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Intermediate Tuning Proposal for Fractional Order PID Controllers with Balanced Servo/Regulation Operation

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Abstract: Many tuning rules for controllers consider each operating mode separately, including servo-control and regulatory-control. This paper considers the problem where an optimal controller for one of these modes has to operate in a different mode than the one for which it was tuned. A weighted performance degradation index is analyzed and an intermediate tuning rule is proposed for fractional-order PID controllers, taking into account a robustness constraint. This rule serves as a proposal for further research on the application of this methodology in the design of tuning rules for FOPID controllers.

Keywords: Fractional Control, Performance Analysis, Robust Control, Tuning Rules, FOPID.

1. INTRODUCTION

With Proportional-Integral-Derivative (PID) controllers being the most widely used option found in most industrial and academic applications Åström and Hägglund (2001), it is very difficult for a different approach to be as popular and influential as PID control.

In recent decades, academia and industry have increasingly focused on the various applications of fractional calculus in control engineering. This has led to numerous proposals for fractional control Zamani et al. (2009), including for fractional order PID controllers Padula and Visioli (2015) and controlled process models that effectively employ fractional order terms to approximate dynamics Meneses et al. (2022).

In general, there has been extensive research exploring the trade-off between system performance and system robustness. However, previous studies have only examined the correlation between controller performance and tuning mode, specifically for either set-point tracking or load-disturbance rejection scenarios.

It is a well known fact that a controller is usually tuned only in the case of servo-control or regulatory-control Visioli (2006), in which case both control modes have very different objectives with respect to the controlled variable of the process. This paper examines the issue of Performance Degradation (PD) that emerges in the scenario where a controller tuned for one control mode needs to perform well in the other control mode.

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This paper is presented as a continuation, for FOPID controllers, of the Weighted Performance Degradation methodology Arrieta et al. (2010, 2011) proposed for PID controllers; presenting a rule for the particular case where it is intended to find a totally intermediate tuning, starting from a tuning rule for FOPID controllers previously studied Padula and Visioli (2011) and validating the proposed rule by means of concrete examples where the behavior in terms of global performance of the control system can be analyzed.

This paper is structured as follows. Section 2 provides a general description of the implemented control system, detailing its relevant characteristics. Section 3 presents the proposed intermediate tuning rule and the considerations taken into account for the optimal parameter search space and the optimization criteria. Section 4 provides concrete examples to support the rule, validating its application. Subsequently, Section 5 presents the research conclusions and proposes future work.

2. PROBLEM FORMULATION

2.1 Control System Configuration

The considered control system is shown in Fig. (1) for a traditional feedback control system.

Where $C(s)$ represents the FOPID controller to be tuned and $P(s)$ is the controlled process model. For the signals presented, it can be defined as: the controlled process output, the set-point of the system, the control signal and the load-disturbance, for $y(s)$, $r(s)$, $u(s)$ and $d(s)$ respectively.

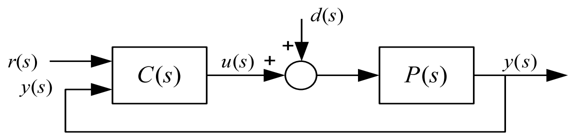

Figure 1. Feedback control system scheme.

Based on the proposed scheme, it is feasible to present the process output as a function of the set-point and disturbance:

$$y(s) = \frac{C(s)P(s)}{1+P(s)C(s)}r(s) - \frac{P(s)}{1+P(s)C(s)}d(s) \quad (1)$$

In this way, it is possible to define the control mode in which the system will operate depending on the system inputs.

2.2 Controlled Process Model

The model used to approximate the controlled process is a first-order plus dead-time (FOPDT) model, which is widely used in industry.

$$P(s) = \frac{K}{Ts+1}e^{-Ls} \quad (2)$$

By normalizing the dead time $\tau = \frac{L}{L+T}$, this model can be used to design the intermediate tuning rule.

2.3 Fractional Order PID 1DoF Series Controller

The proposed controller for the tuning refers to a one-degree-of-freedom (1DoF) FOPID controller in a serial configuration, represented in the following:

$$C(s) = K_p \frac{T_i s+1}{T_i s}\frac{T_d s^{\mu}+1}{\frac{T_d}{\eta}s+1} \quad (3)$$

where K_p represents the proportional gain, T_i the integral time, T_d the derivative time, $\eta = 10T^{\mu-1}$ the derivative filter constant and finally the derivative fractional order μ. The overall structure of the series FOPID controller takes into account the fractional order of the integral action λ. However, for all instances discussed in this work, it is considered $\lambda = 1$, making it the integer case.

