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Improved MOMI tuning method for integrating processes

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Abstract: Integrating processes can be found in various industries. The main characteristic of such processes is that a limited process input can cause an unlimited process output. In general, they are more difficult to control compared to stable processes. The recently developed Magnitude optimum multiple integration tuning method for integrating processes provides very good closed-loop responses. However, it uses a reference-weighting 2-DOF PI(D) controller structure where the weighting parameters for the P and D term of the controller are equal (therefore the user can only change one parameter). Another drawback of the existing method is that it needs to find the roots of the fourth-order algebraic equation. The method proposed here does not require finding these roots and provides better tracking compared to the original method while maintaining optimal disturbance rejection for different integrating process models.

Keywords: PID control, controller tuning, integrating processes, MOMI method, disturbance rejection.

1. INTRODUCTION

Integrating processes can be found in chemical, process and oil industries in a form of level, pressure or concentration control loops (Åström and Hägglund, 1995). Due to the (theoretically) unlimited process outputs to limited process input signals, they are often more challenging for control when compared to stable (self-regulating) processes (Visioli *et al*., 2011). Typical integrating processes are oil–water–gas separator (Visioli *et al*., 2011), distillation column (Chien and Fruehauf, 1990; Fuentes and Luyben, 1983; Ruan *et al*., 2023; Wang and Hang, 2001), polymerisation reactors (Srividya and Chidambaram, 2020), processes with liquid tank systems (Ogunnaike and Ray, 1994), control of airplane position (Filatov *et al*., 1996), injection moulding processes (Liu and Gao, 2012), and similar (Kos *et al*., 2020b).

There are many tuning methods existing for integrating processes (O'Dwyer, 2009; Kumar and Padma Sree, (2016). One of the methods is the Magnitude optimum multiple integration (MOMI) tuning method for integrating processes (Kos *et al*., 2020a,b). The method gives stable and fast closed-loop responses on different integrating processes and does not require an explicit process model. Namely, the controller tuning needs only the process input and output signals during the steady-state change (Kos *et al*., 2020a,b). Naturally, the tuning can also be performed on arbitrary-order and delayed integrating process model. However, one of the weaknesses of the mentioned MOMI method is that it requires calculating the roots of the fourth-order equation. This is not a serious obstacle, since the roots can still be analytically calculated. However, the second weakness is that the reference-weighting factors for the proportional (P) and derivative (D) terms were chosen the same due to simplified derivation. This, inherently, means that the derived 2-degrees-of-freedom (2-DOF) PID controller cannot be optimal.

In this paper we choose another approach where the control is virtually divided into an inner and an outer loop. When the parameters of both controllers are derived, the loops are joined into one 2-DOF controller. Such approach simplifies the derivation of the controller, since it does not require solving the roots of the fourth-order equation and it provides the most optimal tracking response at optimal disturbance-rejection responses, according to the magnitude optimum (MO) control requirements. Note that the tuning can still be performed on either the measurement of the process input and output signals during the steady-state change or on the delayed arbitrary-order integrating process model.

The remaining of the paper is as follows. In section 2, the virtual block scheme of the proposed control method is given. The calculation of the inner controller is given in section 3, while the calculation of the outer controller is carried out in section 4. Few experiments on different integrating process models are given in section 5. The conclusions are given in section 6.

2. THE VIRTUAL CONTROL SCHEME

The proposed control scheme for the integrating process is given in Figure 1. The control structure is frequently used in the literature, where the G_{CI} controller is frequently used for stabilising the process or when designing adaptive controllers (Shiota and Ohmori, 2012). The control scheme is denoted as "virtual", since it will be later on replaced by one 2-DOF PID controller.

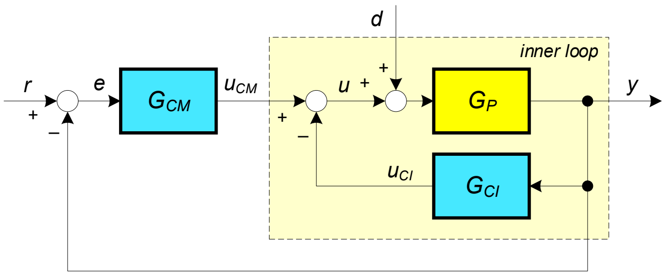

Fig. 1. The "virtual" control scheme consisting of the inner controller (G_{CI}) and the main controller (G_{CM}).

The control strategy is to optimise the disturbance rejection response by an inner controller G_{CI} and then to optimise the tracking response by a main controller G_{CM}.

