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# Fractional-Order PID Optimal Tuning Rule based on a FOPDT Process Model with Robustness Constraints

**S. Madrigal** [*,**], **O. Arrieta** [*,***], **J.D. Rojas** [**,***], **M. Meneses** [***], **R. Vilanova** [***]

[*] *Instituto de Investigaciones en Ingeniería, Facultad de Ingeniería, Universidad de Costa Rica, 11501-2060 San José, Costa Rica.*
[**] *Departamento de Automática, Escuela de Ingeniería Eléctrica, Universidad de Costa Rica, 11501-2060 San José, Costa Rica.*
[***] *Departament de Telecomunicació i d'Enginyeria de Sistemes, Escola d'Enginyeria Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.*

Abstract: This paper introduces a tuning rule design for fractional order PID controllers. The design is based on a first order plus dead time model for the controlled process. The control problem presented, requires minimizing the integrated absolute error for each control mode, then making a different set of rules for set-point and load-disturbance response. For the proposed optimization, a robustness constraint based on the maximum value of the sensitivity function has been considered so that the tuned controllers have an optimal performance and guarantee the stability of the control system at the time of implementation. By performing a curve fitting process towards the optimal controller parameters, the *Model based Robust Tuning* rule was designed and its improvement over other tuning methods is validated through examples.

*Keywords:* Tuning Rules, Robust Control, Fractional Control, FOPID, FOPDT.

## 1. INTRODUCTION

Currently, the PID control algorithm is extensively applied in industrial environments due to its versatility, which makes it a remarkable option due to the number of different tuning methods Åström and Hägglund (2006).
However, in recent years, there has been a significant surge in the study of fractional calculus and its applications. One area of interest is the use of fractional control, specifically fractional order controllers Babu and Chiranjeevi (2016). This topic has garnered increasing attention with published works analyzing the benefits of implementing fractional order PID controllers (FOPID) compared to using the traditional PID controller Vinagre et al. (2007). Recent research has demonstrated the efficacy of utilizing a fractional order model to describe the dynamics of the controlled process Meneses et al. (2018). It has also been analyzed how fractional control can be applied in a way that ensures system stability and robustness Padula and Visioli (2015).
In this context, tuning rules have been developed for FOPID controllers to guarantee a certain level of robustness while obtaining the controller parameters. For example, Sanchez et al. (2017) uses a multi-objective optimization to establish the tuning rules *MsRange* and *MsValue*. Meanwhile, Padula and Visioli (2011, 2012) employ fractional order controllers for integrating, unstable processes, processes modeled by a first-order function plus dead time and create specific tuning rules (P&V rules).

corresponding author: orlando.arrieta@ucr.ac.cr

The *Model based Robust Tuning* (M-RoT) rule, for FOPID controllers, is proposed in this work, which is based on a first order plus dead time (FOPDT) model of the controlled process, this rule is proposed to separate the regulatory-control mode from the servo-control, so that optimal controllers can be found for both cases, additionally it works to achieve different levels of target robustness, in total four different levels are considered for the implementation of the rule. In all cases, in order to make the M-RoT rule easy to use, the aim was to use as few constants as possible for the curve fitting process. The results obtained are validated by comparing the proposed rule with other rules that have been trending for several years in the implementation of fractional order controllers.
The paper is organized in the following sections. Section 2 presents a comprehensive description of the control system, including the process model, the FOPID controllers to be tuned, and the performance and robustness evaluation indices of the system. Section 3 describes the development of the M-RoT rule and presents the results of the rule design and process validation. In Section 4, the comparison of existing rules for FOPID controllers with the proposed M-RoT rule is provided. Finally, Section 5 presents the conclusions of the research.

## 2. PROBLEM FORMULATION

### 2.1 Control System Considered

The main goal of a control system is to achieve the desired output of a controlled process with a given input. The suggested closed-loop control system is the traditional one presented in Fig. (1), where $P(s)$ represents the controlled process model and $C(s)$ is the fractional order PID controller (FOPID) that can be tuned.