2.4 Performance and Robustness

For evaluating system performance, we utilized the integrated absolute error (IAE) as an index. The objective is to use this index to evaluate the performance of the proposed rule against the base rules employed, particularly in the set-point tracking (J_{sp}) and load-disturbance rejection (J_{ld}) modes.

$$J = \int_0^{\infty} |e(t)|dt = \int_0^{\infty} |r(t)-y(t)|dt \quad (4)$$

In the case of achieved robustness, the maximum value of the sensitivity function is used as an index, which can be defined by (5), this index usually lies between the range of values $M_s = \{1.4 - 2.0\}$, in this case it is proposed to use the value $Ms = 2.0$ as the target robustness index for the control system.

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1+C(j\omega)P(j\omega)|} \quad (5)$$

2.5 Optimal Tuning Rules for FOPID Controllers

As base rules are considered those proposed by Padula and Visioli (2011), where a tuning rule for FOPID controllers and its two modes of operation are presented. The tuning functions are presented below:

$$\begin{aligned} K_p &= \frac{1}{K}\left(a\tau^b + c\right) \\ T_i &= T^{\lambda}\left(a\left(\frac{L}{T}\right)^b + c\right) \\ T_d &= T^{\mu}\left(a\left(\frac{L}{T}\right)^b + c\right) \end{aligned} \quad (6)$$

The tuning constants a, b, and c utilized for the $M_s = 2.0$ tuning are presented in the subsequent tables.

Table 1. Tuning parameters for FOPID controller with $M_s = 2.0$ as target robustness.

Operation Mode		a	b	c
K_p	set-point	0.9294	−0.9330	−0.9205
	load-disturbance	0.1804	−1.4490	0.2319
T_i	set-point	−1.0014	−1.0030	1.0310
	load-disturbance	0.6426	0.8069	0.0563
T_d	set-point	0.4203	1.2290	0.0182
	load-disturbance	0.5970	0.5568	−0.0954

Table 2. Fractional coefficient for the FOPID tuning rule.

Derivative fractional order μ	
set-point	load-disturbance
1.0 if $\tau < 0.1$	1.0 if $\tau < 0.2$
1.1 if $0.1 \le \tau < 0.4$	1.1 if $0.2 \le \tau < 0.6$
1.2 if $0.4 \le \tau$	1.2 if $0.6 \le \tau$

These rules will be used during the intermediate tuning rule design process for FOPID controllers.

3. INTERMEDIATE TUNING BY CONSIDERED PERFORMANCE DEGRADATION PROBLEM

3.1 Fractional-Controller search space

The tuning configuration presented by Padula and Visioli (2011) can be considered as extreme cases of operation. The controller parameters are obtained by exclusively considering one of the operation modes, which can result in low performance if both operation modes occur simultaneously.

Based on this premise, this work proposes a search for an intermediate controller to minimize the loss of system performance.

The parameters to be searched belong to a linear combination of the controller parameters of the extreme rules, so it is possible to define the controller configuration in terms of this parameterized family of values as a vector.

$$\bar{\gamma} = [\gamma_1, \gamma_2, \gamma_3, \gamma_4],\ \gamma_i \in [0,1] \tag{7}$$

Each γ_i value is related to a controller parameter (K_p, T_i, T_d, μ), which makes it possible to search for an intermediate tuning.

$$K_p(\gamma_1) = \gamma_1 K_p^{ld} + (1-\gamma_1)K_p^{sp} \tag{8}$$

$$T_i(\gamma_2) = \gamma_2 T_i^{ld} + (1-\gamma_2)T_i^{sp} \tag{9}$$

$$T_d(\gamma_3) = \gamma_3 T_d^{ld} + (1-\gamma_3)T_d^{sp} \tag{10}$$

$$\mu(\gamma_4) = \gamma_4 \mu^{ld} + (1-\gamma_4)\mu^{sp} \tag{11}$$

The first set of parameters $[K_p^{sp}, T_i^{sp}, T_d^{sp}, \mu^{sp}]$, belongs to the extreme set-point tuning and the second set of parameters $[K_p^{ld}, T_i^{ld}, T_d^{ld}, \mu^{ld}]$, to the load-disturbance tuning.

