Preprints, 4th IFAC Conference on
Advances in Proportional-Integral-Derivative Control
Almería, Spain | June 12-14, 2024

Auto-Tuning Software Tool for Inverse Response Model Robust Tuning of 1-DoF PID and FOPID Controllers

S. Madrigal [*,**], **H. Meneses** [*,**], **O. Arrieta** [*,***], **M. Meneses** [***], **R. Vilanova** [***]

[*] *Instituto de Investigaciones en Ingeniería, Facultad de Ingeniería, Universidad de Costa Rica, 11501-2060 San José, Costa Rica.*
[**] *Departamento de Automática, Escuela de Ingeniería Eléctrica, Universidad de Costa Rica, 11501-2060 San José, Costa Rica.*
[***] *Departament de Telecomunicació i d'Enginyeria de Sistemes, Escola d'Enginyeria Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain.*

Abstract: The main objective of this work is to present a software tool that allows to perform the tuning process of PID and Fractional Order PID controllers in an automated method. The tool tunes 1DoF PID and FOPID controllers for a controlled process with a inverse response second-order plus dead time model. This tuning is based on the Inverse Response Model Robust Tuning (IRM-RoT) rule, which aims to achieve optimal control system performance while ensuring a target robustness. Additionally, the tool measures the performance obtained from the control system, as well as the robustness achieved, and displays the control signal obtained by implementing the tuned controller. The examples used show the main characteristics of the tool and conclude that it can be a very useful contribution for educational purposes in fields such as electrical engineering.

Keywords: Auto-Tuning Tool, Inverse Response, PID, FOPID, Fractional Control.

1. INTRODUCTION

In recent years, there has been a significant rise in personal and mobile computer technology. These advancements aim to simplify daily tasks. However, when considering the field of engineering, particularly in education, it has not been adequately addressed. Consequently, most of the developed tools have unrelated approaches to engineering.

Since their introduction in 1940, proportional-integral-derivative (PID) controllers have been the most practical solution for process control in industrial applications. This is because of their effectiveness and wide applicability Åström and Hägglund (2006). However, in recent years, the implementation of fractional order PID (FOPID) controllers has proven to be an innovative solution capable of improving the performance of the control system and is increasingly found in industrial process control applications Meneses et al. (2018), Tepljakov et al. (2021).

Controller tuning tools have been proposed for engineering education and open-source learning of control system topics. These tools aim to provide a better understanding of control systems and their tuning methods Meneses et al. (2022b,a).

Another example is found in Hidalgo et al. (2022), which proposes a tuning tool for FOPID controllers based on a fractional-order process model.

The principal purpose of this paper is to present a tool that has been developed to perform in an automated manner the tuning process of a 1DoF controller, either PID or FOPID, based on a second-order inverse response plus dead time (IRSOPDT) model for the controlled process. Similar tools have been proposed for the analysis of the performance and robustness of a PID controller tune method based on SOPDT models such as Benavides et al. (2013) and Fernandez et al. (2013).

The proposed IRM-RoT automatic tuning tool aims to address the challenges of tuning PID and FOPID 1DoF controllers by applying the *Inverse Response Model Robust Tuning* rule proposed by the same authors (S. Madrigal et al.). However, the tuning rule is complex to use without a tool that automates the proper choice of constants for each tuning function and, if necessary, interpolates the parameters between the values of b, the relative position of the zero in the right half plane.

The paper presents a MATLAB® *AppDesigner* based tool, that allows the user to tune PID and fractional-order PID controllers for a given model. The tool provides diverse features and analysis options including:

- Automated tuning of the three PID controller parameters, or four in the case of FOPID, with only the input of the parameters of the IRSOPDT model of the controlled process.

corresponding author: orlando.arrieta@ucr.ac.cr

- This tuning can be performed for either of the two levels of robustness proposed by the *Inverse Response Model Robust Tuning* (IRM-RoT) rule.
- The tool allows for real-time comparison of the performance of the tuned controllers in both modes of operation.

The paper is organized as follows. In Section 2, the proposed control system scheme is presented, as well the tuning equations of the rule used, the structure of the controllers to be tuned, and the indices to evaluate the performance and robustness of the system. Section 3 presents details on the development of the proposed tool and its components. Subsequently, Section 4 illustrates, by means of examples, the main characteristics and the use of the tool, Section 5 presents feedback on the tool provided by students at University of Costa Rica, and the article ends in Section 6 with the conclusions of this research.

