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Flatness-based hierarchical control of the PM synchronous motor

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***Abstract*— The paper describes a flatness-based control scheme for the permanent magnet synchronous motor. A hierarchical control is developed and the scheme is shown to be compatible with the flatness properties via a detailed time-scale analysis. The proposed control strategy also achieves copper loss minimization at all operating points. Detailed simulations are presented to illustrate stability and robustness properties of the control scheme.**

***Index Terms*— Permanent magnet synchronous motor control, hierarchical control, differential flatness, robust control**

I. INTRODUCTION

The industrial importance of permanent magnet synchronous motor drives has been increasing for a number of years. This class of motors achieves very good efficiency that equals and sometimes even surpasses the efficiency of induction motor drives which have been dominating the industry for a long time. Permanent magnet machines also maximize torque per unit volume and weight, resulting in numerous applications in vehicles and autonomous systems.

While a number of control schemes have been proposed for permanent magnet drives in the literature, our aim in this paper is to explore a new class of nonlinear control algorithms based on the concept of differential flatness. We show that an appropriate choice of the "direct current" allows to minimize copper losses in every mode of operation. Moreover, we established that the flatness-based control is compatible with standard hierarchical control schemes. The stability of the new control scheme, which involve a load torque observer for on-line re-parameterization of trajectories, is established via a precise singular and regular perturbation study of the tracking error equation, coupled with the application of an advance stability result. Since we are interested in industrial applications, we will also evaluate stability robustness of our scheme, in particular with respect to large parametric perturbations that are typical for high-performance applications.

II. SHORT REVIEW OF DIFFERENTIAL FLATNESS

The *differential flatness* is an important structural property of many control systems [1]. Consider a nonlinear control system given by a state-variable representation:

$$\dot{x} = f(x, e) \tag{1}$$

where $e = (e_1, \ldots, e_m)^\top$ is the input and $x = (x_1, \ldots, x_n)^\top$ is the state. System (1) is said to be *(differentially) flat* if and only if there exists a set of m variables $z = (z_1, \ldots, z_m)^\top$ having the following 3 properties:

1) $z = h(x, e, \ldots, e^{(\alpha)})$;
2) every variable of (1) can be expressed in terms of z and a finite number of its time derivatives, in particular:

$$x = A(z, \dot{z}, \ldots, z^{(\beta)}) \tag{2a}$$
$$e = B(z, \dot{z}, \ldots, z^{(\beta+1)}); \tag{2b}$$

3) the components of z are differentially independent.

Such a set of variables $z = (z_1, \ldots, z_m)$ is called a *flat output* or *linearizing output* of the system (1).

The synthesis of control laws using differential flatness or *flatness-based control* is done in two steps:

a) Design of an open-loop *nominal control* corresponding to the predicted trajectory of the flat output;
b) Application of feedback law in order to stabilize the real trajectory around the predicted trajectory of the flat output.

A complete flatness-based control methodology is presented in [2], [3] with stabilization and robustness results.

III. MODEL OF THE MOTOR IN DQ FRAME AND FLATNESS

Consider the DQ model of a synchronous motor with permanent magnets (see for example [6]):

$$L_d \frac{di_d}{dt} = v_d - R_s i_d + n_p L_q \Omega i_q \tag{3a}$$
$$L_q \frac{di_q}{dt} = v_q - R_s i_q - n_p L_d \Omega i_d - n_p \Phi_f \Omega \tag{3b}$$
$$\dot{\theta} = \Omega \tag{3c}$$
$$J\dot{\Omega} = \underbrace{n_p \left[\Phi_f + \Delta L i_d\right] i_q}_{T_e} - T_L \tag{3d}$$

where notation are usual and $\Delta L = L_d - L_q$ is the difference between the direct and quadrature inductances.

The model (3) of the synchronous motor is *flat* with flat output $z = (\theta, i_d, T_L)$. The proof is straightforward and consists of checking the relevant conditions. The main point is the possibility to express every variable of (3) —namely, Ω, i_d, v_d and v_q— in terms of θ, i_d, T_L and their derivative from algebraic manipulation of the equations (3).

IV. EFFICIENCY OPTIMIZATION

This section is devoted to the study of copper losses (Joule's effect) minimization by the control law. The flatness property allows us to derive a simple and elegant solution. The minimum of copper losses is achieved by the appropriate choice of i_d value. This is a component of the flat output and, consequently, one can achieve a control in order to make this current to track a nominal desired trajectory.