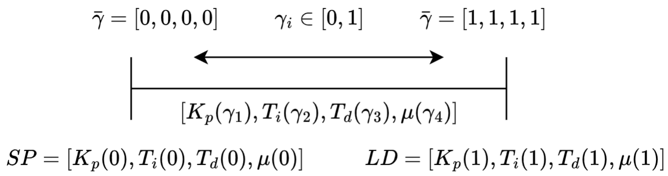

Figure 2. $\bar{\gamma}$-Tuning procedure for an intermediate FOPID controller.

3.2 Performance degradation

According to equation (1), the control system can operate in two modes: servo control and regulatory control. Servo control aims to ensure that the controller output signal accurately tracks the systems set-point signal r, while regulatory control focuses on maintaining the controlled variable at the desired level and rejecting any system disturbances d.

In set-point mode, the system does not consider any disturbances, and the output can be described as follows:

$$y_{sp} = \frac{C(s)P(s)}{1+C(s)P(s)} r(s),\ d(s) = 0 \tag{12}$$

In load-disturbance mode, it is assumed that the set-point remains constant, and the system output is described as:

$$y_{ld} = \frac{P(s)}{1+C(s)P(s)} d(s),\ r(s) = 0 \tag{13}$$

In this situation, the $C(s)$ controller is typically tuned to be optimal for one of the two modes of operation of the system. However, in reality, the controlled process often exhibits dynamic behaviors in the set-point and load-disturbance inputs.

As a result, there may be a loss of performance because the implemented controller will not be optimal for both modes. Performance Degradation (PD) is a measure of the loss of performance that occurs in relation to the optimal operation of the system.

Performance Degradation should be considered for both operating modes. The proposed intermediate $\bar{\gamma}$-tuning, as indicated in (14) and (15), will be evaluated since it does not belong to an extreme operating mode.

$$PD_{sp}(\bar{\gamma}) = \left| \frac{J_{sp}(\bar{\gamma}) - J_{sp}(sp)}{J_{sp}(sp)} \right| \tag{14}$$

$$PD_{ld}(\bar{\gamma}) = \left| \frac{J_{ld}(\bar{\gamma}) - J_{ld}(ld)}{J_{ld}(ld)} \right| \tag{15}$$

Where (14) represents the performance degradation of the $\bar{\gamma}$-tuning in set-point operation mode, and (15) the performance degradation of $\bar{\gamma}$-tuning in load-disturbance operation mode.

$$PD(\bar{\gamma}) = \begin{cases} PD_{ld}(sp)\ \ for\ \bar{\gamma} = [0,0,0,0] \\ PD_{sp}(ld)\ \ for\ \bar{\gamma} = [1,1,1,1] \end{cases} \tag{16}$$

3.3 Criterion for an intermediate mode of operation

To properly evaluate the systems operating mode, it is necessary to redefine the performance index being considered, specifically the Integral of the Absolute Error (IAE).

$$J_x(z) = \int_0^{\infty} |e(t,x,z)| dt \tag{17}$$

In this case, x refers to the operating mode and z refers to the controller tuning mode. Then, $x \in \{sp, ld\}$ and $z \in \{sp, ld\}$.

Finally, Weighted Performance Degradation (WPD) can be introduced by (18), where both indices of PD are considered with a weighting factor Arrieta et al. (2010).

$$WPD(\bar{\gamma};\alpha) = \alpha PD_{ld}(\bar{\gamma}) + (1-\alpha)PD_{sp}(\bar{\gamma}) \tag{18}$$

The α parameter, $\alpha \in [0,1]$, indicates which of the two modes of operation is considered more important or preferred. Identifying the following cases:

- Operation mode in set-point $\alpha = 0$.
- More importance to the set-point mode $\alpha = 0.25$.
- Same importance for both operation modes $\alpha = 0.50$.
- More importance to load-disturbance mode $\alpha = 0.75$.
- Operation mode in load-disturbance $\alpha = 1$.

This work proposes a tuning focused on the intermediate case $\alpha = 0.50$. The objective function of the optimization will be to minimize the value of the $WPD(\bar{\gamma};\alpha)$, subject to a robustness constraint of $M_s = 2.0$ for the control system, in order to find the values of the $\bar{\gamma_{opt}}$ vector.

