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MOMI tuning method based on frequency-response data

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Abstract: The paper presents a modification of the Magnitude Optimum Multiple Integration (MOMI) method process non-parametric data in the frequency domain instead of the time domain. The required frequency data are obtained directly from the filtered amplitude-shifted process step response and have been shown to be relatively insensitive to normally distributed process noise. All calculations, including the calculation of the PID controller parameters, are performed analytically. The closed loop responses to tested processes with added normally distributed noise were relatively fast with small or no overshoot, all according to the Magnitude Optimum (MO) method. The proposed method is not limited to open loop step responses or to the PID controller structure.

Keywords: controller tuning, PID control, Magnitude Optimum, frequency response, nonparametric identification.

1. INTRODUCTION

Tuning of the PID controller parameters is a hot topic for many decades (Visioli, 2006; Vilanova and Visioli, 2012). In general, PID controller tuning methods can be divided into parametric and non-parametric methods. The parametric methods are based on the process models, while the non-parametric methods use only the process measurements in the time or frequency domain.

The representative of the non-parametric tuning method is the Magnitude Optimum Multiple Integration (MOMI) tuning method (Vrančić et al., 1999; Vrančić et al., 2001), which is based on magnitude (amplitude) optimum principle (see, e.g., Veinović et al., 2023). The MOMI method does not require an explicit process model and the tuning can be based solely on process time response during the steady-state change.

The closed loop responses to reference changes are relatively fast with small overshoots for a wide range of process models (lower order, higher order, delayed and non-minimum phase processes). The method was later applied to Smith predictors (Vrečko et al., 2001), optimised for disturbance rejection (Vrančić et al., 2010) and applied to integrating processes (Kos et al., 2020).

While the method has been successfully applied in practice as well, it has one inherent shortcoming. The MOMI method is based on repeated integrations of the process input and output signals in order to calculate the so-called “characteristic areas” or “moments” of the process (Vrančić *et al*., 2001; Vrančić and Huba, 2021). Those moments are then used for the calculation of the controller parameters. However, in practice, the problem arises when the process output and/or input signals contain lower-frequency noise or disturbances. In this case, the calculation of the lower moments may contain some error which is then propagated and amplified when calculating higher moments. Naturally, the process moments can also be calculated directly from the process transfer function, but this defies the advantages of the proposed tuning method (which is based on non-parametric process data). Another way to use non-parametric process data is to use frequency response data (Vrančić et al., 2000). This means that the frequency response of the process transfer function (Nyquist points) at a low frequency and the first two derivatives of this frequency response over the frequency can also be used to tune the PID controller.

However, although the first derivative of the frequency response can be obtained by choosing appropriate excitation signals (DeKeyser *et al*., 2019), the obtained derivative can still be sensitive to process noise. The second-order derivative is even harder to compute reliably. Therefore, the direct application of the proposed tuning method in Vrančić *et al*. (2000) is challenging.

Here, we propose a novel tuning approach in which multiple measurements of the Nyquist points of the process at lower frequencies are used for the calculation without the need for the derivatives of the frequency response. In addition to the proposed tuning approach, we also propose an alternative way to calculate the process Nyquist response from the input and output signals of the process. As shown in this paper, the proposed approach is also relatively insensitive to the process noise.

The rest of the paper will be as follows. The proposed MO tuning method based on the frequency response of the process is explained in Section 2. Section 3 focuses on the measurement of the process frequency response in practice, where an innovative method for measuring the frequency response is presented. Examples on two process models will be carried out in section 4 and concluding remarks are given in section 5.

2. MO FREQUENCY TUNING METHOD

The Magnitude Optimum (MO) tuning method is aiming at achieving flat closed-loop amplitude (magnitude) frequency response in lower-frequency region. The amplitude should remain equal to 1 from ω=0 to as high frequency ω as possible.

$$|G_{CL}(j\omega)| \approx 1, \tag{1}$$

where $G_{CL}(j\omega)$ denotes the closed-loop frequency response.