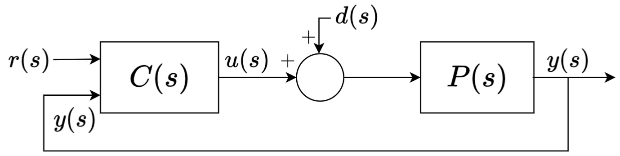


Figure 1. Close-loop control system considered.

For the previously described control system, the following variables were identified: $y(s)$ is the output of the process, $r(s)$ designates the set-point signal for the process output, $u(s)$ represents the output of the tuned controller, and finally $d(s)$ is the load-disturbance signal of the system.

The controlled variable, which is the process output, is described by the input signals $r(s)$ and $d(s)$, as shown:

$$y(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} r(s) + \frac{P(s)}{1 + C(s)P(s)} d(s) \quad (1)$$

This dependence defines the mode of operation for the control system; in the case of servo control, the main objective is that the process output efficiently achieves the set-point tracking task, while in the case of regulatory control, it is necessary to mitigate the effect of disturbances on the controlled variable. The tuning rule is proposed to obtain FOPID controllers which can achieve the two objectives.

### *2.2 Controlled Process Model*

The controlled process $P(s)$ can be modelled by using a first-order-plus-dead-time (FOPDT) transfer function. This model is frequently employed in industry to model process dynamics Visioli (2006).

$$P(s) = \frac{K}{Ts + 1} e^{-Ls} \quad (2)$$

This model includes three parameters: $K$ for the process gain, $T$ for the time constant and $L$ the dead time of the system. By utilizing these parameters, the controlled process dynamics can be represented, allowing for the design of a system controller.

It is possible to normalize the model by means of a transformation $\hat{s} = Ts$, based on the normalized dead time given by (3). This allows for consideration of cases where the system is dominated either by the time constant or the opposite case where the dead time dominates the dynamics of the system.

$$\tau = \frac{L}{T}, \ \tau \in [0.1, 2] \quad (3)$$

### *2.3 Fractional Order PID Controllers*

For this paper, a FOPID controller in standard configuration is considered as a tuned controller, this controller has the form shown below Padula and Visioli (2015):

$$C(s) = K_p \left( 1 + \frac{1}{T_i s^{\lambda}} + \frac{T_d s^{\mu}}{\frac{T_d}{\nu} s + 1} \right) \quad (4)$$

where $K_p$ represents the proportional gain, $T_i$ represents the integral time, $T_d$ represents the derivative time constant, $\nu$ represents the derivative filter parameter, while $\lambda$ and $\mu$ constitute the non-integer parameters of the FOPID controller for the integral and derivative action, respectively.

The expression (5) provides the parameter for the derivative filter, where $\mu$ represents the fractional order of the controller.

$$\nu = 10 T_d^{\frac{\mu - 1}{\mu}} \quad (5)$$

To implement the FOPID controller, it is essential to utilize an integer approximation (Oustaloup et al. (2000)). This recursive approximation is performed on a product of poles and zeros, as shown in the following expression:

$$s^{\mu}_{[\omega l, \omega h]} \cong C_o \prod_{k=1}^{N} \frac{1 + \frac{s}{\omega_{z,k}}}{1 + \frac{s}{\omega_{p,k}}}, \ \mu > 0 \quad (6)$$

When applying this approximation, the valid frequency range is indicated by $\{\omega l, \omega h\} = \{0.001, 1000\}$, on the other hand, the constant $C_o$ is chosen in such a way that the approximation has a unity gain at the cutoff frequency, and in this case an approximation with $N = 8$ is used, this parameter defines the number of poles and zeros for the real-rational transfer function that will approximate the derivative fractional term.

### *2.4 Peformance and Robustness*

As introduced initially, the objective is to develop a tuning rule that seeks an optimal FOPID controller in terms of performance under a given robustness constraint in both servo and regulatory control modes.