The process transfer function (note that it is not explicitly required by the tuning method) is as follows:

$$G_P(s) = \frac{1}{s} G_{P0}^*(s)$$
$$G_{P0}^*(s) = \frac{K_{PR}(1 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots)}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots)} e^{-sTdel}, \quad (1)$$

where $G^*_{P0}(s)$ represents the process without the integrating term.

The process $G^*_{P0}(s)$ can also be described by the following infinite Taylor series around s=0 (Vrančić *et al.*, 2021):

$$G_{P0}^*(s) = (A_0 - A_1 s + A_2 s^2 - A_3 s^3 + \cdots)\ , \quad (2)$$

where A_i denote the so-called i-th characteristic area or moment of the process, which can be analytically calculated from the process steady-state change time response or from the process model (2) (Vrančić *et al.*, 2001; Vrančić *et al.*, 2021):

$$A_0 = K_{PR}$$
$$A_1 = K_{PR}(a_1 - b_1 + T_{del})$$
$$A_2 = A_1 a_1 + K_{PR}\left(b_2 - a_2 - T_{del} b_1 + \frac{T_{del}^2}{2!}\right)$$
$$\vdots$$
$$A_k = \sum_{i=1}^{k-1} (-1)^{k+i-1} A_1 a_{k-i} + (-1)^{k+1} K_{PR}(a_k - b_k) +$$
$$+K_{PR} \sum_{i=1}^{k} \frac{(-1)^{k+i}}{i!} T_{del}^i b_{k-i} \quad (3)$$

Since the process is of integrating type, the inner controller can be a PD controller. Then, the inner control loop transfer function (between signals u_{CM} and y) corresponds to stable process without a pole in the origin. The main controller G_{CM} will be a PID controller.

3. THE INNER CONTROLLER

The proposed inner-loop control strategy is to optimise the disturbance-rejection performance. Since the process $G_P(s)$ contains one pole in the origin, the chosen inner controller G_{CI} is the following PD controller:

$$G_{CI}(s) = \frac{K_{PI} + K_{DI} s}{1 + sT_F}, \quad (4)$$

where K_{PI} is the proportional gain, K_{DI} the derivative gain and T_F the filter time constant of the inner controller. In order to simplify the calculation, the process integrating term is virtually moved into the controller. In this case, the inner loop consists of the stable process $G^*_{P0}(s)$ (1) and the PI controller with filter:

$$G_{CI}^*(s) = \frac{K_{PI} + K_{DI} s}{s(1 + sT_F)}, \quad (5)$$

where now K_{PI} represents the integrating gain and K_{DI} the proportional gain of $G^*_{CI}(s)$.

To simplify the controller parameters derivation, the controller filter is virtually moved to the process. Therefore, the virtual controller transfer function becomes:

$$G_{CI}^{**}(s) = \frac{K_{PI} + K_{DI} s}{s}, \quad (6)$$

and the virtual process becomes:

$$G_{P0}^{**}(s) = \frac{K_{PR}(1 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots)}{(1 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots)(1 + sT_F)} e^{-sTdelay} \quad (7)$$

The controller $G^{**}_{CI}(s)$ parameters for the process $G^{**}_{P0}(s)$ can now be optimised according to the disturbance rejection magnitude optimum (DRMO) tuning method (Vrančić *et al.*, 2004, 2010). The final equation for the PI controller is the following (Vrančić *et al.*, 2004):

$$K_{DI} = \frac{\gamma_2 - sgn(\gamma_2) A_1 \sqrt{A_2^2 - A_1 A_3}}{\gamma_1},$$
$$K_{PI} = \frac{(1 + A_0 K_{DI}^*)^2}{2A_1}$$
$$\gamma_1 = A_0^2 A_3 - 2A_0 A_1 A_2 + A_1^3$$
$$\gamma_2 = A_1 A_2 - A_0 A_3 \quad (8)$$

Note that the areas A_i in (3) should be derived for the process (7) which also contains the controller filter term in the denominator.

Note that the calculation of the inner controller parameters is based on moving the process integrating term inside the G_{CI} and the controller filter inside the process. This is used only to simplify the derivation of the controller parameters. Note that the calculation of the main controller parameters will be based on the actual inner control configuration.