This criterion can be represented by the open-loop Nyquist curve of the controller and the process ($G_{OL}(j\omega)$) which follows vertical line with real value -0.5:

$$Re\{G_{OL}(j\omega)\} \approx -0.5 \tag{2}$$

from ω=0 to the highest frequency possible, where

$$G_{OL}(j\omega) = G_C(j\omega)G_P(j\omega). \tag{3}$$

G_C and G_P in expression (3) stand for the controller and the process transfer function, respectively (Vrančić *et al.*, 1999).

The proposed PID tuning method, therefore, finds such controller parameters that expression (2) approximately holds in the lower frequency region.

The PID controller transfer function, in Laplace domain, is:

$$G_C(s) = \frac{K_I + K_P s + K_D s^2}{s(1 + sT_F)}, \tag{4}$$

where K_I, K_P, and K_D are integral, proportional and derivative gains, respectively, while T_F is the controller filter time constant.

In the proposed method, to simplify the calculations, the denominator of (4) will be considered as a part of the process. Such filtered and integrated process transfer function G^*_P becomes:

$$G_P^*(s) = \frac{G_P(s)}{s(1 + sT_F)}. \tag{5}$$

The modified controller transfer function therefore loses the filter and integrating term:

$$G_C^*(s) = K_I + K_P s + K_D s^2 \tag{6}$$

Note that the open-loop transfer function

$$G_{OL}(s) = G_C(s)G_P(s) = G_C^*(s)G_P^*(s) \tag{7}$$

remains the same when using modified controller and process transfer functions.

The tuning procedure is then based on the following equation:

$$\boldsymbol{Re}\{G_{OL}(j\omega)\} = Re\{G_C^*(j\omega)G_P^*(j\omega)\} \approx -0.5 \tag{8}$$

Therefore,

$$\boldsymbol{Re}\{(K_I - K_D\omega^2 + jK_P\omega)G_P^*(j\omega)\} \approx -0.5 \tag{9}$$

in lower frequency region. Expression (9) can be rewritten into

$$\begin{aligned} Re\{G_{OL}(j\omega)\} = \boldsymbol{Re}\{G_P^*(j\omega)\}\,(K_I - K_D\omega^2)\ - \\ -\boldsymbol{Im}\{G_P^*(j\omega)\}\,(K_P\omega) \approx -0.5 \end{aligned} \tag{10}$$

In matrix form, expression (10) can be rewritten as:

$$M\Theta = \begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \\ \vdots \end{bmatrix} \tag{11}$$

where

$$M = \begin{bmatrix} \boldsymbol{Re}\{G_P^*(j\omega_1)\} & -\boldsymbol{Im}\{G_P^*(j\omega_1)\}\omega_1 & -\boldsymbol{Re}\{G_P^*(j\omega_1)\}\omega_1^2 \\ \boldsymbol{Re}\{G_P^*(j\omega_2)\} & -\boldsymbol{Im}\{G_P^*(j\omega_2)\}\omega_2 & -\boldsymbol{Re}\{G_P^*(j\omega_2)\}\omega_2^2 \\ \boldsymbol{Re}\{G_P^*(j\omega_3)\} & -\boldsymbol{Im}\{G_P^*(j\omega_3)\}\omega_3 & -\boldsymbol{Re}\{G_P^*(j\omega_3)\}\omega_3^2 \\ \vdots & \vdots & \vdots \end{bmatrix}$$

$$\Theta = \begin{bmatrix} K_I \\ K_P \\ K_D \end{bmatrix} \tag{12}$$

Therefore, the controller parameters (vector Θ) can be calculated from three or more Nyquist points of filtered process frequency response $G_P^*(j\omega)$ in lower frequency region. If only three frequency points are used, the calculation is a relatively simple:

$$\Theta = M^{-1}\begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \end{bmatrix} \tag{13}$$

In practice, more robust solution is obtained when using more frequency measurements. In this case the controller parameters can be calculated by the least-squares as follows:

$$\Theta = (M^T M)^{-1} M^T \begin{bmatrix} -0.5 \\ -0.5 \\ -0.5 \\ \vdots \end{bmatrix} \tag{14}$$

3. OBTAINING PROCESS FREQUENCY RESPONSE IN PRACTICE

One of the most common technique for obtaining process frequency response is by applying sinusoidal signals to the process input or by using relay excitation approach (Åström and Hägglund, 1995; de Keyser *et al.*, 2019; Pavković *et al.*, 2021). Such identification procedure can take longer time, especially when the method requires the measurements of the process lower-frequency points. Here we propose another approach which can quite reliably estimate the process frequency response also from the noisy process time responses.