An interesting and useful property of the differentially flat systems is that every variable can be expressed in terms of the flat output components and their derivatives. Therefore, one can

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compute the power dissipated by copper losses P_J in terms of the flat output:

$$P_J = R_s i_d^2 + R_s i_q^2 = R_s \left\{ i_d^2 + \left(\frac{J\ddot{\theta} + T_L}{n_p(\Phi_f + \Delta L i_d)} \right)^2 \right\}$$

To simplify the optimization it is useful to consider P_J as a function of 2 variables only —i_d and T_e— taking into account from (3d) that $J\ddot{\theta} + T_L = T_e$. Consequently, P_J can be considered as a function of i_d and T_e:

$$P_J(i_d, T_e) = R_s \left\{ i_d^2 + \left(\frac{T_e}{n_p(\Phi_f + \Delta L i_d)} \right)^2 \right\} \quad (4)$$

The trajectory $t \mapsto \theta(t)$ will depend on the particular control task or desired motion; the trajectory of the load torque $t \mapsto T_L(t)$ will depend on the operating conditions. Therefore, the trajectory of the electromagnetic torque $t \mapsto T_e(t)$ cannot be freely assigned and depends on the particular dynamic task and operating conditions. The only degree of freedom left is the choice of the trajectory of i_d, which is a component of a flat output. For a given level of torque T_e produced by the motor, the power dissipated by Joule's effect P_J can therefore be considered as a function of i_d only.

A. Non-salient machine ($\Delta L = 0$)

In this case, (4) simplifies into $P_J = R_s \left\{ i_d^2 + \left(\frac{T_e}{n_p \Phi_f} \right)^2 \right\}$ which is obviously minimized for $i_d = i_d^{\text{opt}} \equiv 0$. Notice that our approach allows us to recover the well-known choice $i_d \equiv 0$, often justified in terms of torque efficiency of the motor [6].

B. Salient machine ($\Delta L \neq 0$)

In this case one has generally $\Delta L < 0$ for permanent magnet synchronous motors or $\Delta L > 0$ in the less common case of a wound-field machine operated with constant field voltage.

First notice that P_J is an even function of T_e and therefore the discussion can be focused on $T_e \geq 0$ only. The minimum of P_J can easily obtained by differentiation of the equation (4) with respect to i_d. For notational convenience, introduce $i_{d_o} = -\frac{\Phi_f}{\Delta L}$ which allows to rewrite the expression of the electromagnetic torque and the quadrature currents respectively as: $T_e = n_p \Delta L (i_d - i_{d_o}) i_q$, $i_q = \frac{T_e}{n_p \Delta L (i_d - i_{d_o})}$ Notice that the produced torque is equal to 0 if $i_d = i_{d_o}$. Therefore, we will suppose in everything that follows, that $i_d \neq i_{d_o}$ to avoid the singularity in the expression of i_q. As ΔL is generally small, i_{d_o} can be quite large[1] and in normal operation of the motor the current i_d does not take values in the neighborhood of i_{d_o}. With the new notation (4) becomes:

$$P_J = R_s \left\{ i_d^2 + \left(\frac{T_e}{n_p \Delta L (i_d - i_{d_o})} \right)^2 \right\} \quad (5)$$

For $T_e = 0$ it is obvious, that we have again $i_d = i_d^{\text{opt}} \equiv 0$. For $T_e \neq 0$, one has to differentiate P_J w.r.t. i_d to find its extrema:

$$\frac{\partial P_J}{\partial i_d} = 2R_s \left\{ i_d - \left(\frac{T_e}{n_p \Delta L} \right)^2 \frac{1}{(i_d - i_{d_o})^3} \right\} \quad (6)$$

As long as $i_d \neq i_{d_o}$, $i_d \neq 0$, and $T_e \neq 0$, the equation $\frac{\partial P_J}{\partial i_d} = 0$ is equivalent to:

$$(i_d - i_{d_o})^3 = \left(\frac{T_e}{n_p \Delta L} \right)^2 \frac{1}{i_d} \quad (7)$$

[1] In the case of the parameter set used for the simulation (see values in Table III) $i_{d_o} = 27.8\,\text{A}$ and is quite larger than the nominal current $I_{\text{nom}} = 5.35\,\text{A}$.

For $i_d \mapsto (i_d - i_{d_o})^3 = f(i_d)$ is strictly increasing, and for $i_d \mapsto \left(\frac{T_e}{n_p \Delta L} \right)^2 \frac{1}{i_d} = g(i_d)$ is strictly decreasing on its two intervals of definition, there exists exactly 2 points of intersection between the two graphs, namely i_{d1} and i_{d2}. This mean that P_J has 2 extrema. From the symmetry of f around the point $(i_{d_o}, 0)$ and from the symmetry of g around the point $(0,0)$, it is easy to deduce, whether $\Delta L < 0$ or $\Delta L > 0$, that the inequalities $|i_{d1}| < |i_{d2}|$ and $|i_{d1} - i_{d_o}| > |i_{d2} - i_{d_o}|$ hold. Then, one concludes that $P_J(i_{d1}, T_e) < P_J(i_{d2}, T_e)$ for all $T_e \neq 0$. Consequently the minimum of P_J is reached for $i_d^{\text{opt}}(T_e) = i_{d1}$. The optimal direct axis current can be easily calculated as function of T_e:

$$i_d^{\text{opt}}(T_e) = h(T_e) \quad (8)$$

by numeric resolution of (7). In implementations, the result can be stored in a table that will be used by the controller. Note that if $\Delta L \to 0$ then $i_d^{\text{opt}}(T_e) \to 0$ for every T_e, as expected.

