

Original Benchmark for sensorless induction motor drives at low frequencies and validation of high gain observer

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Abstract—An original benchmark for the validation of sensorless induction motor observers is proposed to evaluate them particularly in the well known case where the motor state could be unobservable. Due to the complexity of observation at low frequencies (specifically on our benchmark) we present an improvement of a high gain observer which has been tested and validated on the reference trajectories of this benchmark.

I. INTRODUCTION

For industrial applications, the reduction of the sensors number is an important problem. Indeed, the sensors contribute to increase the complexity of machineries and the cost of the installation (additional wiring and maintenance). In the field of the induction machine control, the most efficient control strategies such as field oriented control and nonlinear control require velocity measurement. Thus the sensorless control (involving an estimation of speed and position) becomes a major subject of concern. Several approaches for the sensorless control of induction machines have been proposed in the literature. Generally, using the induction motor state equations, the flux and speed can be calculated from the stator voltage and current values [5], [8], [12]. A model reference adaptive system (MRAS) [9], [11] is also an alternative method for sensorless induction motor control. In another proposed scheme [5], the flux is obtained by a full order Luenberger observer. In this case, the adaptation law to estimate the speed uses the cross product of the current error vector and the observed flux vector as input. The methods above perform well except at very low speeds, near zero stator frequency [7]. The main difficulty is the observability problem of the induction machine at low frequencies. Indeed, observability problems at low frequency have not often been taken into account in motor control design. A possibility to circumvent the difficulty is to inject high frequency signals in the stator voltage [6]. Nevertheless, few works have addressed this observability problem. In [1] a sufficient condition for lost of observability is that the excitation voltage frequency is zero and the motor is operating at constant speed. From this point of view, the first purpose of this paper is to propose a dedicated benchmark, in which the reference trajectories are defined to drive the motor from high to low frequencies, with the aim to test and validate observers

without mechanical sensors. The second purpose of this paper is to use this benchmark to test a high gain observer which is an improved version of the one presented in [10]. Robustness tests are defined in the setting of the benchmark with given inductance and resistance variations.

This paper is organized as follows: in section 2, the model of induction machine is reminded. The third section presents our benchmark. In the fourth section we derive a high gain observer and report simulation results. Some conclusions are drawn finally.

II. INDUCTION MOTOR MODEL

The equations of the induction motor model can be written using the Concordia and Park transformations [2]. The resulting dynamic equations are given in the rotor flux reference frame (d-q). Applying this transformation, the model of the motor can be described by (1)

$$\begin{pmatrix} \dot{\Omega} \\ \dot{\rho} \\ \dot{i}_d \\ \dot{i}_q \\ \dot{\psi}_d \end{pmatrix} = \begin{pmatrix} \mu\psi_d i_d - \frac{F_v}{J}\Omega \\ p\Omega + \alpha_r \frac{M_{sr}}{\psi_d} i_q \\ -\Upsilon i_d + \alpha_r \beta \psi_d + p\Omega i_q + \alpha_r \frac{M_{sr}}{\psi_d} i_q^2 \\ -\Upsilon i_q - \beta p\Omega \psi_d - p\Omega i_d + \alpha_r \frac{M_{sr}}{\psi_d} i_d i_q \\ -\alpha_r \psi_d + \alpha_r M_{sr} i_d \end{pmatrix} + \begin{pmatrix} 0 & 0 & -\frac{1}{J} \\ 0 & 0 & 0 \\ \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V_d \\ V_q \\ T_l \end{pmatrix} \quad (1)$$

where i_d , i_q , ψ_d , ρ , V_d , V_q , Ω and T_l denote the stator currents, the rotor flux magnitude, the rotor field frame angle (rotor flux direction), the stator voltage components, the angular speed and the torque load, respectively. The subscripts s and r refer to the stator and rotor. R_s and R_r are the stator and rotor resistances. L_s and L_r are the self-inductances, M_{sr} is the mutual inductance between the stator and rotor windings. p is the number of pole-pairs. J is the inertia of the system (motor and load) and f_v is the viscous damping coefficient. Furthermore, we define $\alpha_r = \frac{R_r}{L_r}$, $\alpha_s = \frac{R_s}{L_s}$, $\beta = \frac{M_{sr}}{\sigma L_s L_r}$, $\gamma = \frac{1}{\sigma L_s}$, $\eta = \frac{1}{\sigma}$, $\mu = \frac{p M_{sr}}{J L_r}$, $\sigma = 1 - \frac{M_{sr}^2}{L_s L_r}$, $\Upsilon = (\alpha_s \eta + \alpha_r \beta M_{sr})$. Only stator currents and stator voltages are measured.