3.4 Intermediate Tuning Proposal

The search for optimal intermediate parameters was performed using MATLAB® *fmincon* solver, followed by a curve-fitting step with *cftool* toolbox, considered the following tuning functions:

$$K_p(\gamma_1) = \frac{1}{K}\left(a\tau^b + c\right) \tag{19}$$

$$T_i(\gamma_2) = T\left(\frac{a\left(\frac{L}{T}\right)^2 + b\left(\frac{L}{T}\right) + c}{\left(\frac{L}{T}\right) + d}\right) \tag{20}$$

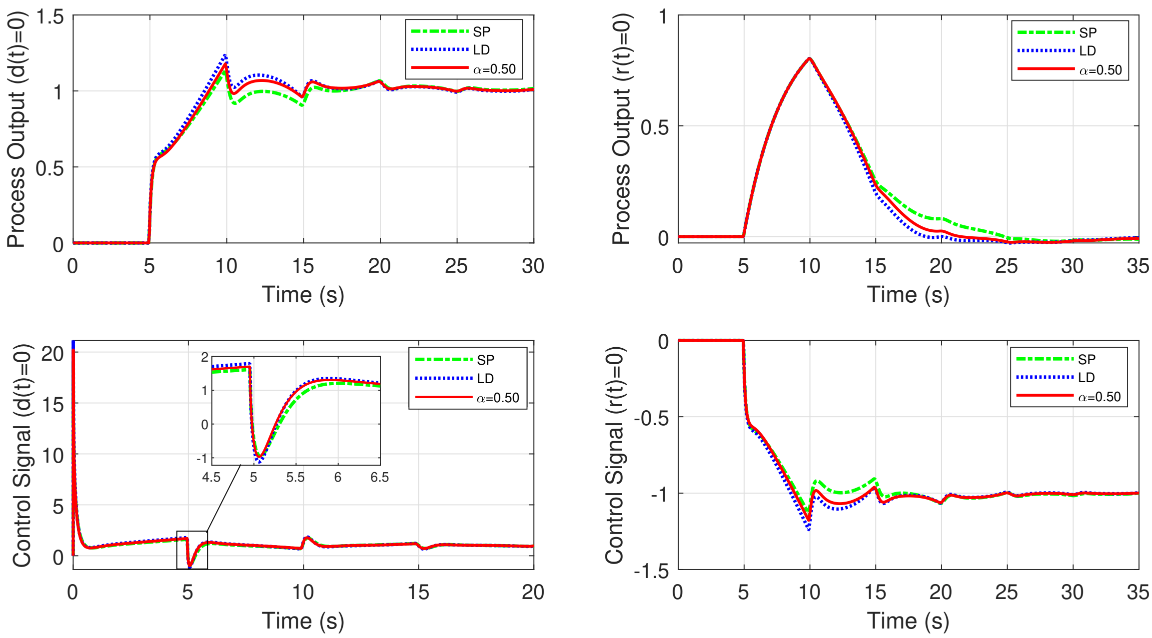

Figure 3. Process and controller output of the extreme rules and the intermediate tuning for the Example 1.

$$T_d(\gamma_3) = T^{\mu}\left(a\left(\frac{L}{T}\right)^b + c\right) \tag{21}$$

$$\mu(\gamma_4) = \left(a\left(\frac{L}{T}\right)^b + c\right) \tag{22}$$

In all cases, the search criteria aims to minimize the Sum Square Error (SSE) by adjusting these functions. After adjusting the intermediate parameters, a validation stage was performed to validate the robustness constraint and the intermediate performance in both operating modes, set-point tracking and load-disturbance rejection, where the adjusted parameters were evaluated iteratively to ensure compliance with the established conditions.

Table 3. Intermediate tuning rule constants for the $\alpha = 0.50$ case.