2. PROCEDURES AND METHODS

2.1 Control System Configuration

Considering the control system shown in Fig. (1), where $P(s)$ is the model of the controlled process and $C_r(s)$, $C_y(s)$ constitute the tuned 1DoF PID or FOPID controller. In this system, the signals $r(s)$, $u(s)$, $d(s)$ and $y(s)$ corresponding to the set-point for the process output, the control signal, the system load-disturbance and the controlled process output are presented.

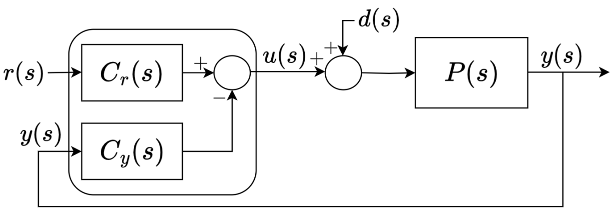

Figure 1. Closed-loop control system.

2.2 Controlled Process Model

The controlled process $P(s)$ is considered as a inverse-response-second-order-plus-dead-time (IRSOPDT) model given by the following transfer function:

$$P(s) = \frac{K(-bTs+1)e^{-Ls}}{(Ts+1)(\alpha Ts+1)} \tag{1}$$

where K represents the process gain, T denotes the dominant time constant, L stands for dead-time, α gives the ratio between the time constants, and b is the relative position of the zero in the right half plane with respect to the dominant time constant.

2.3 1DoF Controllers Structure

The output signal of a one-degree-of-freedom (1DoF) PID or FOPID can be expressed in terms of three different error signals:

$$u(s) = K_p(e_p(s) + e_i(s) + e_d(s)) \tag{2}$$

considering:

$$e_p(s) = r(s) - y(s), \quad e_i(s) = \frac{1}{T_i s}[r(s) - y(s)] \tag{3}$$

$$e_d(s) = -\frac{T_d s}{\frac{T_d}{\eta}s + 1} y(s) \tag{4}$$

Taking the complete form for each error signal, the control signal achieved with the 1DoF control scheme can finally be written as the following expression Åström and Hägglund (2006):

$$u(s) = K_p\left[\left(1 + \frac{1}{T_i s}\right)e(s) - \left(\frac{T_d s}{\frac{T_d}{\eta}s + 1}\right)y(s)\right] \tag{5}$$

where, K_p refers to the proportional controller gain, T_i is the integral time constant, T_d the derivative time constant and T_d/η the constant of the derivative filter.

In the control scheme of Fig. (1), $C_r(s)$ is the transfer function of the setpoint controller and $C_y(s)$ of the feedback controller.

$$C_r(s) = K_p\left(1 + \frac{1}{T_i s}\right) \tag{6}$$

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{\eta}s + 1}\right) \tag{7}$$

Considering this, the output of the process in the 1DoF control system $y(s)$, can be expressed in terms of $C_r(s)$ and $C_y(s)$ using the following expression:

$$y(s) = \underbrace{\frac{C_r(s)P(s)}{1 + C_y(s)P(s)}}_{M_{yr}(s)} r(s) + \underbrace{\frac{P(s)}{1 + C_y(s)P(s)}}_{M_{yd}(s)} d(s) \tag{8}$$

All of the above is applicable when implementing a fractional order PID, except for the feedback controller transfer function, $C_y(s)$, where fractional order is considered for the derivative action μ.

$$C_y(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s^{\mu}}{\frac{T_d}{\eta}s + 1}\right) \tag{9}$$

Finally, the constant η for the derivative filter in both cases is defined by (10), where μ equals 1 for the PID controller.

$$\eta = 10 T_d^{\frac{\mu - 1}{\mu}} \tag{10}$$

When the derivative fractional order term μ is considered in FOPID controllers, it is necessary to use the Oustaloup et al. (2000) approximation for proper implementation. This involves applying the following recursive approximation based on a product of poles and zeros:

$$s^{\mu}_{[\omega l, \omega h]} \cong C_o \prod_{k=1}^{N} \frac{1 + \frac{s}{\omega_{z,k}}}{1 + \frac{s}{\omega_{p,k}}}, \ \mu > 0 \tag{11}$$

The valid frequency range for the approximation is set to $\{\omega l, \omega h\} = \{0.001, 1000\}$, the C_o term is such that the approximation achieves unity gain at the crossover frequency. Also, the parameter N, which denotes the number of poles and zeros for the real-rational transfer function approximating the term s^{μ}, is set to $N = 8$.