III. OBSERVER BENCHMARK

As mentioned in [1], the observability problem of induction motor has been underlined by many authors. In [1], observability issues concerning this problem have been clarified and formally stated. The authors of this paper have

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characterized sufficient conditions leading to observable and unobservable situations. Sufficient conditions of unobservability are that the excitation voltages frequency is zero and that the rotor speed is constant. They have shown that in the particular case where the fluxes are constant or equivalently when the flux angle is constant then $p\Omega + \frac{R_r C_{em}}{p\psi_d^2} = 0$ which defines the unobservability curve (straight line) (Fig. 1).

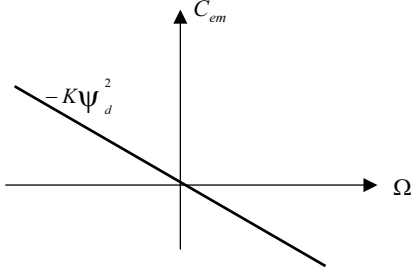


Fig. 1. Unobservability curve in the map (C_{em}, Ω) with $K = \frac{p^2}{R_r}$.

To define a benchmark to test observers on and near the unobservability area, we have defined a scenario (Fig. 2) where the speed Ω and stator pulsation ω_s first starts in such conditions that the motor is observable. Then the pulsation of the stator voltages tends to zero corresponding to constant fluxes (Fig. 3) while the rotor velocity remains constant, making the state unobservable between 4 and 5 seconds and between 6 and 7 seconds. Between 5 and 6 seconds, the rotor moves with a constant acceleration, allowing to check the observer convergence when the state is slightly observable. Finally, the induction motor is driven outside the unobservability curve. In practice, the main difficulty lies in the simultaneous control of speed and stator pulsation so that the slip pulsation $\omega_g = \omega_s - p\omega$ does not exceed a limiting value $\omega_g = R_r M_{sr} i_q / L_r \phi_d$, which corresponds to the highest admissible stator current. The reference slip pulsation is given in Fig. 2.c. In order to respect the above condition, it is necessary to drive the speed of the motor by another connected motor controlled to follow the speed trajectory. At the same time, the frequency of the voltages applied to the stator follows the stator pulsation shown in Fig. 2. This benchmark can be applied on the set-up located at IRCCyN Laboratory [13].

IV. EXTENDED LUENBERGER OBSERVER

Before presenting the high gain observer that we proposed, we show the inherent difficulties at low frequencies of an extended Luenberger observer [3], without speed sensor. This observer is tested on the trajectories of our benchmark. The simulation results that we obtained in the two cases with stator resistance and stator inductances variations respectively are shown in Fig. 4. When the induction motor moves near the unobservability curve, the estimated speed does not follow the reference. The observer presents an important variation and does not manage to converge when the induction motor leaves the unobservability curve. So, the classical observers such as the extended Luenberger

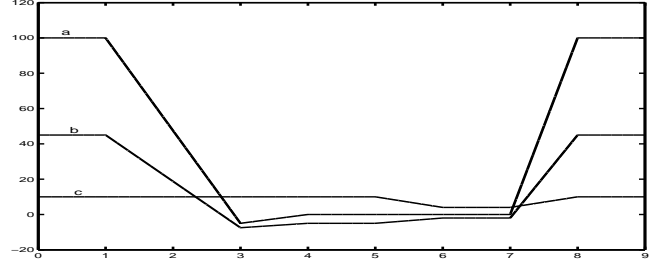


Fig. 2. Observer Benchmark trajectories : a- reference stator voltage pulsation (rd/s), b- reference speed (rd/s), c- reference slip pulsation versus time (s).