Note that (7) is also equivalent to a polynomial equation of degree 4 in the unknown i_d as long as $i_d \neq 0$ that could be explicitly solved to obtain a closed form expression for h.

V. Hierarchical control

To simplify the design of the control, the model (3) can be considered as a cascade of two simple systems which are coupled to each other: an *electrical subsystem* (or *low level subsystem*) and a *mechanical subsystem* (or *high level subsystem*). The electrical subsystem corresponds to (3a) and (3b).

Following standard arguments about the electrical subsystem being substantially faster, we can use a hierarchical control scheme. It will turn out that such scheme is compatible with the flatness property established above, in particular the high level is flat with the same flat output as the whole system. The idea is then as follows: First, design a closed-loop scheme for the low level, which achieve $i_d \longrightarrow i_d^\sharp$, $i_q \longrightarrow i_q^\sharp$, sufficiently fast with respect to the variations of the desired trajectories $t \mapsto \theta^\sharp$, which will be achieved by the high level subsystem. Second, design the control of the high level subsystem, which can be considered as a system with current inputs (i_d, i_q). This scheme works very well if the variations of the currents dictated by the high level subsystem are slower than the rate of convergence of the current errors in the low level subsystem.

Although i_d and i_q are not physical variables of the plant, we will pursue the development as if there were. Indeed, we assume that the angular position θ and the stator windings currents are measurable. With these measurements, it is easy to calculate the values of i_d and i_q.

A. Low level controller

Low level subsystem:

$$L_d \frac{di_d}{dt} = v_d - R_s i_d + n_p L_q \Omega i_q \quad (9a)$$

$$L_q \frac{di_q}{dt} = v_q - R_s i_q - n_p L_d \Omega i_d - n_p \Phi_f \Omega \quad (9b)$$

This is a system with 3 inputs, v_d and v_q as controls and Ω as disturbance input, and with 2 outputs, i_d and i_q the to-be-controlled currents. In the sequel, the controls of the low level subsystem will be called "*actual control*" as they are the control of the plant.

Structure of the low level controller: The closed-loop low level control typically consists of PI controllers, which can be written, using the notations of operational calculus, as:

$$v_d = -K_{\rm p}^d e_d - \frac{K_{\rm i}^d}{s} e_d + v_d^\flat \qquad (10a)$$

$$v_q = -K_{\rm p}^q e_q - \frac{K_{\rm i}^q}{s} e_q + v_q^\flat \qquad (10b)$$

where $v_d^\flat$ and $v_q^\flat$ are reference input detailed below; e_d and e_q are the direct and quadrature current errors defined by:

$$e_d = i_d - i_d^\flat \qquad (11a)$$

$$e_q = i_q - i_q^\flat \qquad (11b)$$

We propose to use a *feedforward based on reference current ($i_d^\flat$ and $i_q^\flat$) and measured speed* Ω expressed as:

$$v_d^\flat = R_s i_d^\flat - n_p L_q \Omega i_q^\flat \qquad (12a)$$

$$v_q^\flat = R_s i_q^\flat + n_p L_d \Omega i_d^\flat + n_p \Phi_f \Omega \qquad (12b)$$

B. High level controller

High level subsystem: Assuming that the low level controller ensures, that $i_d \longrightarrow i_d^\flat$ and $i_q \longrightarrow i_q^\flat$ sufficiently fast, the mechanical part or high-level subsystem of (3) reads

$$\dot\theta = \Omega \qquad (13a)$$

$$J\dot\Omega = n_p \left[\Phi_f + \Delta L i_d^\flat\right] i_q^\flat - T_L \qquad (13b)$$

and is nothing else than the *reduced model* obtained with $i_q = i_q^\flat$ and $i_d = i_d^\flat$ in (3) under the control defined by (10), (11), (12). Subsystem (13) can be seen as a system with control $(i_d^\flat, i_q^\flat)$ and with disturbance T_L. Note that $(i_d^\flat, i_q^\flat)$ constitutes a "*virtual control*" as it does not correspond to the physical control variables. Subsystem (13) is flat with flat output (θ, i_d, T_L), that is the same flat output than the whole system (3). However, there is only one control objective, either to regulate the position, the speed or even the torque produced by the motor. Therefore, one of the two control channels can be remain unused. The best choice is not to use $i_d^\flat$ in the high level controller. There are two justifications for this choice: First, if we want to minimize copper losses, $i_d^\flat$ must be chosen to be equal to $i_d^\sharp = i_d^{\rm opt}$ from (8). This can be achieved by the low-level controller. Second, if $\Delta L = 0$, there is only one control $i_q^\flat$ in (13), and even if $\Delta L \neq 0$ the system (13) presents a lower level of controllability w.r.t. $i_d^\flat$ than $i_q^\flat$: $\Phi_f \gg |\Delta L|$ implies that in most of the operations, at least in the the subset $-I_{\rm nom} \le i_d, i_q \le I_{\rm nom}$, one has $\left|\frac{\partial T_e}{\partial i_d}\right| \le \left|\frac{\partial T_e}{\partial i_q}\right|$. In summary, $i_d^\flat$ can thus even be thought as a (known) disturbance for the high level subsystem. The high level controller has only to calculate the appropriate value of $i_d^\flat$ corresponding to copper loss minimization.

