the model of the regulated output (constant references of ω_R and Q) and thus (15) will still regulate the system with good performances in case of not too large model errors (errors on the terms of matrices A, B, C and D). It is why good results are still obtained with control gains of (15) which are not necessarily synthesized with the exact X_{sc} value of the grid. For example, Fig. 5 contains a comparison of the dynamics (speed and stator voltage) obtained on the high impedance grid (X_{sc} = 30%) when the control law (15') is implemented with gains tuned for X_{sc} = 30% (curves in solid lines) with the dynamics obtained when the same control law is used with the gains tuned for X_{sc} = 40% (curves in dashed lines). The very small difference between the two responses proves that the controller is robust enough not to retune the gains each time the grid changes. It has been found that two sets of gains corresponding to X_{sc} = 10% and 30% could cover all the grid connection situations with satisfactory performances.

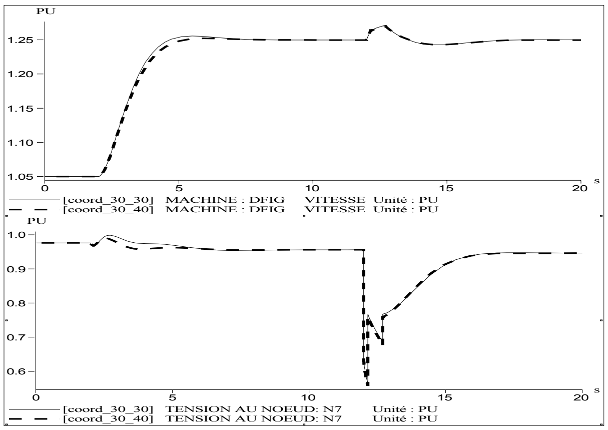

Fig. 5: Parametric robustness

V. CONCLUSION

A novel control approach of the DFIG machine for wind-power plants has been presented. It consists of a coordinated control of rotor speed and reactive power. The control law is a linear state-feedback and it is synthesized using a linearized model of the machine connected to an infinite bus through a variable reactance. The closed-loop behavior is improved especially in case of grid short-circuit and the regulator is proven to be robust for parametric uncertainties, and thus compatible with grids with different equivalent impedances.

The paper focus on the DFIG machine control only and the mechanical torque provided by the turbine has been considered as an exogenous signal. This is usually the case, since the control of the turbine is apart from the DFIG machine control (see, *e.g.*, [15], [10] and related references). However, the framework used here can be extended to also incorporate the turbine control, *i.e.*, the pitch control of the turbine blades.

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