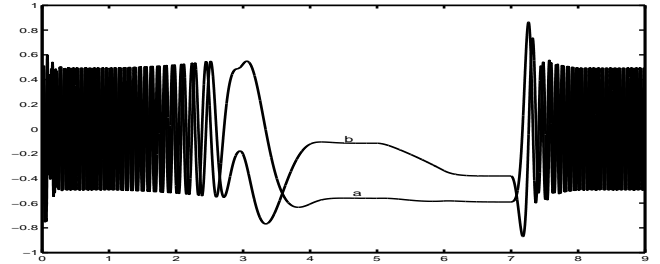


Fig. 3. Components of rotor flux : a- $\Phi_{r\alpha}$, b- $\Phi_{r\beta}$ (Wb) versus time (s).

observer perform well except near zero stator frequency.

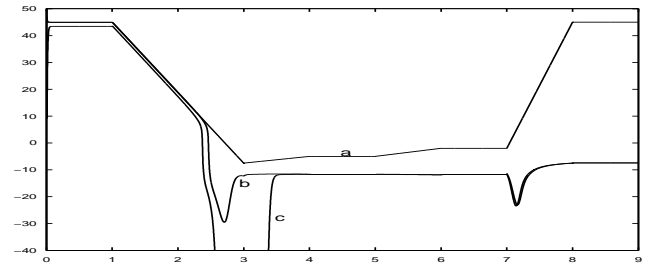


Fig. 4. Observer Benchmark: estimated speed (rd/s): a- reference speed, b- resistance variation +50%, c- induction variation +20% versus time (s).

V. HIGH GAIN OBSERVER

A. Introduction

As introduced in this paper, several observers such as the high gain observer have been developed to estimate rotor speed. High gain observer appears as an important technique for the design of feedback controllers of nonlinear systems. We start to illustrate the main ideas of this technique [10] and thereafter, we expose the observer that we propose and test on our benchmark.

B. High gain observer

Consider the nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \quad (2)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input and $y \in \mathbb{R}$ is the output.

Theorem [4]

Assume that the system (2) is uniformly observable for all input u , then the system (2) is equivalent to the system (3):

$$\dot{\xi} = \begin{pmatrix} \xi_2 \\ \xi_3 \\ \dots \\ \xi_n \\ \Phi(\xi) \end{pmatrix} + \begin{pmatrix} g'_1(\xi_1) \\ g'_2(\xi_1, \xi_2) \\ \dots \\ g'_{n-1}(\xi_1, \dots, \xi_{n-1}) \\ g'_n(\xi) \end{pmatrix} u = f'(\xi) + g'(\xi)u \quad (3)$$

$$y = \xi_1 := C\xi$$

in which the function $g'(x)$ is globally Lipschitzian with respect to x , uniformly with respect to u . Moreover for all input u uniformly bounded, the system (4)

$$\dot{\hat{\xi}} = f'(\hat{\xi}) + g'(\hat{\xi})u + S_\infty^{-1}C^T(y - C\hat{\xi}) \quad (4)$$

with S_∞ solution of (5):

$$0 = -\theta S_\infty - A^T S_\infty - S_\infty A + C^T C \quad (5)$$

where (C, A) is in the canonical form of observability, is an observer of (3), i.e, for all sufficient large θ the estimation error satisfies:

$$\| \hat{\xi}(t) - \xi(t) \| \leq K(\theta) \exp\left(-\frac{\theta t}{3}\right) \| \hat{\xi}_0 - \xi_0 \|.$$

C. Application to induction motor

In [10] a high gain observer is given to estimate the rotor speed. This observer robustly estimates the derivative of the currents. The boundedness of the control protects the state of the plant from peaking phenomenon when the observer estimates are used instead of the true state. We have checked this observer on our benchmark and since the results were not satisfying near the unobservability curve, we have improved this observer to overcome this difficulty and increase its performances.

Using the (d-q) equations of the induction motor model (1) in the (d-q) frame, we follow the design in [10] to write flux, position and currents estimations:

$$\dot{\hat{\rho}} = p\hat{\Omega} + \alpha_r \frac{M_{sr}}{\hat{\psi}_d} \hat{i}_q \quad (6)$$

$$\dot{\hat{i}}_d = -\Upsilon \hat{i}_d + \alpha_r \beta \hat{\psi}_d + p\hat{\Omega} \hat{i}_q + \alpha_r \frac{M_{sr}}{\hat{\psi}_d} \hat{i}_q^2 + \gamma \hat{V}_d \quad (7)$$

$$\dot{\hat{i}}_q = -\Upsilon \hat{i}_q - \beta p \hat{\Omega} \hat{\psi}_d - p\hat{\Omega} \hat{i}_d + \alpha_r \frac{M_{sr}}{\hat{\psi}_d} \hat{i}_d \hat{i}_q + \gamma \hat{V}_q \quad (8)$$

$$\dot{\hat{\psi}}_d = -\alpha_r \hat{\psi}_d + \alpha_r M_{sr} \hat{i}_d \quad (9)$$

where $\hat{\Omega}$ is given by the change of variables using estimated flux rather than itself.