Structure of the controller: From the flatness property of (13), the nominal control based on differential are:

$$i_d^\flat = i_d^\sharp = h(J\ddot\theta^\sharp + \widehat{T_L}) \quad \text{(see (8))} \qquad (14a)$$

$$i_q^\flat = i_q^\sharp = \frac{J\ddot\theta^\sharp + \widehat{T_L}}{n_p(\Phi_f + \Delta L i_d^\sharp)} \qquad (14b)$$

Following the philosophy of "exact feedforward based on differential flatness" [2], one has to add to $\ddot\theta^\sharp$ a correcting term depending on the position error to close the loop. The position is stabilized around its nominal value by a simple PI controller:

$$i_q^\flat = \frac{J\left[\ddot\theta^\sharp - (K_{\rm P} + \frac{K_{\rm I}}{s})e_\theta\right] + \widehat{T_L}}{n_p(\Phi_f + \Delta L i_d^\sharp)} \qquad (15)$$

where $e_\theta = \theta - \theta^\sharp$. Of course, having the knowledge of the flatness of the complete model it is useful of chose a 3-differentiable reference trajectory $\theta^\sharp$ for θ for the calculation in (8).

C. Load torque and high level observer

We have shown previously the need to have an on-line estimation of the load torque T_L. Consequently, we will design an observer. As we need to estimate the instantaneous value of the load torque, we can make the hypothesis $\dot T_L = 0$. The to-be-observed system can be rewritten as:

$$\dot\theta = \Omega \qquad (16a)$$

$$\dot\Omega = \frac{1}{J}\left[n_p(\Phi_f + \Delta L i_d) i_q - T_L\right] \qquad (16b)$$

$$\dot T_L = 0 \qquad (16c)$$

and a possible observer based on the position measurement is:

$$\dot{\widehat\theta} = \widehat\Omega + \ell_1(\theta - \widehat\theta) \qquad (17a)$$

$$\dot{\widehat\Omega} = \frac{1}{J}\left[n_p(\Phi_f + \Delta L \widehat\imath_d)\widehat\imath_q - \widehat{T_L}\right] + \ell_2(\theta - \widehat\theta) \qquad (17b)$$

$$\dot{\widehat{T_L}} = \ell_3(\theta - \widehat\theta) \qquad (17c)$$

where $\widehat\imath_d = i_d^\flat$ or $\widehat\imath_d = i_d$ and $\widehat\imath_q = i_q^\flat$ or $\widehat\imath_q = i_q$. Denote the observation errors as $\hat e_\theta = \theta - \widehat\theta$, $\hat e_\Omega = \Omega - \widehat\Omega$, and $\hat e_{T_L} = T_L - \widehat{T_L}$:

$$\dot{\hat e}_\theta = -\ell_1 \hat e_\theta + \hat e_\Omega \qquad (18a)$$

$$\dot{\hat e}_\Omega = -\ell_2 \hat e_\theta - \frac{1}{J}\hat e_{T_L} + c(i_d, i_q, \widehat\imath_d, \widehat\imath_q) \qquad (18b)$$

$$\dot{\hat e}_{T_L} = -\ell_3 \hat e_\theta \qquad (18c)$$

With the usual choice, $\widehat\imath_d = i_d$ and $\widehat\imath_q = i_q$, $c(i_d, i_q, \widehat\imath_d, \widehat\imath_q) = 0$ and the observation error equation is linear and homogeneous. However, an interesting choice can be:

$$\begin{cases} \widehat\imath_d = i_d \\ \widehat\imath_q = i_q^\flat \end{cases}$$

which leads to

$$c(i_d, i_q, \widehat\imath_d, \widehat\imath_q) = \frac{n_p}{J}(\Phi_f + \Delta L i_d) e_q$$

As $|\Delta L i_d| \ll \Phi_f$ and $|\Delta L i_d^\flat| \ll \Phi_f$, it is not useful to choose $\widehat\imath_d = i_d^\flat$ since only second order effects will result.

We remark here that it is useful to put some information from trajectories of low-level variables in the high-level observer.

D. Summary of notations

Before to pursue the analysis of the control scheme, it seems useful to recall all notations introduced so far. They are summarized in Table I and Table II. Note that Table I also sets the units of all the signals as they will appear in Fig. 2 to 7 at the end of the paper. The detailed hierarchical controller exposed in the preceding section is summarized on Fig. 1.