With this purpose, we introduce an index to measure the performance acquired by the system after the implementation of the tuned controller, the Integrated Absolute Error (IAE), defined as follows:

$$J = IAE = \int_0^{\infty} |e(t)| dt = \int_0^{\infty} |r(t) - y(t)| dt \quad (7)$$

This index will be transformed into the cost function after conducting the optimization process, taking into account each control mode separately as appropriate ($J_{sp}$ with $d(t) = 0$ or $J_{ld}$ with $r(t) = 0$).

To ensure that the tuned controller implementation can handle process non-linearities in reality, it is recommended to evaluate a robustness index based on the sensitivity function. The index, defined in (8), indicates how robust the tuned controller is.

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C(j\omega)P(j\omega)|} \quad (8)$$

This metric indicates the relative stability of the system and typically falls in the range of $M_s = \{1.4, 2.0\}$. A value of 1.4 denotes robust tuning, while a value of 2.0 indicates more aggressive tuning. The metric will serve as a constraint during the optimization process. In this work, four levels of robustness are considered in order to present different options for tuning:

$$M_s = \{1.4, 1.6, 1.8, 2.0\} \quad (9)$$

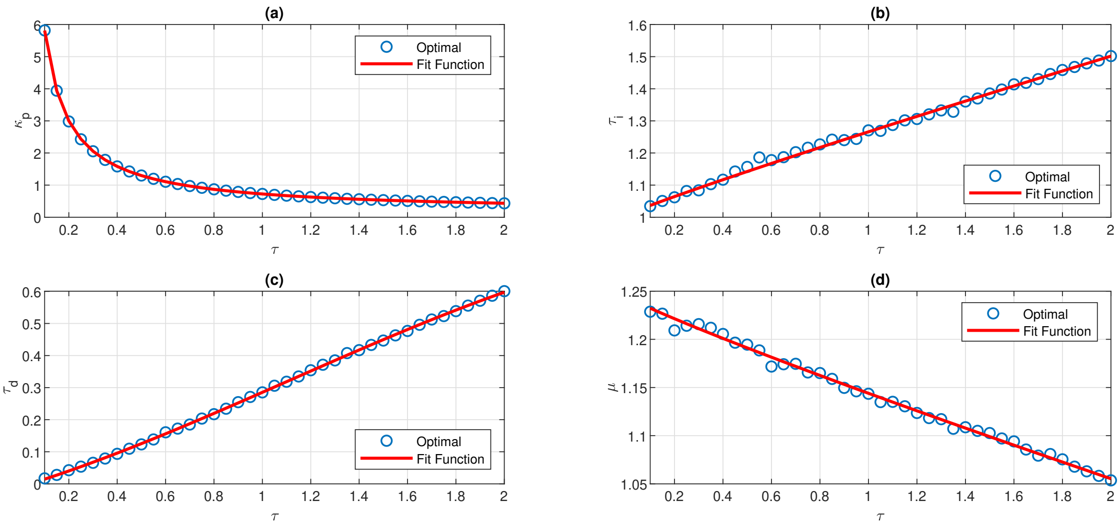


Figure 2. Tuning parameters for the M-RoT rule FOPID controller with target robustness $M_s = 1.4$.

## 3. TUNING RULE DESIGN

The design process of the rule starts with the search of the optimal parameters of the FOPID controller, this procedure is carried out through optimizations using the MATLAB® *fmincon* solver, where the $J_{sp}$ or $J_{ld}$ index is minimized according to the case and considering the robustness constraint of the target level.