C.1. Estimation of the dq frame angle

The measurements of the motor are given in the classical fixed stator frame (a,b,c). The observer is written in the

frame of the rotating rotor field (d-q). It is thus necessary to carry out a change of reference from the measures. Initial measurements are transformed from the three-phase reference frame to a diphasic reference frame using the Concordia equations:

$$V_\alpha = \sqrt{\frac{2}{3}}(V_a - \frac{1}{2}V_b - \frac{1}{2}V_c)$$

$$V_\beta = \sqrt{\frac{2}{3}}(V_b - V_c)$$

$$i_\alpha = \sqrt{\frac{2}{3}}(i_a - \frac{1}{2}i_b - \frac{1}{2}i_c)$$

$$i_\beta = \sqrt{\frac{2}{3}}(i_b - i_c)$$

The next step consists in passing in the turning reference frame by the Park transformation. This transformation requires the calculation of rotor field frame angle with respect to the fixed reference frame. This calculation is carried out starting from the equations of (10) to (12), just as the calculation of the new measurements to the frame (d-q).

$$\frac{d\hat{\rho}}{dt} = p\hat{\Omega} + \alpha_r \frac{M_{sr}}{\hat{\psi}_d} \hat{i}_q \quad (10)$$

$$\hat{V}_d = \cos(\hat{\rho})V_\alpha + \sin(\hat{\rho})V_\beta \quad (11)$$

$$\hat{V}_q = -\cos(\hat{\rho})V_\alpha + \sin(\hat{\rho})V_\beta \quad (12)$$

C.2. Observer structure

The speed observer of [10] is designed in several stages of calculation:

Step1. First speed estimation

From equation (8), one draws a first equation of speed while taking: $\hat{\Omega} = \hat{\Omega}$:

$$\hat{\Omega} = \frac{-\dot{\hat{i}}_q - \Upsilon \hat{i}_q - \beta p \hat{\Omega} \hat{\psi}_d - p \hat{\Omega} \hat{i}_d + \alpha_r \frac{M_{sr}}{\hat{\psi}_d} \hat{i}_d \hat{i}_q + \gamma \hat{V}_q}{\beta p \hat{\psi}_d - p \hat{i}_d} \quad (13)$$

This requires the calculation of the derivate of i_q which is performed by a high gain observer in the next step.

Step2. High gain observer

Equation (13) requires the derivative of \hat{i}_q . To compute the latter, Strangas [10] uses a high gain observer based on the following state:

$$y_1 = \hat{i}_q, \quad y_2 = \dot{\hat{i}}_q.$$

It can be verified that y_1 and y_2 satisfy the state equations

$$\dot{y}_1 = y_2 \quad (14)$$

$$\dot{y}_2 \stackrel{def}{=} \frac{d^2 \hat{i}_q}{dt^2} \quad (15)$$

Notice that the state equations (14) and (15) are written in

the form (3) (with $n = 2$, $\Phi(y) = 0$ and $u = 0$). Then a high gain observer (4) can be designed

$$\begin{bmatrix} \dot{\hat{y}}_1 \\ \dot{\hat{y}}_2 \end{bmatrix} = \begin{bmatrix} \hat{y}_2 \\ 0 \end{bmatrix} + S_\infty^{-1} C^T (y_1 - \hat{y}_1) \quad (16)$$

with S_∞ is solution of (5), where (C, A) is in the canonical form of observability: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $C = [1 \ 0]$.