VI. Stability analysis

A. Observer

The error equation (18) of the observer can be rewritten as

$$\frac{d}{dt}\begin{pmatrix}\hat e_\theta \\ \hat e_\Omega \\ \hat e_{T_L}\end{pmatrix} = \begin{pmatrix} -\ell_1 & 1 & 0 \\ -\ell_2 & 0 & -1/J \\ -\ell_3 & 0 & 0 \end{pmatrix}\begin{pmatrix}\hat e_\theta \\ \hat e_\Omega \\ \hat e_{T_L}\end{pmatrix} + \begin{pmatrix} 0 \\ n_p(\Phi_f + \Delta L i_d)(i_q - i_q^\flat)/J \\ 0 \end{pmatrix} \qquad (19)$$

TABLE I

NOTATIONS FOR SIGNALS

Name	Variable (actual value)	High level reference or nominal	low level reference	observed or input of obs.	Unit
Position	θ	$\theta^\sharp$	—	$\widehat{\theta}$	rad
Speed	Ω	$\Omega^\sharp$	$\Omega^\flat$	$\widehat{\Omega}$	rad/s
Load torque	T_L	—	—	$\widehat{T_L}$	N m
Direct cur.	i_d	$i_d^\sharp$	$i_d^\flat$	$\widehat{\imath}_d$	A
Quadr. cur.	i_q	$i_q^\sharp$	$i_q^\flat$	$\widehat{\imath}_q$	A
Direct volt.	v_d	$v_d^\sharp$	$v_d^\flat$	—	V
Quadr. volt.	v_q	$v_q^\sharp$	$v_q^\flat$	—	V

TABLE II

DEFINITION OF ERROR SIGNALS

Error	High level	Low level	Observation
Position	$e_\theta = \theta - \theta^\sharp$	—	$\hat{e}_\theta = \theta - \widehat{\theta}$
	$\xi_\theta = \int e_\theta$		
Speed	$e_\Omega = \Omega - \Omega^\sharp$	—	$\hat{e}_\Omega = \Omega - \widehat{\Omega}$
Load torque	—	—	$\hat{e}_{T_L} = T_L - \widehat{T_L}$
Direct cur.	—	—	$e_d = i_d - i_d^\flat$
			$\xi_d = \int e_d$
Quadr. cur.	—	—	$e_q = i_q - i_q^\flat$
			$\xi_q = \int e_q$

This is a linear-time invariant system whose characteristic polynomial is $s^3 + \ell_1 s^2 + \ell_2 s - \ell_3/J$. A simple Routh's table show that one has stability if and only if

$$\ell_1 > 0, \ell_2 + \frac{\ell_3}{J} > 0, \ell_3 < 0$$

Recall that to achieve efficient copper loss minimization, we need a quickly converging observer w.r.t. the dynamics of the other variables. In theory, an appropriate choice of the gains ℓ_1, ℓ_2, and ℓ_3 allows us to tune the observer to be as fast as desired. In practice, the gain of the observer should be limited to avoid saturations.

B. Low level controller

Denote by $\tau_d = \frac{L_d}{R_s}$ and $\tau_q = \frac{L_q}{R_s}$ the stator winding time-constants associated with the direct and the quadrature axis. The time-constant of the direct and quadrature modes can be set approximately to $\epsilon_d \tau_d > 0$ and $\epsilon_q \tau_q > 0$, where $\epsilon_d, \epsilon_q \in \mathbb{R}$ are dimensionless parameters, respectively, by choosing[2]

$$K_\mathrm{p}^d = \frac{2L_d}{\epsilon_d \tau_d} - R_s, \quad K_\mathrm{i}^d = \frac{L_d}{\epsilon_d^2 \tau_d^2} \tag{20a}$$

$$K_\mathrm{p}^q = \frac{2L_q}{\epsilon_q \tau_q} - R_s, \quad K_\mathrm{i}^q = \frac{L_q}{\epsilon_q^2 \tau_q^2} \tag{20b}$$

The low level control can thus be rewritten as:

$$\dot{\xi}_d = \frac{e_d}{\epsilon_d \tau_d} \tag{21a}$$

$$v_d = -(\frac{2L_d}{\epsilon_d \tau_d} - R_s) e_d - \frac{L_d}{\epsilon_d \tau_d} \xi_d \quad R_s i_d^\flat - n_p L_q \Omega i_q^\flat \tag{21b}$$

$$\dot{\xi}_q = \frac{e_q}{\epsilon_q \tau_q} \tag{21c}$$

$$v_q = -(\frac{2L_q}{\epsilon_q \tau_q} - R_s) e_q - \frac{L_q}{\epsilon_q \tau_q} \xi_q \quad R_s i_q^\flat + n_p L_d \Omega i_d^\flat + n_p \Phi_f \Omega \tag{21d}$$

[2]Note that this choice is only made to simplify the stability analysis, but does not mean that the controller is sensitive the the parameter values. Only the order of magnitude is important in this analysis and any other choice with same order of magnitude will permit to draw the same conclusion.