Table 1. Load-Disturbance rejection FOPID tuning $M_s = \{1.4, 1.6, 1.8\}$.

| | $K_p$ | $T_i$ | $T_d$ | $\mu$ |
|---|---|---|---|---|
| Target robustness $M_s = 1.4$ $0.1 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.5831 | 4.6390 | −0.0193 | −0.0540 |
| $b$ | −0.9512 | 0.0842 | 0.0556 | 1.4121 |
| $c$ | 0.1191 | −3.5150 | 0.2870 | 1.1612 |
| $d$ | - | - | −0.0019 | - |
| Target robustness $M_s = 1.6$ $0.2 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.7173 | 2.2233 | −0.0077 | −0.0315 |
| $b$ | −0.9978 | 0.2536 | 0.0036 | 1.2507 |
| $c$ | 0.2384 | −0.9993 | 0.2925 | 1.1751 |
| $d$ | - | - | −0.0055 | - |
| Target robustness $M_s = 1.8$ $0.2 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.8768 | 1.9658 | −0.0050 | −0.0324 |
| $b$ | −0.9934 | 0.3276 | −0.0042 | 1.2812 |
| $c$ | 0.2774 | −0.6829 | 0.2906 | 1.1850 |
| $d$ | - | - | −0.0103 | - |

After finding the set of optimal parameters, the next procedure is the curve fitting, the intention with this process is to find a fitting function that describes the trend of each parameter of the controller, in order to find an approximation of the optimal value as a function of the normalized dead time, this procedure is performed with the cftool toolbox searching to a fit function with the minimum Sum Square Error (SSE) respect to the optimal parameters.

Finally, a validation of the fitted parameters determined by the proposed functions is performed to ensure that the imposed robustness constraint is achieved over the entire range of normalized dead time values.

### *3.1 M-RoT rule target robustness $M_s = \{1.4, 1.6, 1.8\}$*

*Set-Point and Load-Disturbance tuning:* For M-RoT rule in the $M_s = \{1.4, 1.6, 1.8\}$ target levels, the set of tuned parameters is described by the following equations for normalized proportional gain, normalized derivative time, derivative integral time, integral order and derivative order.

$$K_p K = \kappa_p \quad \frac{T_d}{T^{\mu}} = \tau_d \quad \frac{T_i}{T^{\lambda}} = \tau_i \tag{10}$$

$$\kappa_p = a\tau^b + c \tag{11}$$
$$\tau_i = a\tau^b + c \tag{12}$$
$$\tau_d = a\tau^3 + b\tau^2 + c\tau + d \tag{13}$$
$$\mu = a\tau^b + c \tag{14}$$
$$\lambda = 1 \tag{15}$$

Tables (1) and (2) provide the values of constants $a$, $b$, $c$, and $d$ for the tuning rule in the both modes of control.

Table 2. Set-Point tracking task FOPID tuning $M_s = \{1.4, 1.6, 1.8\}$.

| | $K_p$ | $T_i$ | $T_d$ | $\mu$ |
|---|---|---|---|---|
| Target robustness $M_s = 1.4$ $0.1 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.5818 | 0.2596 | −0.0277 | −0.1006 |
| $b$ | −0.9889 | 0.9322 | 0.0918 | 0.9120 |
| $c$ | 0.1421 | 1.0063 | 0.2306 | 1.2446 |
| $d$ | - | - | −0.0094 | - |
| Target robustness $M_s = 1.6$ $0.2 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.7558 | 0.4082 | −0.0075 | −0.0498 |
| $b$ | −0.9735 | 0.8719 | 0.0037 | 0.9063 |
| $c$ | 0.2172 | 0.9961 | 0.2860 | 1.2031 |
| $d$ | - | - | −0.0116 | - |
| Target robustness $M_s = 1.8$ $0.2 \leq \tau \leq 2.0$ | | | | |
| $a$ | 0.9015 | 0.4704 | −0.0078 | −0.0257 |
| $b$ | −0.9115 | 0.8264 | −0.0030 | 1.5405 |
| $c$ | 0.2224 | 0.9654 | 0.3128 | 1.1538 |
| $d$ | - | - | −0.0037 | - |