By solving (5), we find S_∞ as follows:

$$S_\infty = \begin{bmatrix} \frac{1}{\theta} & -\frac{1}{\theta^2} \\ -\frac{1}{\theta^2} & \frac{2}{\theta^3} \end{bmatrix}$$

then we deduced the gains of the observer (16):

$$S_\infty^{-1} C^T = \begin{bmatrix} 2\theta \\ \theta^2 \end{bmatrix}$$

where θ is positive parameter. By choosing it sufficiently large, one obtains great gains for the observer. This choice can be established by writing the transfer function of this high gain observer as a second order system:

$$\frac{\hat{y}_1(s)}{y_1(s)} = \frac{2\theta s + \theta^2}{s^2 + 2\theta s + \theta^2} \equiv \frac{2\zeta\omega_n s + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where the damping value is 1 and the pulsation value is equal to θ .

Step3. Final estimation of speed

Once the derivative of \hat{i}_q is used to calculate a first estimated speed $\hat{\Omega}$ deduced from (13) by substituting the derivate of \hat{i}_q with \hat{y}_2 .

The first speed estimation $\hat{\Omega}$ is reinjected in a second equation to obtain an improved estimation $\hat{\hat{\Omega}}$.

$$\hat{\hat{\Omega}} = -\frac{\hat{y}_2 + p\hat{\Omega}\hat{i}_d(\alpha_s\eta + \alpha_r\beta M_{sr})\hat{i}_q + \alpha_r \frac{M_{sr}}{\psi_d} \hat{i}_d \hat{i}_q - \gamma \hat{V}_q}{\beta p \hat{\Omega} \hat{\psi}_d}$$

C.3. Observer improvement

When using equation (7) to estimate \hat{i}_d as proposed by Strangas [10], the observer results were very bad due to high frequency oscillations on the benchmark trajectories. So we have developed a high gain observer to estimate this current as follows:

$$\text{Setting } z_1 = \hat{i}_d, \quad z_2 = \dot{\hat{i}}_d.$$

It can be verified that z_1 and z_2 satisfy the state equations

$$\dot{z}_1 = z_2 \quad (17)$$

$$\dot{z}_2 \stackrel{def}{=} \frac{d^2 \hat{i}_d}{dt^2} \quad (18)$$

The state equations (17) and (18) are written in the form (3) (with $n = 2$, $\Phi(z) = 0$ and $u = 0$). As previously a high gain observer can be designed to estimate the current \hat{i}_d and its derivative:

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + 2\theta(z_1 - \hat{z}_1) \\ \dot{\hat{z}}_2 &= \theta^2(z_1 - \hat{z}_1) \end{aligned}$$

where θ is positive parameter which is chosen in the same way that in the case of estimate current \hat{i}_q .

The flux $\hat{\psi}_d$ is deduced from equation (9) and the stator voltage \hat{V}_q is given by (12).

C.4. Simulation results with the proposed benchmark

In this section, the high gain observer we have developed is tested on the Observer Benchmark. The speed of the induction motor is controlled by another connected motor using speed measurement. In Fig. 5 are shown the results obtained in the nominal case, in the case with stator resistance variation of 50% and in the case of stator inductance variation (+20%).

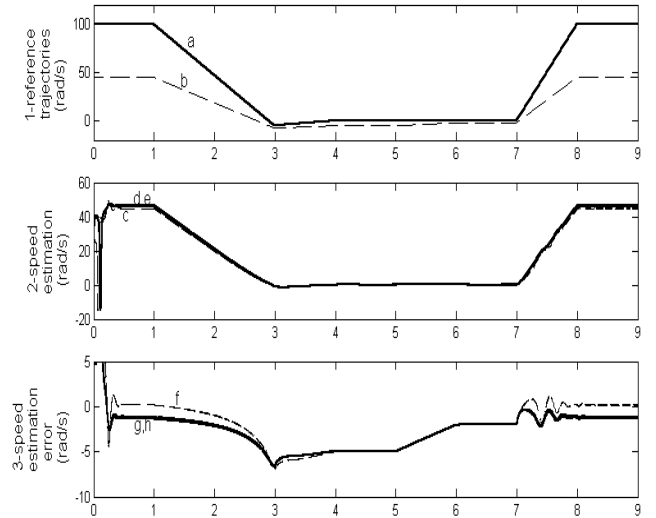


Fig. 5. Observer Benchmark: speed estimation, a- speed, b-stator voltage pulsation; d,h: nominal case; c,f: induction variation +20%; e,g: resistance variation +50% versus time (s).