First, we have to obtain the process open-loop step response, which, in Laplace form, is as follows:

$$Y(s) = \frac{G_P(s)}{s}, \tag{15}$$

where $Y(s)$ represents the Laplace transform of the process output signal. The measured process step-response is then filtered by the a-priori chosen controller filter with time constant T_F. The filtered step-response, in Laplace domain, can then be represented as follows:

$$Y^*(s) = \frac{G_P(s)}{s(1+sT_F)}. \tag{16}$$

As can be seen, the Laplace form of the filtered response (16) corresponds to expression (5). Since both expressions are the same, the integrated and filtered process responses in frequency domain may be calculated by applying Fourier transform on the time-domain signal (16).

However, the Fourier transform cannot be applied to signals that are not absolutely integrable. Unfortunately, the filtered process step-response (16) is such a signal. The usual approach in such a situation is to differentiate process output response and then apply the Fourier transform. However, by differentiating the noisy process response, the noise in the differentiated signal is significantly amplified, leading to unreliable Fourier transform results.

In order to solve the mentioned problem, we propose a different approach. The filtered step-response should be subtracted from the final value of the step-response:

$$y^{**}(t) = y(\infty)h(t) - y^*(t), \tag{17}$$

where $h(t)$ denotes the unity-step signal which changes from 0 to 1 at t=0 and $y(\infty)$ denotes the final value of the filtered step-response. The signal $y^{**}(t)$ is absolutely integrable, so the Fourier transform can be applied on it. The Laplace transform of signal $y^{**}(t)$ is:

$$Y^{**}(s) = G_P^{**}(s) = \frac{y(\infty)}{s} - \frac{G_P(s)}{s(1+sT_F)}. \tag{18}$$

Therefore, when the Fourier transform of the signal $y^{**}(t)$ is already calculated, the following transformation should be applied to get the Fourier transform of the filtered step-response of the process:

$$Y^*(j\omega) = G_P^*(j\omega) = -j\frac{y(\infty)}{\omega} - Y^{**}(j\omega). \tag{19}$$

The calculation of $Y^*(j\omega)$ should be performed at three or more frequencies ω and then it can be applied to expression (12).

Remark 1. The duration of the process step-response experiment should be chosen so that the process output settles down. This is usually achieved after the sum of the process time delay and several process time constants.

Remark 2. The calculation of the final value of the process filtered step-response $y(\infty)$ can be estimated by averaging the last 5-10 % of the process step-response.

Remark 3. The filtered process step-response should be divided by the amplitude of the process input step change.

Remark 4. Selecting the highest frequency is not an easy task. In practice, it can be based on the duration of the experiment, as will be shown in the next section.

4. EXAMPLES

The tuning procedure will be illustrated on two different process models, although the authors tried it on several process models. The first example is the following first-order process model with time delay:

$$G_P(s) = \frac{e^{-s}}{1+2s}. \tag{20}$$

In both experiments, the sampling time is T_S=0.002 s and time of experiment is T_{exp}=15 s. The normally distributed noise signal (e.g. with Matlab function randn) with amplitude 0.05 and sampling time T_S=0.002 s is added to the process output for both tested process models. The PID controller filter time constant was a-priori chosen as T_F=0.02 s (holds for both tested processes).

The process time response, when applying step signal at the process input, is shown in Figure 1.

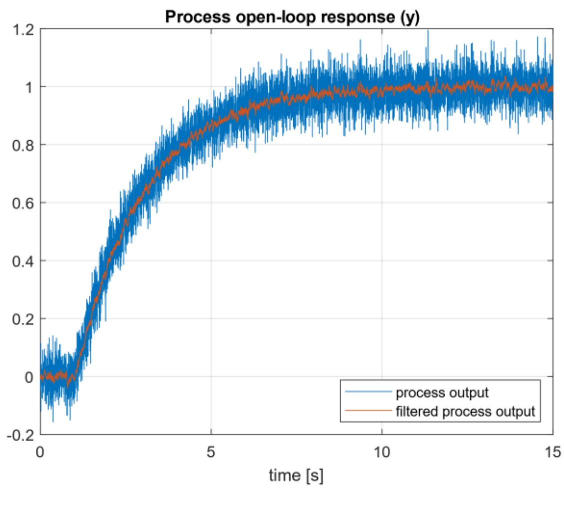

Fig. 1. The open-loop step response of the process (20) (blue line) and the filtered response (orange line).