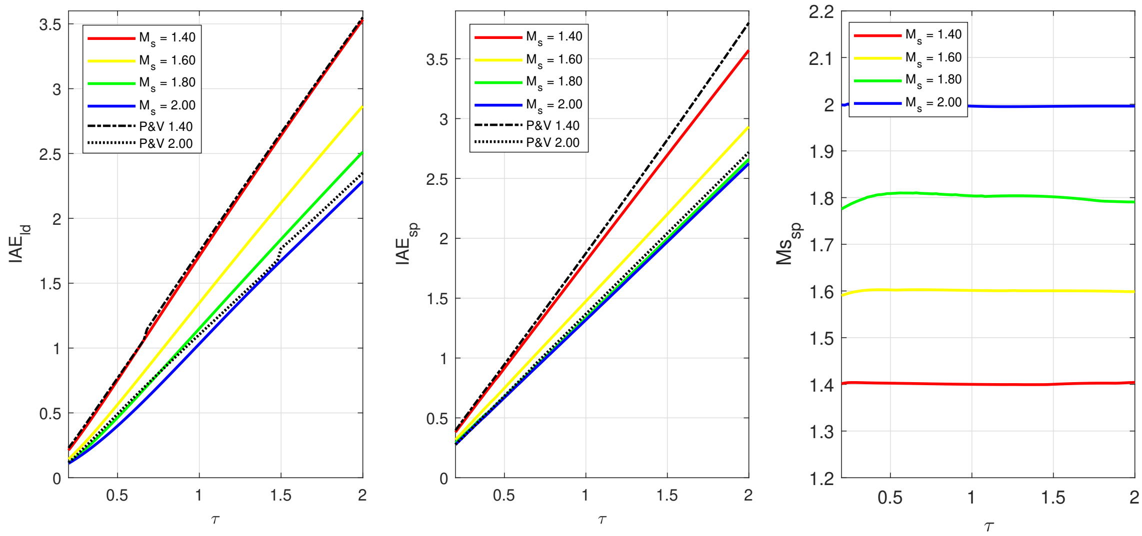


Figure 3. Performance and robustness indexes for a normalized FOPDT process model for the M-RoT and P&V rules.

### *3.2 M-RoT rule with target robustness $M_s = 2.0$*

For the final instance of the M-RoT rule, the desired level of robustness is considered to be $M_s = 2.0$, resulting in the following set of fitting functions.

*Set-Point tuning:* In this control mode, the same functions (11)-(15), are used for the $M_s = 2.0$ case, as in the previous tuning.

Table 3. Set-Point tracking FOPID tuning with target robustness $M_s = 2.0$.

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| | Normalized dead-time range $0.1 \leq \tau \leq 0.2$ | | | |
| $K_p$ | $-142.2470$ | 2.4068 | 7.5521 | - |
| $T_d$ | $-10.1304$ | 6.1024 | $-0.8391$ | 0.0644 |
| $\mu$ | $2.1660e^{-4}$ | $-2.2738$ | 1.1559 | - |
| | $0.2 < \tau \leq 0.4$ | | | |
| $K_p$ | 0.7505 | $-1.0606$ | 0.4559 | - |
| $T_d$ | 2.5364 | $-2.1958$ | 0.9569 | $-0.0641$ |
| $\mu$ | $-0.0370$ | 1.2102 | 1.1667 | - |
| | $0.4 < \tau \leq 2$ | | | |
| $K_p$ | 0.9001 | $-0.9509$ | 0.2866 | - |
| $T_d$ | 0.0016 | $-0.0388$ | 0.3681 | $-0.0114$ |
| $\mu$ | $-0.0370$ | 1.2102 | 1.1667 | - |
| | $0.1 \leq \tau \leq 2.0$ | | | |
| $T_i$ | 0.4608 | 0.8826 | 0.9828 | - |

*Load-Disturbance tuning:* For this particular mode, the normalized proportional gain fit function transforms into a rational function type given by the following equation:

$$\kappa_p = \frac{a\tau^2 + b\tau + c}{\tau + d} \tag{16}$$

However, the other four normalized parameters of the FOPID controller are also tuned utilizing the same tuning functions as in the previous case.