Speed estimation (Fig. 5)

On Fig. 5 the speed responses for nominal case and cases with stator resistance and stator inductances variations are shown. For both robustness tests cases ($R_s + 50\%$, $L_s + 20\%$), the static error is the same when the observer is near unobservable conditions. After leaving the unobservable area, the static error tends to zero for the case ($L_s + 20\%$)(Fig. 5.3.f) in opposition to the case ($R_s + 50\%$) for which a static error remains (Fig. 5.3.g).

- Stator resistance variation

The speed error is merged with the nominal case (Fig.5.3.g and Fig.5.3.h) even if under unobservable condition. On Fig.5.2.e and Fig.5.3.g, it clearly appears that the 50% variation of stator resistance does not affect the observer stability.

- Stator inductance variation

the augmentation of the stator inductance amplifies peaking effect at beginning time (Fig.5.3.f), but decreases static

error under steady conditions.

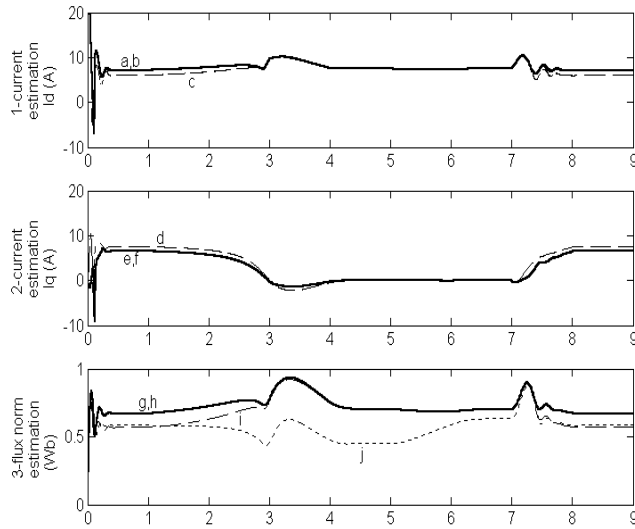


Fig. 6. Observer Benchmark trajectories: i_d, i_q currents and flux observation; j: right value; a,e,g: nominal case; c,d,i: induction variation +20%; b,f,h: resistance variation +50% versus time (s).

Id, Iq currents and flux observation (Fig. 6)

- Variation of stator resistance

In the nominal case and in the case of +50% variation on R_s the current responses are very similar respectively: Fig.6.1.a and Fig.6.3.h, where it appears a static error under unobservable condition. Moreover, in the two cases (nominal and +50% variation on R_s), an error on the observed flux arises even in the observable conditions.

- Variation of stator inductance

There is a small increase at beginning time for currents responses. Thereafter it appears a current error of tracking lower than 5% (Fig.6.1.c and Fig.6.2.d.). On the other hand, the tracking in flux is improved with this positive inductance variation in the observable conditions but the static error remains in the unobservable area (Fig.6.3.i).

By comparing the simulation results obtained with an extended Luenberger observer (see IV), we can remark that the high gain observer is stable near unobservable curve and manages to converge when the induction motor leaves this unobservable area. The main reason for this difference between the behavior of the two observers near and after the unobservable curve lies in their estimation error gains. The high gain observer uses gains which are preliminary fixed. The extended Luenberger observer gains are computed at each iteration of the observer by fast poles placement. When the induction motor moves near zero stator frequency, some components of extended Luenberger observer gains become very large, then the part of feedback due the measure is important and insignificant (unobservable conditions). Thus, we observe bad estimates and the observer diverges (Fig. 4). On the other hand, in the case of high gain observer,

the gains remain fixed throughout the reference trajectory tracking: the estimation is good and the observer is stable near and after unobservable conditions (Fig. 5).

VI. CONCLUSION

An original Benchmark for sensorless induction motor observer validation is proposed. It is specially suited to test observers particularly in the well known case for which the motor could be unobservable. This benchmark evaluates the performances of observers at low frequencies. We have enhanced this fact by Luenberger observer results. Moreover we have improved a high gain observer which has been tested and validated on the reference trajectories of our benchmark. This result is verified by the various robustness tests carried out. However the flux estimation needs improvement. Indeed the estimated speed is used to calculate the rotor frame angle. If an error appears on the latter, the observer is destabilized. To perform the observer, an estimation of the angle can be obtained by using the flux in the fixed reference frame "alpha, beta".

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