The filtering of the process output signal was performed by the first-order filter with the time constant equivalent to T_F=0.02 s. The Nyquist curve of the filtered process response is calculated by expression (19) and shown in Figure 2. It is clear that the calculated frequency response is very similar to the theoretical one in spite of a considerable process noise.

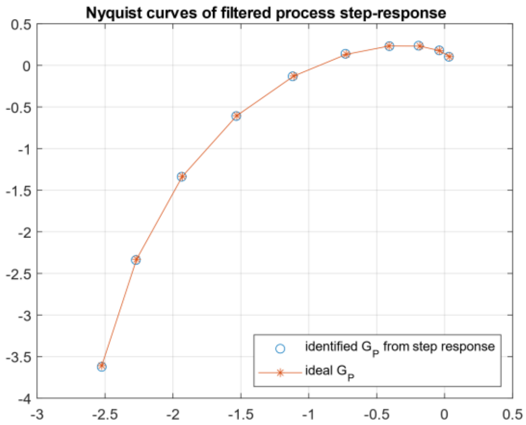

Fig. 2. Nyquist curve of the filtered process (20) step-response

The 10 frequency points in the Nyquist plot were chosen to span logarithmically equidistantly over one frequency decade, where the lowest and the highest frequencies are

$$f_{min} = \frac{1}{2T_{exp}}\ [Hz],\ \ f_{max} = \frac{10}{2T_{exp}}[Hz] \qquad (21)$$

Naturally, the process Nyquist curve can be easily calculated by multiplying the Nyquist curve of the filtered process step-response (Figure 2) by $s\cdot(1+sT_F)$ in Laplace space or by $j\omega\cdot(1+j\omega T_F)$ in frequency-domain. Just for illustration purposes, the calculated Nyquist curve of the original process is shown in Figure 3.

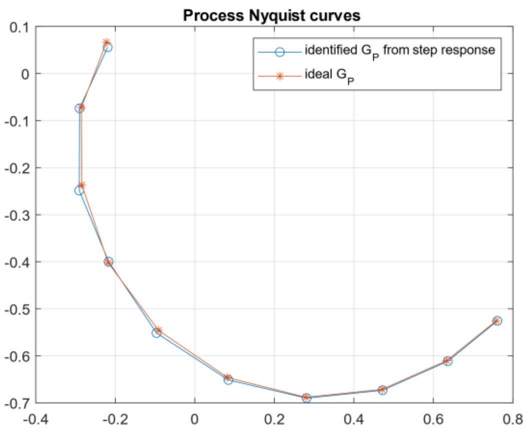

Fig. 3. Nyquist curve of the identified (calculated) process (20) (circles) and the actual Nyquist curve of the process (stars).

The lowest 7 Nyquist frequency points in Figure 2 were fed to expressions (12) and (14) to calculate the controller parameters. The resulting PID controller parameters were:

$$K_I = 0.789,\ K_P = 1.87,\ K_D = 0.64 \qquad (22)$$

The open-loop Nyquist curve of $G_C G_P$ is then shown in Figure 4. It can be seen that the open-loop frequency response tightly follows vertical line with real value of -0.5 at low frequencies, that corresponds to the MO optimum criterion.

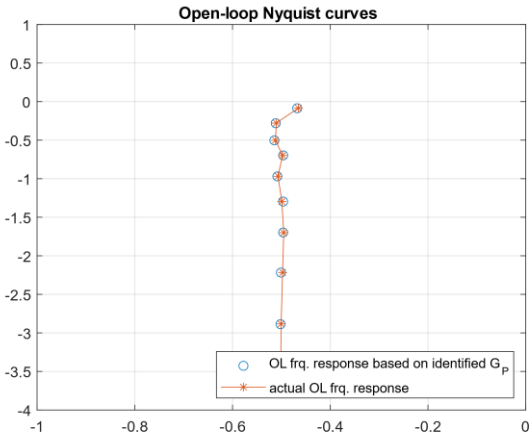

Fig. 4. The identified (circles) and the actual (stars) open-loop Nyquist curve $G_C(j\omega)G_P(j\omega)$.