In all cases, we validated the tuned parameters to find the optimal fit for each specific trend, as illustrated in Fig. (2), to finally determine all cases of the M-RoT rule, validating the performance index obtained and the robustness of the normalized system for the entire range of normalized dead time worked, as shown in Fig. (3).

Table 4. Load-Disturbance rejection FOPID Tuning with target robustness $M_s = 2.0$.

| | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|
| | Normalized dead-time range $0.1 \leq \tau \leq 0.4$ | | | |
| $T_d$ | $-3.1504$ | 2.9868 | $-0.6132$ | 0.0694 |
| | $0.4 < \tau \leq 2.0$ | | | |
| $T_d$ | $-0.0054$ | $9.8542e^{-4}$ | 0.3025 | $-0.0215$ |
| | $0.1 \leq \tau \leq 2.0$ | | | |
| $K_p$ | 0.1784 | $-0.2455$ | 1.5107 | 0.1155 |
| $T_i$ | 2.0947 | 0.3136 | $-0.7890$ | - |
| $\mu$ | $-0.0418$ | 1.3629 | 1.1884 | - |

## 4. EXAMPLES

### *4.1 Example 1*

As an initial case study to evaluate the performance of the M-RoT rule, the FOPDT process model given by (17) is proposed.

$$P_1(s) = \frac{1}{s+1} e^{-0.67s} \tag{17}$$

The model parameters are $K = 1$, $T = 1$, and $L = 0.67$, giving a normalized dead time, $\tau = 0.67$, in which the M-RoT rule can be applied and the tuned FOPID controllers can be obtained.

In this case, a comparison is made with the MsValue rule proposed in Sanchez et al. (2017) and with the rule proposed by Padula and Visioli (2011), in both cases there are target robustness values of $M_s = \{1.4, 2.0\}$, for fractional order controllers.

MsValue performs the rule by applying a multi-objective optimization without separating the servo and regulatory control modes; on the other hand, the Padula-Visioli rule (P&V rule) considers a specific optimization for both servo-regulatory control modes.

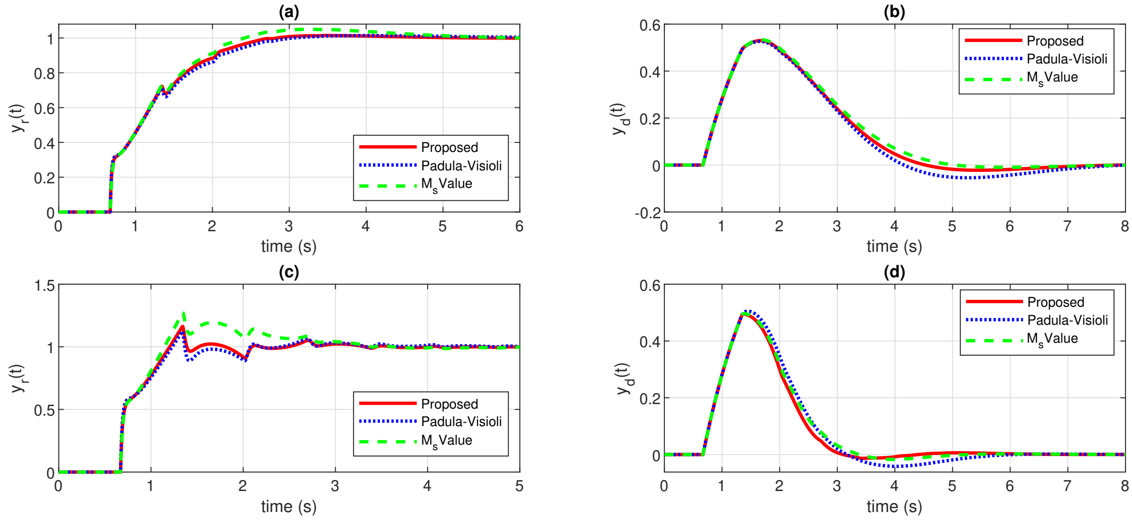


Figure 4. System response shown in both control modes for Example 1. Figures (a) and (b) represent the $M_s = 1.40$ target tuning, while Figures (c) and (d) represent the $M_s = 2.00$ target case.