4. THE MAIN CONTROLLER

The main controller G_{CM} controls the inner loop. By using block manipulations, it can be shown that the inner loop transfer function G_{PI} (from signal u_{CM} to y), when applying (2) and (4) is the following:

$$G_{PI}(s) = \frac{f_0 - f_1 s + f_2 s^2 - f_3 s^3 + \cdots}{e_0 - e_1 s + e_2 s^2 - e_3 s^3 + \cdots}\ , \tag{9}$$

where

$$\begin{gathered} e_0 = A_0 K_{PI} \\ e_1 = A_1 K_{PI} - A_0 K_{DI} + 1 \\ e_2 = A_2 K_{PI} - A_1 K_{DI} + T_F \\ e_k = A_k K_{PI} - A_{k-1} K_{DI}\ ; k \geq 3 \\ f_0 = A_0 \\ f_k = A_k - A_{k-1} T_F\ ; k \geq 1\ , \end{gathered} \tag{10}$$

and where A_i are the characteristic areas of the process $G^*_{P0}(s)$ (2), T_F is the controller filter, and the inner controller parameters are K_{PI} and K_{DI} (8).

The main controller is the filtered PID controller:

$$G_{CM}(s) = \frac{K_I + K_P^* s + K_D^* s^2}{s(1 + sT_F)}, \tag{11}$$

where the filter time constant T_F is fixed to the same value as in inner controller (4). The main PID can then be calculated for process (9), where the MOMI tuning method can be used (Vrančić *et al.*, 2021). Note that the filter term in the denominator of (11) should be considered as a part of the process (9) when calculating the process characteristic areas A_i. The calculation of the PID controller parameters is as follows (Vrančić *et al.*, 2010):

$$\begin{bmatrix} K_I \\ K_P^* \\ K_D^* \end{bmatrix} = \begin{bmatrix} -A_1 & A_0 & 0 \\ -A_3 & A_2 & -A_1 \\ -A_5 & A_4 & -A_3 \end{bmatrix}^{-1} \begin{bmatrix} -0.5 \\ 0 \\ 0 \end{bmatrix}, \tag{12}$$

Note that areas Ai in (12) differ from the ones in (10).

By considering the inner (4) and the main (11) controller structures the following overall control equation is obtained:

$$\begin{gathered} u = \frac{K_I + K_P^* s + K_D^* s^2}{s(1 + sT_F)}(r - y) - \frac{K_{PI} + K_{DI} s}{1 + sT_F} y = \\ \frac{K_I + bK_P s + cK_D s^2}{s(1 + sT_F)} r - \frac{K_I + K_P s + K_D s^2}{s(1 + sT_F)} y \\ K_P = K_P^* + K_{PI} \\ K_D = K_D^* + K_{DI} \\ b = \frac{K_P^*}{K_P}, \qquad c = \frac{K_D^*}{K_D} \end{gathered} \tag{13}$$

Therefore, the controller in Figure 1 can be realised by the 2-DOF PID controller with reference weighting factors b (for P term) and c (for D term), as given in (13) and Figure 2.

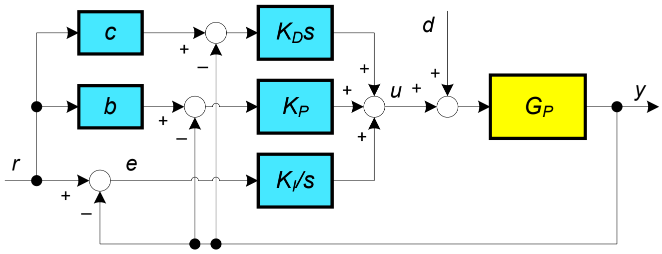

Fig. 2. The 2-DOF control realisation of the scheme in Figure 1.

5. EXAMPLES

Let us illustrate the proposed design on the following process models:

$$\begin{gathered} G_{P1}(s) = \frac{e^{-0.2s}}{s(1+s)(1+0.2s)} \\ G_{P2}(s) = \frac{e^{-s}}{s} \\ G_{P3}(s) = \frac{1}{s(1+s)^4} \end{gathered} \tag{14}$$

The chosen models are the same as the ones used in Kos *et al.* (2020b), since the results will be compared to the mentioned method. For all the processes, the controller filter time constant was T_F=0.01 s, since the same value was used by Kos *et al.* (2020b).