2.4 Performance and Robustness

The IRM-RoT rule is a integrated absolute error based optimal tuning rule for IRSOPDT processes, the IAE is given by (12) and this index can define the performance achieved by the control system in both servo-control (J_{sp}) and regulatory-control (J_{ld}) modes of operation.

$$J_e = IAE = \int_0^{\infty} |e(t)| \, dt = \int_0^{\infty} |r(t) - y(t)| \, dt \tag{12}$$

Therefore, the index that optimizes the IRM-RoT rule is the unweighted sum of J_{sp} for the set-point tracking task and J_{ld} for the load-disturbance rejection case.

$$J_{erd} = J_{sp} + J_{ld} \tag{13}$$

In consequence, implementing the tuning rule controllers, the performance for both control modes, servo-control, and regulatory-control can be balanced.

To fulfill the robustness criteria, the sensitivity function maximum value serves as an indicator:

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C(j\omega)P(j\omega)|} \tag{14}$$

For stable processes such as the one studied, it is recommended that the M_s index remain in the range of $1.40 \leq M_s \leq 2.00$, the IRM-RoT rule only considers the bounds of this range as specific cases.

2.5 IRMRoT Tuning Equations

For tuning PID and FOPID controllers with the IRM-RoT method, specific fixed values for the time constant ratio α are employed, which correspond to the following:

$$\alpha = \{0.10, 0.25, 0.50, 0.75, 1.00\} \tag{15}$$

For both types of controllers, the normalized dead time range is typically specified as $0.10 \leq \tau \leq 2.0$ Visioli (2006). Regarding b, the typical values used by the rule are:

$$b = \{0.25, 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5\} \tag{16}$$

PID Tuning Equations:

$$\kappa_p = K_p K = \frac{a_1\tau^2 + a_2\tau + a_3}{\tau + a_4} \tag{17}$$

$$\tau_i = \frac{T_i}{T} = k_1\tau^3 + k_2\tau^2 + k_3\tau + k_4 \tag{18}$$

$$\tau_d = \frac{T_i}{T^{\mu}} = c_1\tau^3 + c_2\tau^2 + c_3\tau + c_4 \tag{19}$$

FOPID Tuning Equations: The normalized proporcional gain κ_p has been tuned with the same equation (17) used for the PID controller.

$$\tau_i = \frac{T_i}{T} = k_1\tau^4 + k_2\tau^3 + k_3\tau^2 + k_4\tau + k_5 \tag{20}$$

$$\tau_d = \frac{T_i}{T^{\mu}} = c_1\tau^4 + c_2\tau^3 + c_3\tau^2 + c_4\tau + c_5 \tag{21}$$

$$\mu = d_1\tau^4 + d_2\tau^3 + d_3\tau^2 + d_4\tau + d_5 \tag{22}$$

The normalized parameters of both controllers depend on the values of alpha, b, and the normalized dead time of the model, for both robustness levels of the closed-loop control system $M_s = 1.40$ for a smooth tuning or $M_s = 2.00$ for an agressive tuning.

3. IRM-ROT AUTO-TUNING SOFTWARE TOOL DESCRIPTION

The software tool was created and developed based on the MATLAB® *AppDesigner* toolbox, which provides the capability to design specialized applications with an interactive graphical user interface, and allows the packaging of code dependencies so the tool can be freely distributed to any user who has a MathWorks license to use MATLAB®.

The main screen of the automated tuning tool for the IRM-RoT rule is shown in Fig. (2) and the sections of which are described below:

(1) Controlled Process Model Input Data
In this initial section of the tool, the user obtains information regarding to the IRSOPDT model, which is being used to model the controlled process and, therefore, to tune PID and FOPID controllers based on the IRM-RoT rule. The user can enter the model parameters, including the process gain K, dominant time constant T, dead time L, the relative location of the non-minimum phase zero b, and the ratio of time constants α.

To correctly input this information, the user must click on the *Load Parameters* button and ensure that the normalized dead time ranges, as outlined in the IRM-RoT rule, are met.

(2) Controller 1DoF Tuning Mode Selector
The IRM-RoT rule employs 1DoF PID and FOPID controllers to control second order processes with inverse response; this description is provided to the user to help them in the practical implementation if they need it after obtaining the controller parameters from the tool.