Table 5. Results in regard to Example 1.

| Tuning Rule | $K_p$ | $T_i$ | $T_d$ | $\mu$ | $\lambda$ | $J_{sp}$ | $J_{ld}$ | $M_s$ |
|---|---|---|---|---|---|---|---|---|
| MsValue 1.4 | 0.97 | 1.04 | 0.20 | 1.12 | 1 | 1.26 | 1.10 | 1.40 |
| MsValue 2.0 | 1.67 | 1.02 | 0.21 | 1.11 | 1 | 1.05 | 0.64 | 1.97 |
| P&V SP 1.4 | 0.83 | 0.98 | 0.22 | 1.20 | 1 | 1.26 | 1.20 | 1.43 |
| P&V SP 2.0 | 1.26 | 1.03 | 0.28 | 1.20 | 1 | 0.92 | 0.82 | 2.17 |
| P&V LD 1.4 | 0.61 | 0.54 | 0.33 | 1.20 | 1 | 1.38 | 1.12 | 1.44 |
| P&V LD 2.0 | 0.91 | 0.52 | 0.38 | 1.10 | 1 | 1.15 | 0.70 | 1.95 |
| M-RoT SP 1.4 | 1.00 | 1.18 | 0.18 | 1.17 | 1 | 1.22 | 1.17 | 1.40 |
| M-RoT LD 1.4 | 0.97 | 0.97 | 0.21 | 1.13 | 1 | 1.28 | 1.07 | 1.40 |
| M-RoT SP 2.0 | 1.60 | 1.30 | 0.22 | 1.14 | 1 | 0.88 | 0.81 | 2.00 |
| M-RoT LD 2.0 | 1.81 | 1.06 | 0.18 | 1.16 | 1 | 1.05 | 0.60 | 2.03 |

Table (5) presents the results obtained from the response of the process to a step input in both set-point and load-disturbance of the system, in this table all the mentioned cases of the three rules are compared, a graphical analysis is shown in Fig. (4) where the performance of the system in both control modes can be analized.

Results presented in (5), illustrates the superiority of the M-RoT rule compared to the MsValue and Padula-Visioli rules in terms of accuracy in achieving robustness target levels and for the general performance of the system. The M-RoT rule achieved the required robustness levels for all four cases, and outperformed in both the set-point tracking task and load-disturbance rejection performance index.

Graphically, the improvement of the system response is evident with the implementation of M-RoT rule controllers. In all cases, the system demonstrates a more accurate and smoother response compared to the other rules used in the example.

### *4.2 Example 2 High-order process*

For this example, a high-order process presented by Åström and Hägglund (2000), is used as one of the processes presented as a benchmark to analyze different closed-loop system responses.

$$P_2(s) = \frac{1}{(s+1)^8} \tag{18}$$

After reduction to an FOPDT process model, the parameters of the reduced model are $K = 1$, $T = 3.06$, $L = 4.95$ to obtain a normalized dead time $\tau = 1.62$. This controlled process model is characterized by a highly dominant dead time.

Table 6. Results in regard to Example 2.

| Tuning Rule | $K_p$ | $T_i$ | $T_d$ | $\mu$ | $\lambda$ | $J_{sp}$ | $J_{ld}$ | $M_s$ |
|---|---|---|---|---|---|---|---|---|
| MsValue 1.6 | 0.67 | 5.16 | 1.47 | 1.08 | 1 | 7.62 | 7.60 | 1.60 |
| MsValue 1.8 | 0.80 | 4.89 | 1.48 | 1.08 | 1 | 6.67 | 6.13 | 1.81 |
| M-RoT SP 1.6 | 0.69 | 4.95 | 1.51 | 1.12 | 1 | 7.25 | 7.14 | 1.60 |
| M-RoT LD 1.6 | 0.68 | 4.63 | 1.55 | 1.11 | 1 | 7.30 | 7.01 | 1.60 |
| M-RoT SP 1.8 | 0.80 | 5.09 | 1.58 | 1.10 | 1 | 6.57 | 6.33 | 1.80 |
| M-RoT LD 1.8 | 0.82 | 4.95 | 1.50 | 1.13 | 1 | 6.73 | 6.10 | 1.80 |

For this case, the M-RoT and MsValue rules were applied in the cases with robustness objective $M_s = \{1.6, 1.8\}$, in order to analyze all the instances of the proposed rule. Table (6) presents the obtained results for this example.