The calculated characteristic areas, according to the process $G_{PI}(s)$ (14), without the controller filter, are according to (3):

$$A_0 = 1,\ A_1 = 1.4,\ A_2 = 1.5,\ A_3 = 1.52. \tag{15}$$

The calculated specific areas, according to the process $G_{PI}(s)$ (14), including the controller filter, are, according to (3):

$$A_0 = 1,\ A_1 = 1.41,\ A_2 = 1.514,\ A_3 = 1.537. \tag{16}$$

The inner controller parameters (8) are:

$$K_{PI} = 2.04,\ K_{DI} = 1.40. \tag{17}$$

The calculated characteristic areas of the inner loop (9), including the main controller filter, are:

$$\begin{gathered} A_0 = 0.49,\ A_1 = 0.577,\ A_2 = 0.339,\ A_3 = 0.114, \\ A_4 = 0.01682,\ A_5 = 0.00152 \end{gathered} \tag{18}$$

The main PID controller parameters (12) are:

$$K_I = 1.346,\ K_P^* = 0.563,\ K_D^* = 0.065. \tag{19}$$

This corresponds to the following 2-DOF PID controller:

$$\begin{gathered} K_I = 1.346,\ K_P = 2.603,\ K_D = 1.463, \\ b = 0.22, c = 0.04. \end{gathered} \tag{20}$$

The obtained 2-DOF controller (20) will be compared with the controller provided in Kos *et al.* (2020b), which is set to the most optimal disturbance rejection response (b=c=0):

$$K_I = 1.399,\ K_P = 2.63,\ K_D = 1.466,$$
$$b = 0, c = 0. \qquad (21)$$

The closed-loop results are given in Figure 3. At t=0, the reference changes to r=1 and at the half time of the experiment, the disturbance signal changes to d=1 for all three examples. Since the controller parameters, without considering the weighting factors, are virtually the same, the disturbance rejection performance is almost the same, as seen in Figure 3. The only difference is slightly smaller undershoot of the proposed method. However, due to different weighting factors b and c, the reference tracking response of the proposed method is faster with slightly smaller overshoot.

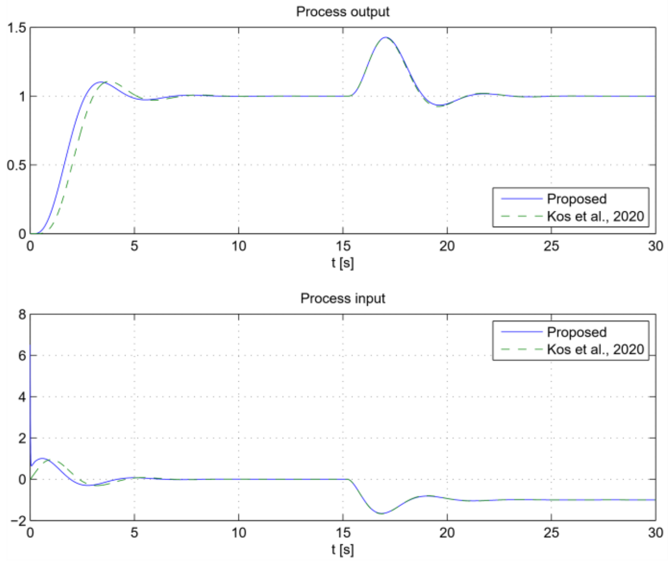

Fig. 3. The closed-loop process responses with integrating process $G_{P1}(s)$.

Similarly, the same procedure is repeated for the process $G_{P2}(s)$. The inner controller parameters (8) are:

$$K_{PI} = 0.796,\ K_{DI} = 0.268. \qquad (22)$$

The main PID controller parameters (12) are:

$$K_I = 0.441,\ K_P^* = 0.304,\ K_D^* = 0.0649. \qquad (23)$$

This corresponds to the following 2-DOF PID controller:

$$K_I = 0.441,\ K_P = 1.10,\ K_D = 0.333,$$
$$b = 0.276, c = 0.195. \qquad (24)$$

The obtained 2-DOF controller (24) will be compared with the controller calculated in Kos *et al.* (2020b), set to the most optimal disturbance rejection response:

$$K_I = 0.475,\ K_P = 1.128,\ K_D = 0.339,$$
$$b = 0, c = 0. \qquad (25)$$

The closed-loop results are given in Figure 4. Again, the disturbance rejection performance of the proposed method has slightly smaller undershoot, while the reference tracking response of the proposed method is significantly improved with faster response and slightly smaller overshoot.

The same tuning procedure has been applied to the process $G_{P3}(s)$. The calculated (proposed) 2-DOF PID controller is:

$$K_I = 0.0438,\ K_P = 0.375,\ K_D = 0.576,$$
$$b = 0.226, c = 0.086. \qquad (24)$$

The obtained 2-DOF controller (24) will be compared with the controller calculated in Kos *et al.* (2020b), set to the most optimal disturbance rejection response:

$$K_I = 0.0455,\ K_P = 0.379,\ K_D = 0.578,$$
$$b = 0, c = 0. \qquad (25)$$

The closed-loop results are given in Figure 5. Again, the disturbance rejection performances are almost the same with slightly smaller undershoot of the proposed method. The reference tracking response of the proposed method is visibly improved.