In this scenario, four distinct types of tuning are provided, for integer order controllers at the maximum sensitivity levels $M_s = \{1.40, 2.00\}$ and the same levels for fractional order controllers, in addition, the operating ranges of the rule for the relative position of the non-minimum phase zero b are shown, with the intention that the user can verify that the process model data has been correctly completed according to the type of tuning to be performed.

(3) Display of the Tuned Controller Parameters
Once the user selects the controller and the type of tuning required, by pressing the corresponding button, the tool will perform the necessary computations to obtain the tuned parameters, these will be displayed in different slots labeled with the representation symbol for each parameter.

(4) Performance and Robustness Analysis
After conducting all necessary procedures to obtain the parameters of the tuned controller of the IRM-RoT rule, the tool additionally allows the user to evaluate the performance of the control system with the model of the controlled process submitted by the user and the tuned controller.

The closed-loop control system is then simulated to obtain the Integrated Absolute Error (IAE) indices for both operating modes, set-point tracking and load disturbance rejection, while the robustness is analyzed from the maximum sensitivity index M_s of the control system. These metrics are based on a step input for both the set-point signal and the load-disturbance.

(5) Closed-Loop System Graphical Analysis
Finally, this final section of the tool can be used to obtain plots of the system response and the control signal for the control system with the tuned controllers and the process model entered by the user.

The section aims to visually confirm the efficacy of the IRM-RoT rule tuned controllers, by displaying certain dynamic characteristics of the control system.

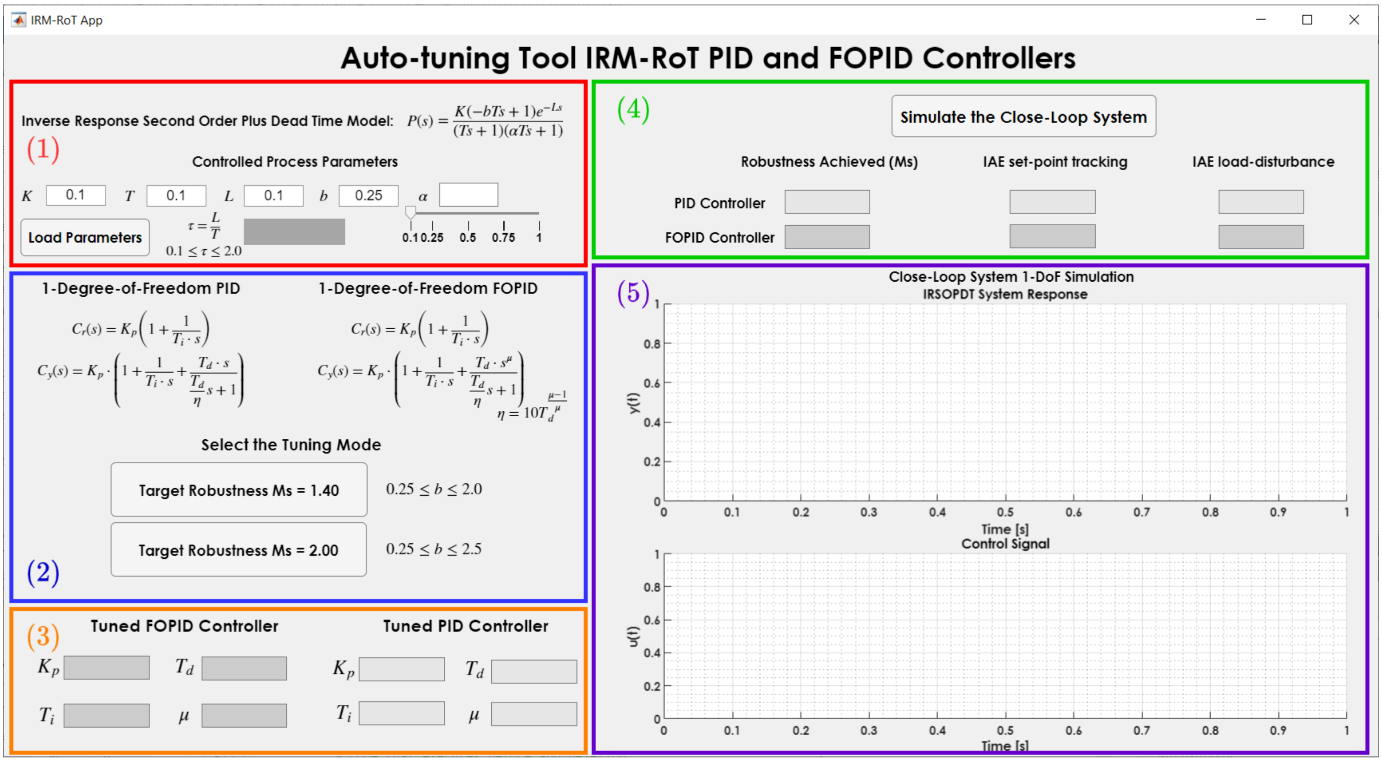

Figure 2. IRM-RoT Auto-Tuning Tool Main Screen.

