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Prediction method for closed-loop data for emulated linear time-varying system

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Abstract: In this study, data-driven control methods, namely VRFT, FRIT, and estimated response iterative tuning (ERIT), are proposed for controlled plants in linear systems. Notably, the ERIT method allows the prediction of input/output data in advance and the adjustment of feedforward controllers. A method is proposed in this study for predicting closed-loop data for an arbitrary controller with a linear and time-invariant system as the controlled plant.

Keywords: Database-driven control, data prediction, linear time-varying system

1. INTRODUCTION

In recent years, data-driven control system design methods have been proposed for systems where the structure or parameters are not well understood, allowing for the direct calculation of control parameters from operational data without relying on modeling [Hjalmarsson et al. (1998); Campi et al. (2002); Kaneko (2013); Kaneko et al. (2018); Sakatoku et al. (2021); Yamamoto et al. (2009); Wakitani et al. (2019)]. In methods such as the Estimated Response Iterative Tuning (ERIT) [Kaneko et al. (2018)] and those based on the superposition principle [Sakatoku et al. (2021)], it is possible not only to adjust the controller but also to predict input/output data when the controller is applied in advance. Additionally, the Database-Driven control method [Wakitani et al. (2019)], proposed for nonlinear systems, has reported examples of practical applications.

The Database-Driven control method [Wakitani et al. (2019)] relies on a database-centric approach, adjusting PID gains by referencing stored data. Therefore, greater control performance improvement is expected with the ability to store more operational data. To achieve this, the authors have proposed a method in which input/output data is predicted in advance using the ERIT method, and the predicted data is stored in a database to enhance control performance [Okada et al. (2019)]. However, the ERIT method provides predictions only when the controller is modified and cannot predict the response when the controlled system undergoes a system variation.

This paper proposes a method based on the approach outlined in [Sakatoku et al. (2021)] to predict input/output data when the controlled system undergoes a system variation. Here, a linear time-varying system is emulated with a finite impulse response (FIR)-type filter structure. In addition, a technique is proposed that can improve control performance using a database-driven control method utilizing data predicted by the proposed scheme, even if the time of the system variation is unknown.

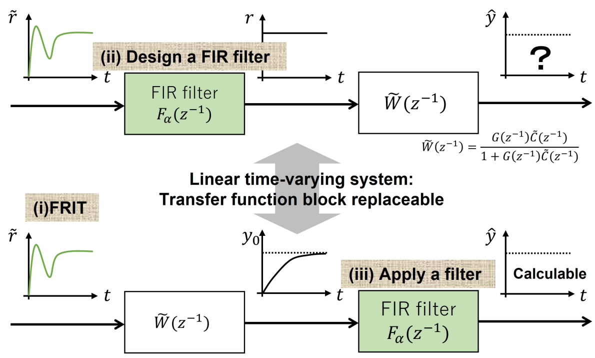

Fig. 1. Closed-loop data prediction using FIR filter.

2. PREDICTION OF CLOSED-LOOP RESPONSE FOR ARBITRARY CONTROLLER $\tilde{C}(z^{-1})$

In a previous study [Sakatoku et al. (2021)], the pseudo-reference input $\tilde{r}(t)$ in the FRIT method was used to predict the closed-loop response based on the "principle of superposition."

In this study, similar to the previous work, the pseudo-reference input $\tilde{r}(t)$ is utilized. However, as illustrated in Fig. 1, the focus is on the interchangeability of the transfer function blocks of a linear time-invariant system. Data prediction is achieved using a FIR filter $F_\alpha(z^{-1})$. The prediction of closed-loop response data follows the steps outlined below.

(i) Calculation of $\tilde{r}(t)$ using the FRIT method
(ii) Design of FIR filter $F_\alpha(z^{-1})$
(iii) Prediction of closed-loop response using $F_\alpha(z^{-1})$

Firstly, the controlled plant $G(z^{-1})$ considered in this study is an "unknown single-input/single-output discrete-time linear time-invariant (LTI) system." Here, the initial input/output data of the closed-loop and reference signal $\{u_0(t), y_0(t), r(t)\}_{t=0,1,2\cdots,N-1}$ (where N is the number of data) are assumed to have been obtained in advance, and the controller here is denoted as $C_0(z^{-1})$.