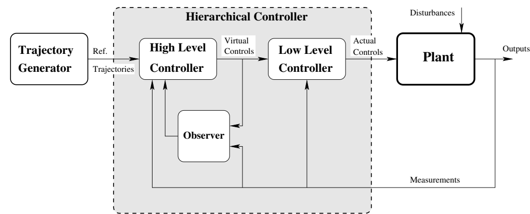

Fig. 1. Detailed hierarchical controller

where ξ_d and ξ_q are the integrals of $\frac{e_d}{\epsilon_d \tau_d}$ and $\frac{e_q}{\epsilon_q \tau_q}$ respectively.

The error equation of (9) together with (21) is:

$$\epsilon_d \tau_d \dot{\xi}_d = e_d \tag{22a}$$

$$\epsilon_d \tau_d \frac{de_d}{dt} = -2e_d - \xi_d + \epsilon_d \tau_d n_p \frac{L_q}{L_d} \Omega e_q - \frac{di_d^\flat}{dt} \tag{22b}$$

$$\epsilon_q \tau_q \dot{\xi}_q = e_q \tag{22c}$$

$$\epsilon_q \tau_q \frac{de_q}{dt} = -2e_q - \xi_q - \epsilon_q \tau_q n_p \frac{L_d}{L_q} \Omega e_d - \frac{di_q^\flat}{dt} \tag{22d}$$

The necessary and sufficient of exponential stability of the homogeneous linear differential equation associated to (22) is satisfied as soon as $\epsilon_d > 0$ and $\epsilon_q > 0$ which is the case.

An advantageous choice of ϵ_d and ϵ_q is such that $\epsilon_d \tau_d$ and $\epsilon_q \tau_q$ achieve a "high gain" feedback, which ensures a quick asymptotic convergence of i_d and i_q to $i_d^\flat$ and $i_q^\flat$ for any bounded Ω. A more detailed analysis can be performed by studying the *boundary layer associated* [5] to (22). A regular perturbation argument on (22) rewritten in fast times $t_d = \frac{t}{\epsilon_d \tau_d}$ and $t_q = \frac{t}{\epsilon_q \tau_q}$ show that for $\epsilon_d \tau_d = 0$ and $\epsilon_q \tau_q = 0$, e_d and e_q become insensitive to Ω.

A coarser analysis shows that the characteristic polynomial of (22) is approximatively the same as the one of

$$\frac{d}{dt}\begin{pmatrix} \xi_d \\ e_d \\ \xi_q \\ e_q \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{\epsilon_d \tau_d} & 0 & \\ -\frac{2}{\epsilon_d \tau_d} & -\frac{1}{\epsilon_d \tau_d} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\epsilon_q \tau_q} \\ 0 & 0 & -\frac{2}{\epsilon_q \tau_q} & -\frac{1}{\epsilon_q \tau_q} \end{pmatrix} \begin{pmatrix} \xi_d \\ e_d \\ \xi_q \\ e_q \end{pmatrix} \tag{23}$$

provided that $|\Omega| \ll \min(\frac{L_d}{n_p L_q \epsilon_d \tau_d}, \frac{L_q}{n_p L_d \epsilon_q \tau_q})$ which can be transformed into upper bounds for ϵ_d and ϵ_q:

$$\epsilon_d \ll \frac{L_d}{n_p L_q \tau_d |\Omega|_{\max}}, \qquad \epsilon_q \ll \frac{L_q}{n_p L_d \tau_q |\Omega|_{\max}} \tag{24}$$

A more general way to study the error equation is to use Kelemen's theorem [4]. This has also the advantage of being applicable in cases where the choice of gains is not as clear as in (20).

C. High-level

The error equation associate to the high level controller is:

$$\dot{\xi}_\theta = e_\theta \quad (25a)$$

$$\dot{e_\theta} = e_\Omega \quad (25b)$$

$$\dot{e_\Omega} = \left(\frac{\Phi_f + \Delta L i_d^\flat}{\Phi_f + \Delta L i_d^\sharp}\right)(-K_\mathrm{D} e_\Omega - K_\mathrm{P} e_\theta - K_\mathrm{I}\xi_\theta) - \frac{\hat{e}_{T_L}}{J} \quad (25c)$$

The stability is easy to establish for bounded and sufficiently slow-varying $i_d^\flat$ and $i_d^\sharp$ using Kelemen's Theorem [4].

VII. Simulations

A. Parameters

TABLE III

Parameters for simulations.

Name	Notation	Value	Unit
Stator resistance	R_s	0.97	Ω
Direct inductance	L_d	5.4	[mH]
Quadrature inductance	L_q	9.0	[mH]
Number of pole pairs	n_p	8	[1]
Flux constant	Φ_f	0.1	[Wb]
Rotor inertia	J	1.1e-3	[kg m^2]
Nominal current	I_nom	5.35	[A]
Nominal power		1480	[W]
Maximal speed		4600	[rpm]
Nominal torque		3.32	[Nm]

The values are shown in Table III. As stated before, the tuning of the low level is done using the two dimensionless parameters ϵ_d and ϵ_q. In order to simplify the tuning of the observer and the high level controller, let us introduce two other dimensionless parameters α and β which the represent the ratio of the time constant of the observer and high level controller w.r.t. the low level time constant τ_d and τ_q.