The obtained controller was then tested in the closed-loop configuration with the process. The closed-loop response is shown in Fig. 5. As expected, the closed-loop response is relatively fast with small overshoot, all according to the MO tuning method.

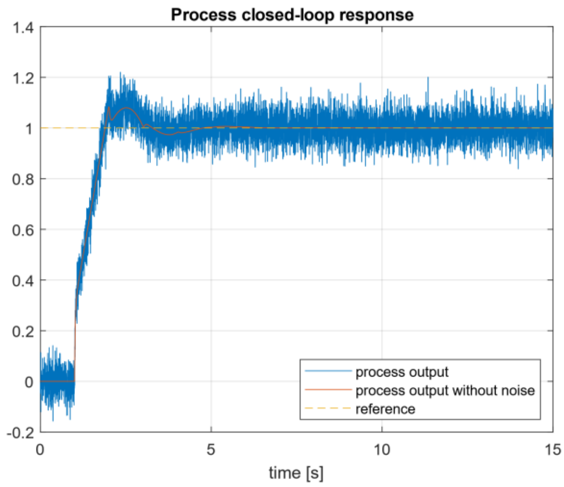

Fig. 5. The closed-loop time response of the process (20) output.

The second example is the third-order process:

$$G_P(s) = \frac{1}{(1+s)^3}. \qquad (23)$$

The process time response, when applying step signal at the process input, is shown in Figure 6.

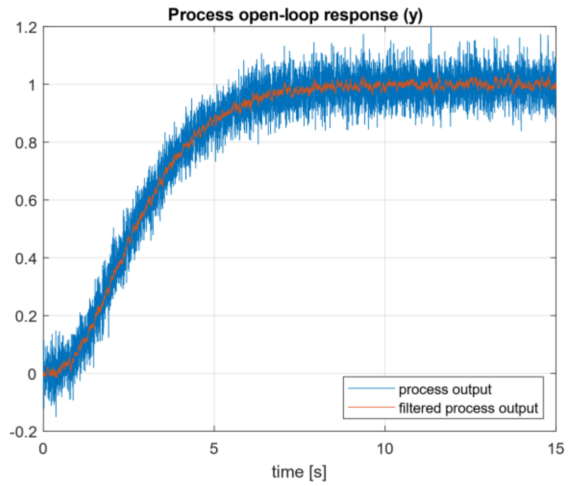

Fig. 6. The open-loop step response of the process (23) (blue line) and the filtered response (orange line).

All the filtering and calculation steps were the same as in the first example. The Nyquist curve of the filtered process response is calculated by expression (19) and shown in Figure 7. Again, the calculated frequency response is very similar to the theoretical one.

Like in the previous example, the lowest 7 Nyquist frequency points in Figure 11 were fed to expressions (12) and (14) to calculate the controller parameters. The resulting PID controller parameters were:

$$K_I = 1.11,\ K_P = 2.84,\ K_D = 2.07 \tag{24}$$

The open-loop Nyquist curve of $G_C G_P$ is then shown in Figure 8. It can be seen that the open-loop frequency response again tightly follows vertical line with real value of -0.5.

The obtained controller was then tested in the closed-loop configuration with the process. The closed-loop response is shown in Fig. 9. As expected, the closed-loop response is relatively fast with small overshoot, all according to the MO tuning method.