The integral of absolute error (IAE) values for all three tested processes for tracking (IAE_t) and disturbance rejection (IAE_d) responses are given in Table 1. It can be seen that all the IAE values are lower (better) when using the proposed method (note that the IAE_d values are comparable).

Due to space limitations, the fourth process (a non-minimum phase process) in Kos et al. (2020) is not presented herein. However, the proposed method again results in better IAE_t value and in comparable IAE_d value.

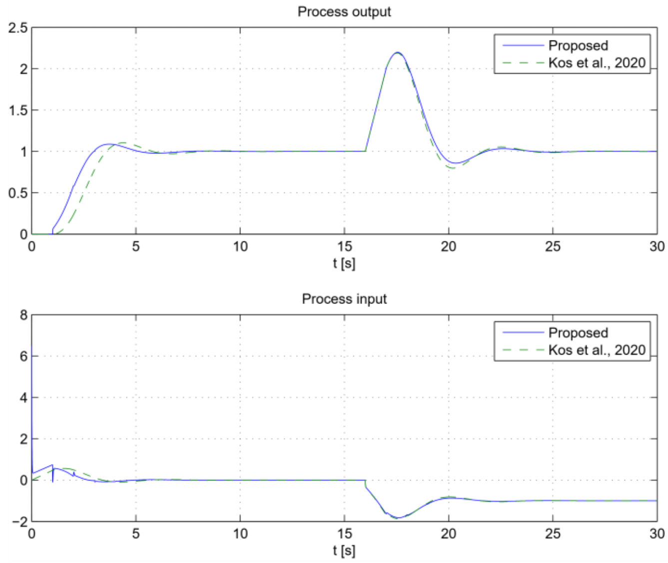

Fig. 4. The closed-loop process responses with integrating process $G_{P2}(s)$.

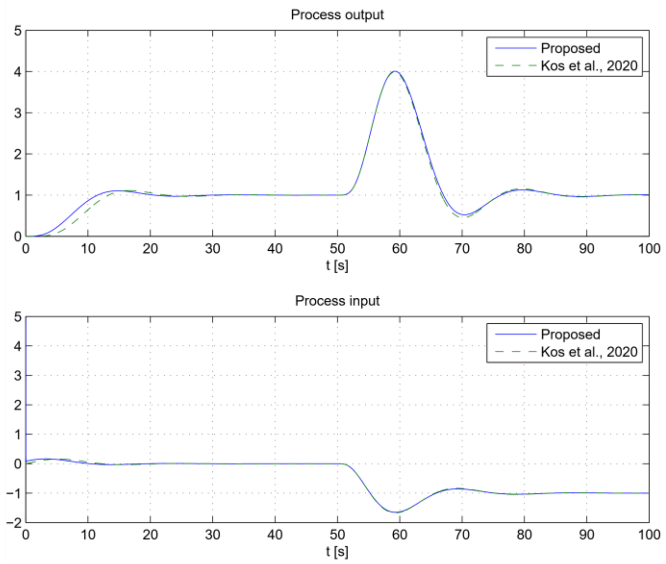

Fig. 5. The closed-loop process responses with integrating process $G_{P3}(s)$.

Table 1. The IAE values for tracking (IAE_t) and disturbance (IAE_d) rejection response for all three processes.

	proposed		Kos et al., 2020	
process	IAE_t	IAE_d	IAE_t	IAE_d
G_{P1}	1.79	0.92	2.18	0.93
G_{P2}	2.06	2.67	2.68	2.71
G_{P3}	7.87	28.57	9.67	28.69

6. CONCLUSIONS

The proposed simplified method for the calculation of the 2-DOF PID controller parameters gives virtually the same (optimal) disturbance rejection performance when compared to the original MOMI tuning method for three different integrating processes. At the same time, the proposed method visibly improves the tracking performance. The method was tested on three very different integrating process models where the results suggests that it is overall more optimal than the existing MOMI tuning method for integrating processes (Kos *et al*., 2020b). Another advantage of the proposed modification is that it does not require finding roots of the fourth-order equation.

The sensitivity and stability analyses of the proposed method are not performed, since the control performance is effectively the same as in Kos *et al*., 2020b, where both analyses were already carried out extensively. Currently, the implementation of the proposed method to higher-order controller structures is under investigation. The second line of research is adding user-defined parameter to define a trade-off between tracking and disturbance rejection performance. The online tool for the calculation of the controller parameters is also under preparation.

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