(i) Computation of $\tilde{r}(t)$ using the FRIT Method

Introduce the pseudo reference input $\tilde{r}(t)$ in the FRIT method with the following equation:

$$\tilde{r}(t) = \tilde{C}^{-1}(z^{-1})u_0(t) + y_0(t), \tag{1}$$

where $\tilde{C}(z^{-1})$ is a controller with parameters different from the initial controller $C_0(z^{-1})$. As shown in Fig. 1, $\tilde{C}(z^{-1})$ and the initial output $y_0(t)$ are expressed by the following equations:

$$y_0(t) = \tilde{W}(z^{-1})\tilde{r}(t) \tag{2}$$

$$\tilde{W}(z^{-1}) := \frac{G(z^{-1})\tilde{C}(z^{-1})}{1 + G(z^{-1})\tilde{C}(z^{-1})}, \tag{3}$$

where $\tilde{W}(z^{-1})$ is the closed-loop transfer function constructed by $\tilde{C}(z^{-1})$.

(ii) Design of FIR Filter $F_\alpha(z^{-1})$

a FIR filter $F_\alpha(z^{-1})$ satisfying the relation in Equation (4) is introduced:

$$r(t) = F_\alpha(z^{-1})\tilde{r}(t) \tag{4}$$

$$F_\alpha(z^{-1}) := \alpha_0 + \alpha_1 z^{-1} + \cdots + \alpha_{N-1} z^{-(N-1)}. \tag{5}$$

Each parameter α_i $(i = 0, 1, \cdots, N-1)$ of $F_\alpha(z^{-1})$ can be calculated using Equation (6):

$$\boldsymbol{\alpha} = \tilde{R}^{-1}\boldsymbol{r_0} \tag{6}$$

$$\boldsymbol{\alpha} := [\, \alpha_0 \ \alpha_1 \ \cdots \ \alpha_{N-1} \,]^\top \tag{7}$$

$$\tilde{R} := \begin{bmatrix} \tilde{r}(0) & 0 & \cdots & 0 \\ \tilde{r}(1) & \tilde{r}(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}(N-1) & \tilde{r}(N-2) & \cdots & \tilde{r}(0) \end{bmatrix} \tag{8}$$

$$\boldsymbol{r_0} := [\, r(0) \ r(1) \ \cdots \ r(N-1) \,]^\top . \tag{9}$$

Here, if $\tilde{r}(0) \neq 0$, $\tilde{R}$ becomes regular, and Equation (6) is computable.

(iii) Prediction of Closed-Loop Response Using $F_\alpha(z^{-1})$

By using the aforementioned FIR filter $F_\alpha(z^{-1})$, the predicted response $\hat{y}(t)$ when implementing the controller $\tilde{C}(z^{-1})$, as shown in Fig. 1, can be calculated using the following equation:

$$\hat{y}(t) = F_\alpha(z^{-1})y_0(t). \tag{10}$$

Similarly, the predicted input signal $\hat{u}(t)$ can be calculated using the following equation.

$$\hat{u}(t) = F_\alpha(z^{-1})u_0(t). \tag{11}$$

3. CLOSED-LOOP DATA PREDICTION CONSIDERING SYSTEM VARIATION $G_\Delta(z^{-1})$

In this study, closed-loop data prediction is considered for a linear time-varying system that undergoes system variation over time. Specifically, a multiplicative system variation $G_\Delta(z^{-1})$ is assumed, as given by the following equation:

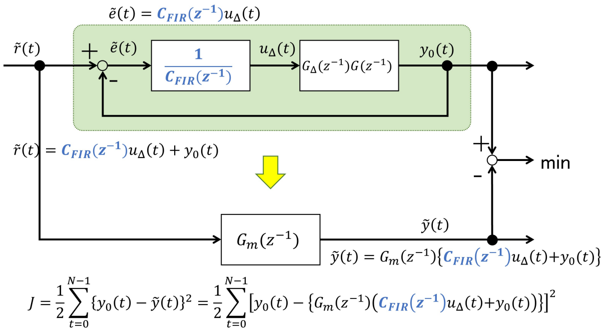

Fig. 2. FRIT method using FIR controller.

$$\tilde{G}(z^{-1}) := G_\Delta(z^{-1}) \cdot G(z^{-1}), \tag{12}$$

where $\tilde{G}(z^{-1})$ represents the controlled plant after system variation, and $G_\Delta(z^{-1})$ is the transfer function of the user-defined multiplicative variation.