	K_p	T_i	T_d	μ
		$0.1 \leq \tau < 0.4$		
a	1.3871	-10.4884	0.4412	$-2.3609e^{-7}$
b	-0.7696	14.0961	1.0250	-5.6271
c	-1.6248	-0.0542	0	1.1000
d	-	5.0471	-	-
		$0.4 \leq \tau < 0.6$		
a	0.6332	2.2914	0.8253	$-9.3768e^{-11}$
b	-1.1365	-2.0503	0.5736	-48.6626
c	-0.5450	15.2380	-0.3667	1.1567
d	-	16.6971	-	-
		$0.6 \leq \tau \leq 0.67$		
a	-8.4447	-1.8037	0.0082	0
b	9.8500	9.9340	4.8290	0
c	0.6413	0.8609	0.6078	1.200
d	-	10.4274	-	-

Table (3) shows the constants of the intermediate tuning a, b, c, and d for the different ranges of normalized dead time τ considered. Additionally, Fig. (4) confirms that the tuning complies with the stipulated by showing the behavior of the $WPD(\bar{\gamma}; \alpha)$ with respect to $PD_{sp}(\bar{\gamma})$ and $PD_{ld}(\bar{\gamma})$.

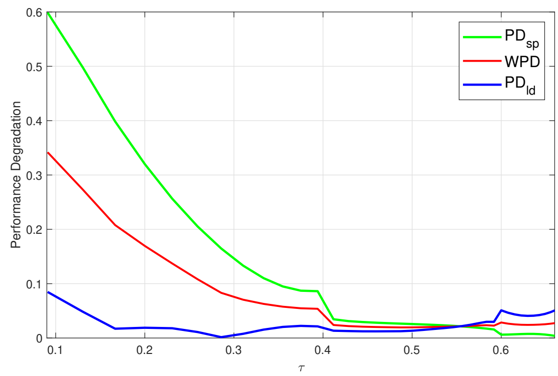

Figure 4. Weighted Performance Degradation for the intermediate rule and the normalized process.

4. EXAMPLES

4.1 *Example 1. High-order process*

As a first example of the proposed intermediate tuning, consider the high-order process used as a benchmark in Åström and Hägglund (2000):

$$P_1(s) = \frac{1}{(s+1)^8} \tag{23}$$

After reducing the model order, a FOPDT with the following parameters $K = 1$, $T = 3.06$ and $L = 4.95$ can be identified, which is a process characterized by dead time dominance. Table (4) presents the results obtained and Fig. (3) shows the response to a step input in both control modes.

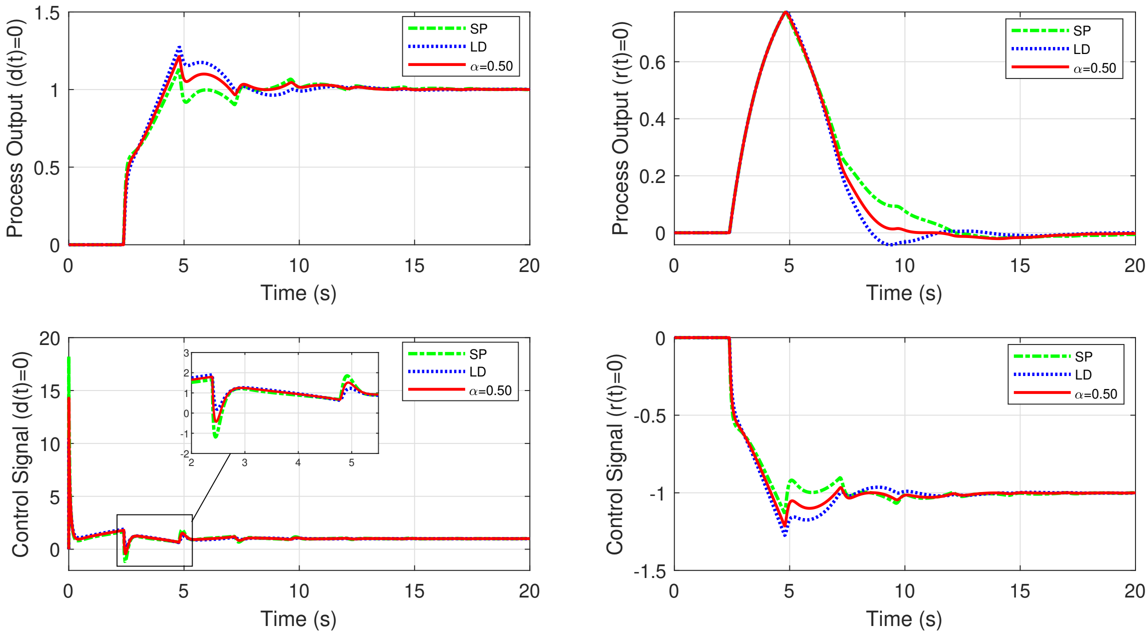

Figure 5. Process and controller output for the extreme rules and the intermediate tuning proposed for the Example 2.