The characteristic polynomial associated with the observer error dynamic (19) is $s^3 + \ell_1 s^2 + \ell_2 s - \ell_3/J$ and can be identified with $(s + \frac{1}{\tau_\mathrm{obs}})^3$ by an appropriate choice of ℓ_1, ℓ_2 and ℓ_3 and τ_obs is the time constant associated with the observer. The parameter α is defined as $\alpha = \frac{\tau_\mathrm{obs}}{\max(\tau_d,\tau_q)}$ and allow to tune the dynamics of the observer w.r.t. the low level controller.

Under the hypothesis $i_d^\flat = i_d^\sharp$, the high level error dynamics (25a) becomes linear and in this case admits $s^3 + K_\mathrm{D}s^2 + K_\mathrm{P}s + K_\mathrm{I}$ as characteristic polynomial. It is useful to choose the gains K_P, K_I, and K_D in order to identify the latter polynomial with $(s + \frac{1}{\tau_\mathrm{high}})^3$ where τ_high is the time constant associated with the high level controller. The parameter β is thus defined as $\beta = \frac{\tau_\mathrm{high}}{\max(\tau_d,\tau_q)}$. Hence, the complete tuning of the hierarchical control rely only on the four parameters ϵ_d, ϵ_q, α, β.

B. Nominal tuning

Consider the following tuning $\epsilon_d = 0.5$, $\epsilon_q = 0.25$, $\alpha = 5$, $\beta = 10$, that will be our "nominal" case for comparison with other cases. The respective simulations are reported in Fig. 2 and Fig. 3. We remark a good tracking control.

Reducing the values of the tuning parameters allows to obtain a faster response: for example with $\epsilon_d = 0.2$, $\epsilon_q = 0.1$, $\alpha = 3$, $\beta = 6$ it becomes almost impossible to distinguish between the reference and the actual curves. (Not reported here for the sake of conciseness.)

The choice of the 4 parameters allows to exhibit easily wrong behavior of the closed-loop: low level controller too slow w.r.t to variations of low level reference signals, high level controller too fast w.r.t the low level controller, load torque observer too slow. All these cases are not reported here.

C. Robustness of the control law

In this section, we illustrate the properties of robustness of the control law with respect to initial conditions and uncertainties on parameters. See [2] and [3] for a theoretical study of this problems for flatness based control laws. All simulations in this section have been performed using the nominal tuning ($\epsilon_d = 0.5$, $\epsilon_q = 0.25$, $\alpha = 5$, $\beta = 10$).

Robustness w.r.t. position initial condition: Fig. 4 shows the case in which $\theta(0) = -\pi/4$. In the two cases we observe a good behavior of the control law w.r.t. to the uncertainty on initial position. This implies a disturbance on the behavior of the observer during the transient.

Robustness w.r.t. inductance uncertainties: For this simulation we use the nominal values of L_d and L_q in the (low level) controller but 50% of the nominal value in the plant model. These case may be similar to a saturation of the machine (the actual fluxes are less than the value obtained with the nominal values of L_d and L_q). We remark that the low level is not too disturbed and that the high level is almost insensitive to this parameter uncertainty. See Fig. 5 and Fig. 6.

Robustness w.r.t. inertia uncertainties: We suppose here that we have a wrong knowledge of the value of the inertia of the shaft. The nominal inertia is used in the high level control and in the observer while the model is supposed to have 1.5 times the nominal inertia. This implies that the feedforward part of the control law is not powerful enough during the transients to achieve the desired trajectory and this implies more effort from the feedback part of high level controller. However the control scheme is quite able to cope with this kind of uncertainty as illustrated by Fig. 7.

VIII. Conclusion

In this paper we have presented a hierarchical flatness-based control for the permanent magnet synchronous motor. The hierarchical controller mainly relies on physical properties of the model, and is simple to tune with the proposed selection of gains. We also illustrated the stabilization and robustness properties of the control scheme. This class of control schemes can easily be modified for other types of motors that share the flatness property.

IX. References

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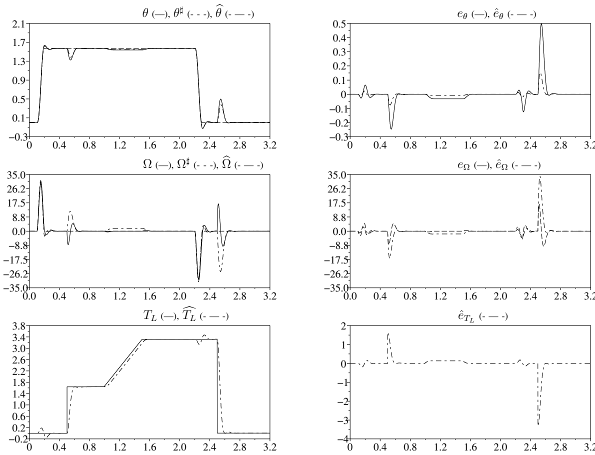

Fig. 2. Nominal control (high level variables).