In this study, the methodology outlined in Section 2 is employed to predict the response of the system after variation, denoted as $\tilde{G}(z^{-1})$. Specifically, the $\tilde{r}(t)$ term in Eq. (8) is replaced with the following equation:

$$\tilde{r}(t) = \tilde{C}^{-1}(z^{-1})u_\Delta(t) + y_0(t) \tag{13}$$

$$u_\Delta(t) := G_\Delta^{-1}(z^{-1})u_0(t). \tag{14}$$

Here, it should be noted that $y_0(t) = \tilde{G}(z^{-1})u_\Delta(t)$.

By using Eq. (13) to construct Eq. (8) and applying Eqs. (6), (10), the closed-loop predicted output $\hat{y}(t)$ for the controlled plant $\tilde{G}(z^{-1})$ and controller $\tilde{C}(z^{-1})$ can be calculated. Additionally, the predicted input is computed using the following equation:

$$\hat{u}(t) = F_\alpha(z^{-1})u_\Delta(t) \tag{15}$$

It should be noted that the methodology in Section 2 is applicable to linear time-invariant systems. Therefore, in this study, $G_\Delta(z^{-1})$ is given in the form of a FIR model. For example, assuming system variation occurs at a certain time t_n, $G_\Delta(z^{-1})$ is set as follows:

$$G_\Delta(z^{-1}) = 1 + g_{t_n} z^{-t_n}, \tag{16}$$

where g_{t_n} is a parameter provided by the user.

4. DESIGN OF CONTROLLER TO ACHIEVE DESIRED CONTROL PERFORMANCE

In Section 3, a method was proposed for predicting the closed-loop response when an arbitrary controller $\tilde{C}(z^{-1})$ is given. In this section, the method for designing a controller to achieve the desired reference trajectory $y_{ref}(t)$ is explained (Fig. 2).

Firstly, the reference trajectory $y_{ref}(t)$ is expressed using a reference model $G_m(z^{-1})$ as follows [Yamamoto and Shah (2004)]:

$$y_{ref}(t) = G_m(z^{-1})r(t) \tag{17}$$

$$G_m(z^{-1}) := \frac{z^{-1}P(1)}{P(z^{-1})}, \tag{18}$$

where $P(z^{-1})$ is the polynomial of the reference model, given by:

$$P(z^{-1}) := 1 + p_1 z^{-1} + p_2 z^{-2} \tag{19}$$

$$\begin{cases} p_1 = -2\exp\left(-\dfrac{\rho}{2\mu}\right)\cos\left(\dfrac{\sqrt{4\mu-1}}{2\mu}\rho\right) \\ p_2 = \exp\left(-\dfrac{\rho}{\mu}\right) \\ \rho := T_s/\sigma \\ \mu := 0.25(1-\delta) + 0.51\delta \end{cases}. \tag{20}$$

Here, T_s is the sampling time, and σ, δ are parameters related to the rise time and damping characteristics of the control system, respectively, set arbitrarily by the user.

In the previous section, a multiplicative system variation $G_\Delta(z^{-1})$ was provided as a FIR-type transfer function. Therefore, to correspond to the system variation of $G_\Delta(z^{-1})$, it is desirable to have the controller also in FIR type ideally. Here, as shown in Fig. 2, since the FRIT method requires the inverse system of the controller, its inverse system is expressed in the following FIR type equation:

$$C_{FIR}(z^{-1}) := c_0 + c_1 z^{-1} + \cdots + c_{N-1} z^{-(N-1)}. \tag{21}$$

The performance evaluation criterion J_{FIR} for the FRIT method in Fig. 2 is given by the following equation:

$$\begin{aligned} J_{FIR} &= \frac{1}{2}\sum_{t=0}^{N-1}\{y_0(t) - \tilde{y}(t)\}^2 \\ &= \frac{1}{2}(\boldsymbol{\nu} - \Psi\boldsymbol{\theta})^\top(\boldsymbol{\nu} - \Psi\boldsymbol{\theta}), \end{aligned} \tag{22}$$

where each variable in the above equation is as follows:

$$\tilde{y}(t) = G_m(z^{-1})\left\{C_{FIR}(z^{-1})u_\Delta(t) + y_0(t)\right\} \tag{23}$$

$$\boldsymbol{\theta} := [c_0, c_1, \cdots, c_{N-1}]^\top \tag{24}$$

$$\Psi := [\boldsymbol{\psi}(0), \boldsymbol{\psi}(1), \cdots, \boldsymbol{\psi}(N-1)]^\top \tag{25}$$

$$\boldsymbol{\psi}(t) := \begin{bmatrix} G_m u_\Delta(t) \\ G_m u_\Delta(t-1) \\ \vdots \\ G_m u_\Delta(t-N+1) \end{bmatrix}^\top \tag{26}$$

$$\begin{aligned} \boldsymbol{\nu} := [&(1 - G_m(z^{-1}))y_0(0), (1 - G_m(z^{-1}))y_0(1), \\ &\cdots, (1 - G_m(z^{-1}))y_0(N-1)]^\top \end{aligned} \tag{27}$$