Due to the process having a large dead time, the simulation time was set at $t = 35s$ for this case. Additionally, we observe the system response for a step input in both control modes in Fig. (5) in the both rules used in the comparison.

The obtained results indicate again an outperformance of the M-RoT rule for FOPID controllers against the MsValue rule, but it can be highlighted that both rules present a great precision to reach the target robustness levels proposed in each case, but with the same level of robustness reached, the M-RoT rule is able to obtain a performance index even lower than that of MsValue, in both control modes.

Including the scenario where the controller obtained by tuning specifically for set-point tracking with $M_s = 1.6$ of the M-RoT rule outperforms the MsValue controller with the same robustness target working in load-disturbance rejection.

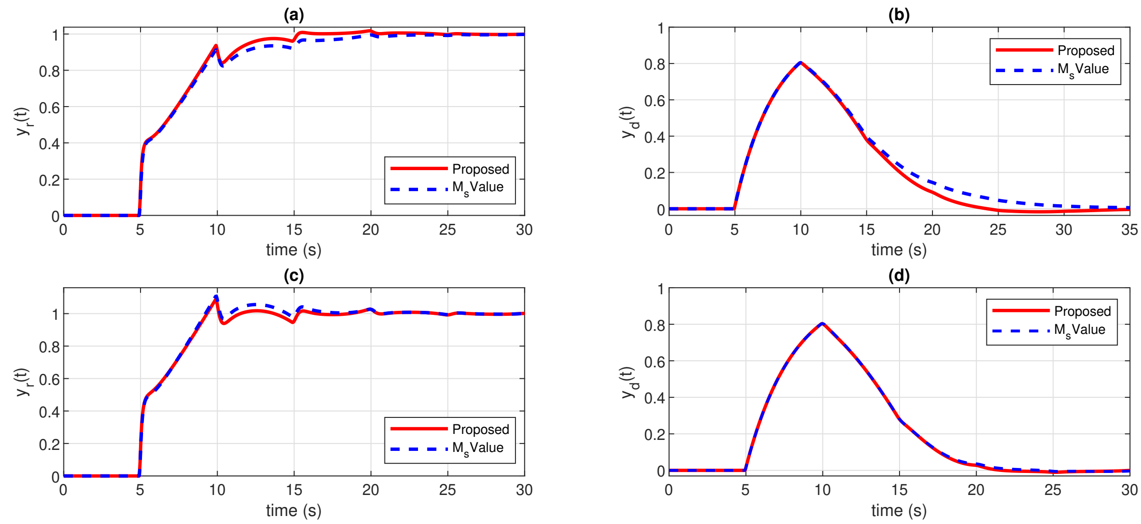


Figure 5. System response shown in both control modes for Example 2. Figures (a) and (b) represent the $M_s = 1.6$ target tuning, while Figures (c) and (d) are the $M_s = 1.8$ target case.

## 5. CONCLUSIONS

The M-RoT rule was validated in terms of performance and robustness, with cases where the performance of the rule is analyzed for the tuning of both control modes, against rules such as MsValue, which uses a more robust optimization method to be designed, or the Padula-Visioli rule, which has been a reference rule for fractional order PID controllers for a considerable time.

The proposed tuning rule is easy to use since it avoids the use of a large number of constants for the tuning functions and can provide optimal performance for controlling first-order plus dead-time models, which are widely used in industry and academia. The M-RoT rule also can achieve the desired level of robustness for the control system.

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