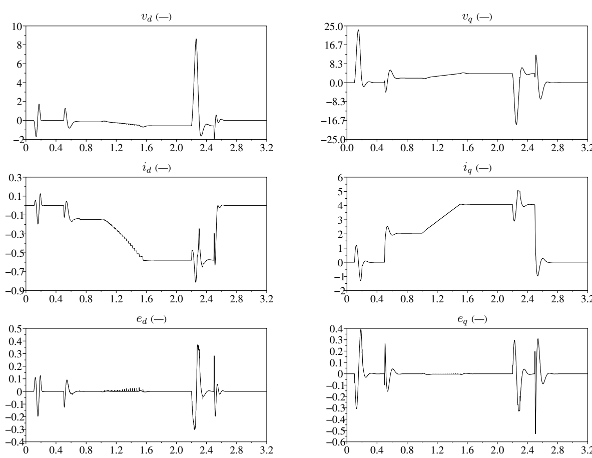

Fig. 3. Nominal control (low level variables).

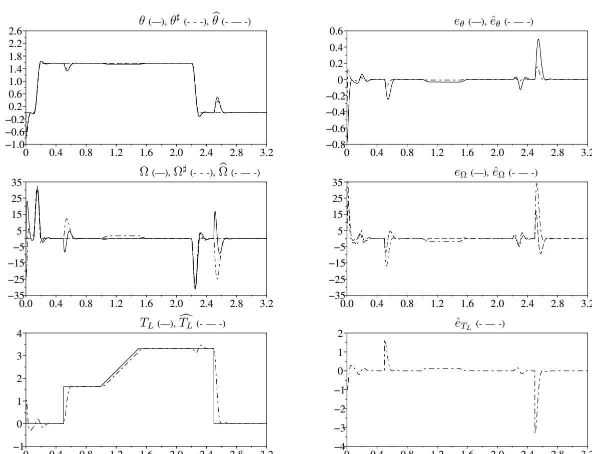

Fig. 4. Uncertainty of $\theta(0)$ (high level variables).

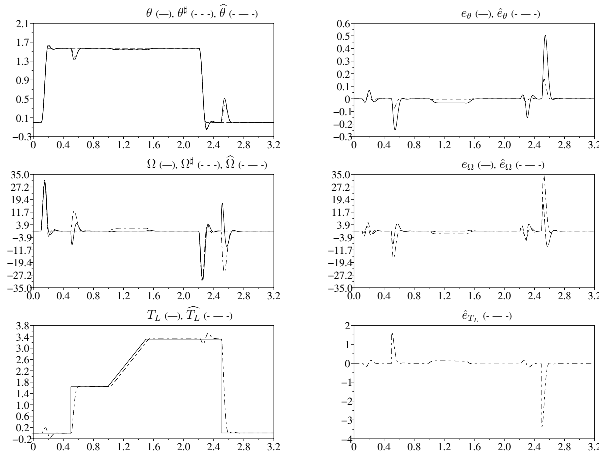

Fig. 5. Uncertainty on inductances (high level variables).

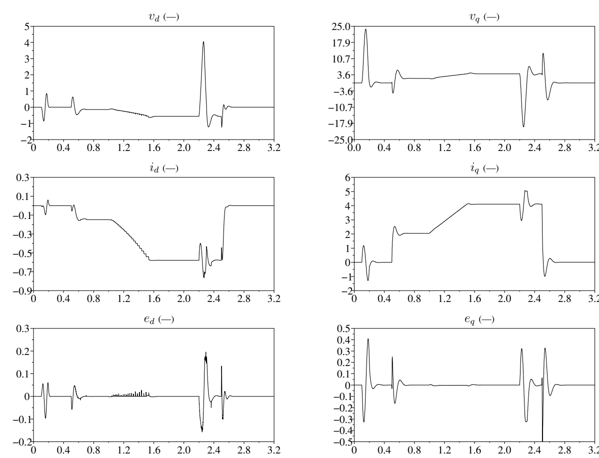

Fig. 6. Uncertainty on inductances (low level variables).

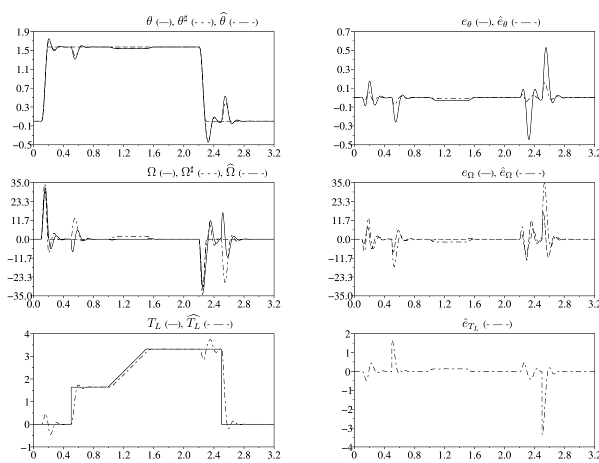

Fig. 7. Uncertainty on inertia (high level variables).