4. EXAMPLES OF THE TOOL USAGE

To analyze the tool operation, the following model of the controlled process is considered:

$$P_1(s) = \frac{(-2.34s + 1)e^{-1.50s}}{(1.2s + 1)(0.12s + 1)} \tag{23}$$

As a first step, the input parameters are identified based on the process model, including the process gain $K = 1.00$, the dominant time constant $T = 1.20$, the dead time $L = 1.50$, the ratio between time constants $\alpha = 0.10$, and finally the relative location of the zero $b = 1.95$.

The IRM-RoT rule tunes both PID and FOPID controllers for values within the range of $0.25 \leq b \leq 2.00$ while the target robustness value is $M_s = 1.40$, therefore for this example the tuning of both controllers is proposed to analyze the output data of the tool.

When selecting the type of tuning to be performed, the tool displays the result of the calculation of the parameters for the tuned controller, K_p, T_i, T_d and μ, in the case of the PID controller, the value of the derivative fractional order is always unitary.

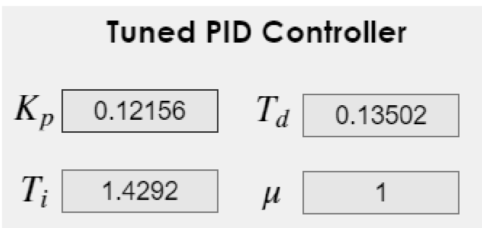

Figure 3. Example 1 display PID controller tuned parameters.

After tuning the controllers, the automatic tuning tool allows real-time evaluation of controller performance in both control modes using a step input for set-point tracking and load-disturbance rejection.

In this instance, two plots are provided by the tool to evaluate the system response and the output signal of the tuned controllers.

The first graph represents the set-point step input and the system load-disturbance with dashed black lines, while the system response to both inputs implementing the tuning controllers is shown with blue and red colored lines. The second graph displays the control signal generated by the tuned controller over the simulation time.

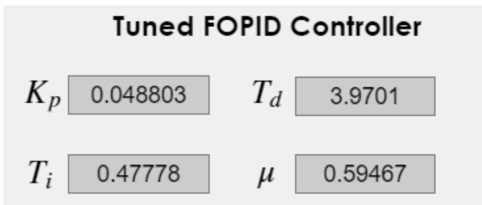

Figure 4. Example 1 display FOPID controller tuned parameters.

IAE performance indices for the servo and regulatory control cases, as well as the M_s for the robustness, are presented at the top of both graphs to enable users to observe controller performance information and assess the achieved robustness target for the selected tuning.

For this example, Fig. (5) show the main screen of the tool during the tuning process of the PID and FOPID controllers using the process model presented in (23).