Table 4. Performance and robustness index regard to the Example 1.

Tuning Rule	K_p	T_i	T_d	μ	J_{sp}	J_{ld}	M_s
SP	0.59	1.67	1.24	1.20	6.68	6.30	2.07
$\alpha = 0.50$	0.60	1.53	1.16	1.15	6.74	5.99	2.01
LD	0.61	1.52	1.10	1.10	6.80	5.75	2.10

The obtained results demonstrate the expected performance of the FOPID controller achieved by the intermediate tuning, according to what has been explained above, the proposal was to find a tuning that would work properly in both control modes, seeking to minimize the weighted performance degradation, and also meet the desired level of robustness $M_s = 2.0$. In each instance of performance index consideration (J_{sp} and J_{ld}), the suggested tuning is within the upper and lower bounds of performance. Moreover, it provides a greater level of control system robustness than the extreme rules employed.

Table 5. Performance Degradation index for the Example 1.

Tuning Rule	PD_{sp}	PD_{ld}	WPD
SP	-	0.0962	-
$\alpha = 0.50$	0.0082	0.0409	0.0245
LD	0.0170	-	-

The performance results allow for a comparison between the base extreme rules and the proposed intermediate balance tuning in terms of Performance Degradation. Table (5) shows the results of the PD for Example 1, and it is clear that the proposed intermediate rule presents the lowest degradation in both control modes operation.

4.2 Example 2. Inverse-response process

Finally, an additional example is proposed, in this case working with a process with inverse response dynamics:

$$P_2(s) = \frac{1-s}{(s+1)^3} \tag{24}$$

After first order approximation, the model parameters $K = 1$, $T = 1.62$ and $L = 2.39$ are obtained and after applying the extreme tuning rules and the proposed intermediate tuning rule, the results can be analyzed using Table (6) and the set-point and load-disturbance step response in Fig. (5).

Table 6. Performance and robustness evaluation for Example 2.

Tuning Rule	K_p	T_i	T_d	μ	J_{sp}	J_{ld}	M_s
SP	0.59	1.67	1.24	1.20	3.24	3.06	2.09
$\alpha = 0.50$	0.60	1.53	1.16	1.15	3.29	2.78	1.99
LD	0.61	1.51	1.10	1.10	3.46	2.69	2.17

The graphical results indicate satisfactory performance, demonstrating strong intermediate behavior for both set-point tracking and load-disturbance rejection modes. In addition, Table (7) validates the purpose of the balance tuning rule by showing the PD indexes for the extreme rules and the proposed rule, making the overall improvement in the system clearer.

Table 7. Performance Degradation index for the Example 2.

Tuning Rule	PD_{sp}	$\text{PD}_{l}d$	WPD
SP	-	0.1398	-
$\alpha = 0.50$	0.0142	0.0306	0.0224
LD	0.0667	-	-

The proposed intermediate tuning rule for $\alpha = 0.50$ is a valid option to improve overall control system performance while ensuring the established level of robustness. Additionally, it provides a high degree of accuracy for reaching the $M_s = 2.0$ target value.

5. CONCLUSIONS AND FUTURE WORK

The study presents an intermediate tuning approach for FOPID controllers, particularly when both control modes need a efficient performance. The suggested WPD tuning method for $\alpha = 0.50$ proves to be a practical choice for multiple scenarios. The rule development method sacrifices a degree of optimality in the performance of the controller tuned by the extreme rules, but the tuned intermediate controller outperforms the extremes in a generalized degree of performance, measured by the weighted performance degradation and de robustness index.

The extreme rules (SP and LD) employ very little freedom in the tuning of the derivative fractional order, since it is fixed along the proposed normalized dead time, which became a limitation to find an adequate tuning in a general way in the whole τ range, since the curve fitting procedure did not adapt well to the abrupt changes of the fractional order.

As a future work, it is proposed to implement this methodology for a FOPID tuning rule that allows more freedom in the tuning of the fractional order, since in this work the performance of the design procedure has been validated by means of different examples.

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