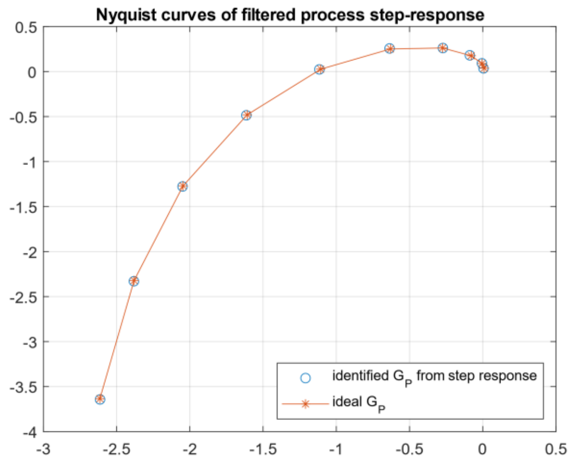

Fig. 7. Nyquist curve of the filtered process (23) step-response

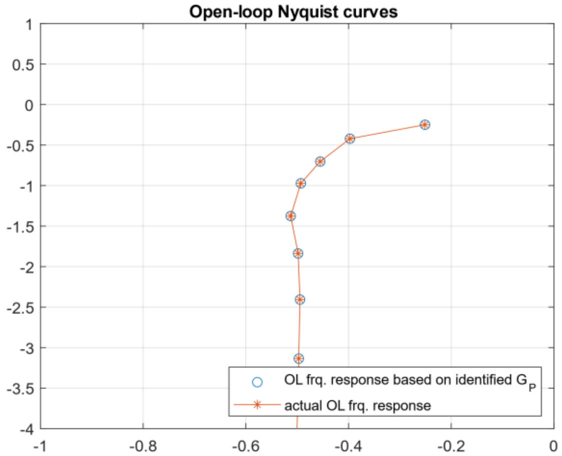

Fig. 8. The identified (circles) and the actual (stars) open-loop Nyquist curve $G_C(j\omega)G_P(j\omega)$.

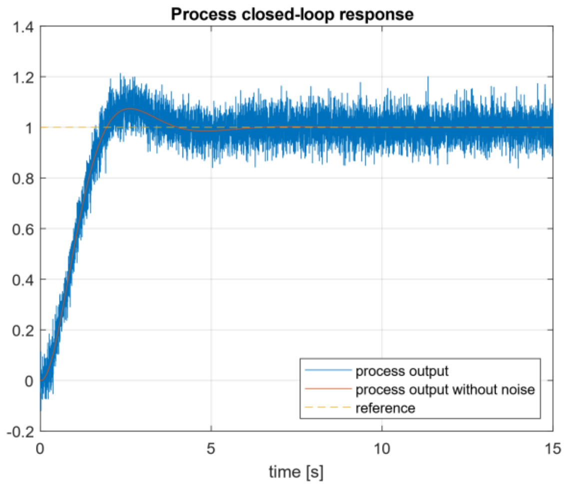

Fig. 9. The closed-loop time response of the process (23) output.

Due to the lack of space, only two examples were shown. However, the proposed method was also tested on other types of processes, including the non-minimum phase processes. The results were similar to the ones presented above. The identified process frequency responses were very close to the actual ones and the closed-loop responses were fast and non-oscillatory, all according to the MO tuning method.

5. CONCLUSIONS

The proposed PID tuning method is based on the open-loop process step responses. The open-loop time-response is first transferred into the frequency-domain by Fourier transform of the shifted process open-loop response. Then the process frequency response is obtained by simple algebraic manipulation in the frequency domain. The obtained frequency responses were very similar to the ideal ones in spite of a relatively high levels of the process noise. The PID controller parameters were also determined analytically and without any optimisation. The closed-loop time responses were relatively fast with small or no overshoot, all according to the Magnitude Optimum (MO) method.

The proposed method can be therefore considered as an extension of the MOMI method into the frequency domain and can also be used when the process output signal contains a higher level of noise. Moreover, frequency-domain data opens up new possibilities for additional optimisation of the closed-loop response. It is also worth mentioning that the proposed method is not limited to the process step-response measurement and to the PID controller structure.

In future research, the sensitivity of the method to process noise and the duration of the experiment will be investigated in more detail. Since the proposed method is not limited to PID controllers, the tuning of higher-order controllers and the series realisation of the control algorithm will be investigated as well. The time-varying systems identification is also investigated. Modification of the proposed tuning method for finer, user-defined, adjustment of the closed-loop response is also foreseen. Future research will also include the optimisation of disturbance rejection performance, since 1 degree of freedom (1-DOF) controller structure can lead to sub-optimal disturbance rejection responses.

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