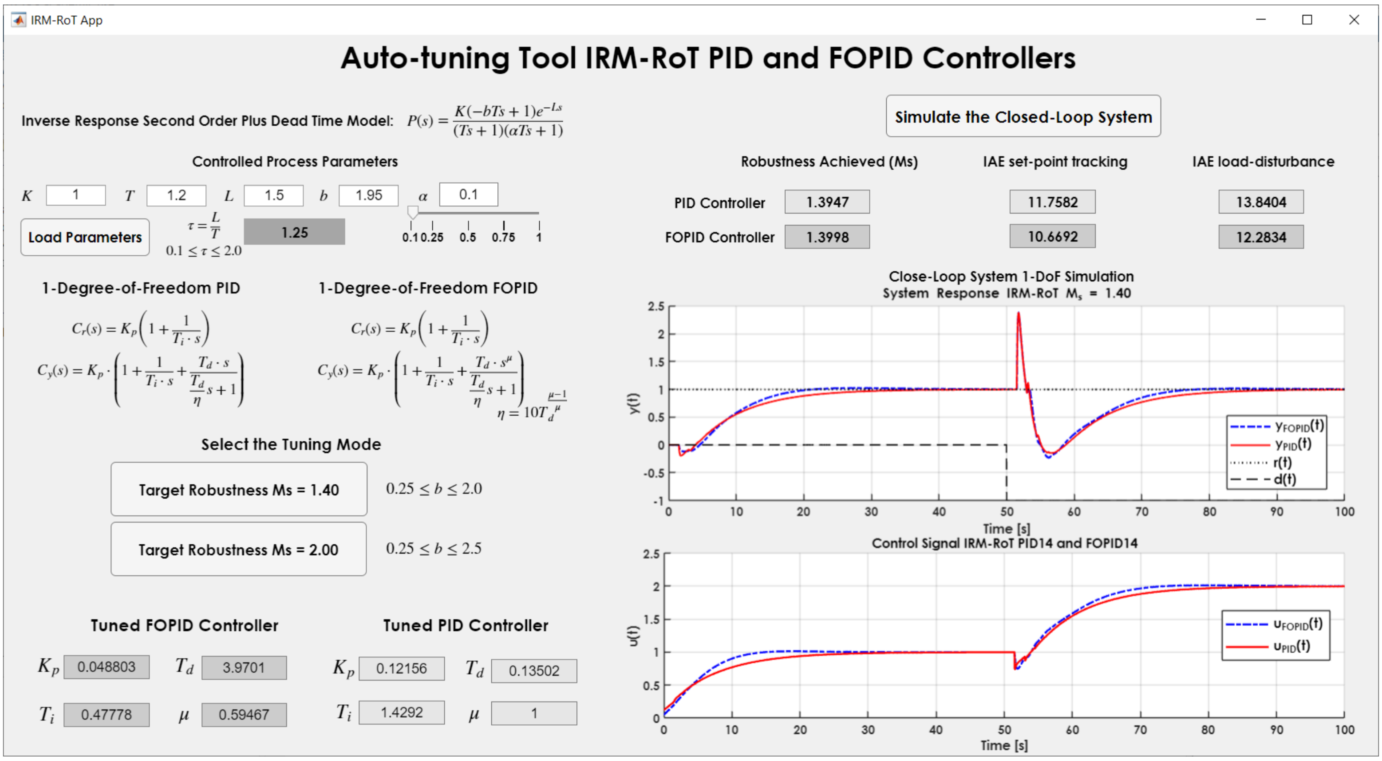

Figure 5. Example 1. IRM-RoT PID and FOPID tuned controllers main screen based on $P_1(s)$.

5. STUDENTS FEEDBACK

The software tool has been tested by students of Electrical Engineering at the University of Costa Rica, looking for students who had already passed an entry level course in control systems, to ensure that they have a fundamental understanding of the tuning process of a controller. It was also presented during the final work presentations for the attainment of a bachelors degree.

The tool has certainly received many positive comments, highlighting its usefulness for the tuning process of controllers using the IRM-RoT rule, which focuses on processes controlled with inverse response dynamics, as well as the information available to the user for the tuning process and evaluation of system performance.

One of the aspects that were outstanding was that this tool can be installed in any device that has a MathWorks license to use the MATLAB® software, which makes it portable to a certain extent.

The tool is available for download from the **MathWorks File Exchange** to any user with a license for MATLAB® software:

`https://www.mathworks.com/matlabcentral/fileexchange/160966-auto-irmrot-tool`

6. CONCLUSIONS

The tuning process of the IRM-RoT rule can be challenging to use at times due to the various tables dependencies caused by the tuning function constants. However, the presented tool offers a practical solution for the automatic tuning IRM-RoT method of PID and FOPID controllers.

The software tool was developed to efficiently manage IRSOPDT model parameters used for tuning. This accelerates the tuning process to just a few seconds for a given model.

The evaluation of the system performance by implementing the tuned controllers through the use of plots, makes the tool more useful, since the user can check whether the performance level and the robustness objective of the system are satisfactorily met and, in turn, compare the performance of the fractional order PID controller with the integer PID.

ACKNOWLEDGEMENTS

The financial support from the University of Costa Rica, under the grant 731-B9-265, is greatly appreciated. Also, this work has received support from the Catalan Government under Project SGR 2021 00197, and also by the Spanish Government under MICINN projects PID2019-105434RBC33 and TED2021-806 129134B-I00 co-funded with the European Union ERDF funds.

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