Therefore, the control parameters $\boldsymbol{\theta}^*$ that minimize Eq. (22) can be obtained by applying the least squares method as follows:

$$\boldsymbol{\theta}^* = (\Psi^\top\Psi)^{-1}\Psi^\top\boldsymbol{\nu}. \tag{28}$$

It should be noted that using the controller $C^*_{FIR}(z^{-1})$ composed of $\boldsymbol{\theta}^*$, the predicted input/output data is calculated by constructing Eq. (8) using the $\tilde{r}(t)$ in the following equation and applying Eqs. (6), (10), (15):

$$\tilde{r}(t) = C^*_{FIR}(z^{-1})u_\Delta(t) + y_0(t). \tag{29}$$

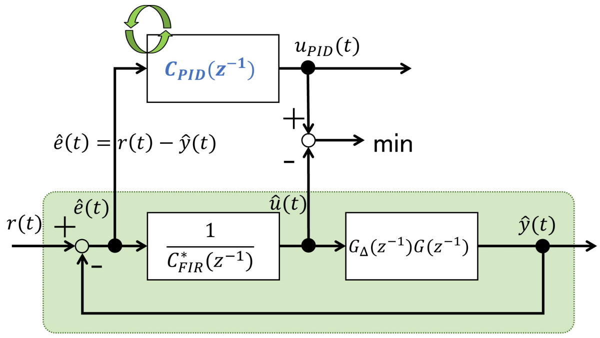

Fig. 3. Design of a PID controller.

5. DESIGN OF A PID CONTROLLER MIMICKING THE CHARACTERISTICS OF $C^*_{FIR}(z^{-1})$

In the previous section, considering the system variation $G_\Delta(z^{-1})$, the controller $C^*_{FIR}(z^{-1})$ was designed. However, $C^*_{FIR}(z^{-1})$ has a high order, leaving challenges on the implementation front. Therefore, in this section, a PID controller $C_{PID}(z^{-1})$ is designed to mimic the characteristics of $C^*_{FIR}(z^{-1})$ (Fig. 3).

Firstly, as the reference signal $r(t)$ is known, the predicted deviation $\hat{e}(t)$ is obtained using the predicted output $\hat{y}(t)$ calculated in the previous section.

$$\hat{e}(t) = r(t) - \hat{y}(t). \tag{30}$$

Here, $u_{PID}(t)$ in Fig. 3 is calculated as follows:

$$u_{PID}(t) = C_{PID}(z^{-1})\hat{e}(t) \tag{31}$$

$$C_{PID}(z^{-1}) := K_P + \frac{K_I}{\Delta} + K_D\Delta, \tag{32}$$

where K_P, K_I, K_D represent the proportional gain, integral gain, and derivative gain, respectively. To optimize these PID gains, the performance criterion J_{PID} given by the following equation is minimized:

$$J_{PID} = \sum_{t=0}^{N-1}\{\hat{u}(t) - u_{PID}(t)\}^2. \tag{33}$$

Here, since J_{PID} can be linearly described with respect to PID gains, the least squares method is applicable. However, considering the assumed time-varying system variation $G_\Delta(z^{-1})$, applying methods such as gradient descent or sequential least squares might also be useful.

For example, considering a system variation as expressed in Eq. (16), where a system variation happens at time t_n, the data can be divided into two intervals: (i) the time interval from 0 to $(t_n - 1)$ and (ii) the time interval from t_n to $(N-1)$. By separately applying the least squares method to each interval, PID gains can be calculated, which are appropriate for their respective intervals.

6. DATABASE-DRIVEN PID CONTROL USING PREDICTIVE DATA

In the technique described above, the time for the system variation is assumed in advanced. However, in the actual industrial world, the time for the system variation is unknown, hence there is a need for a mechanism to

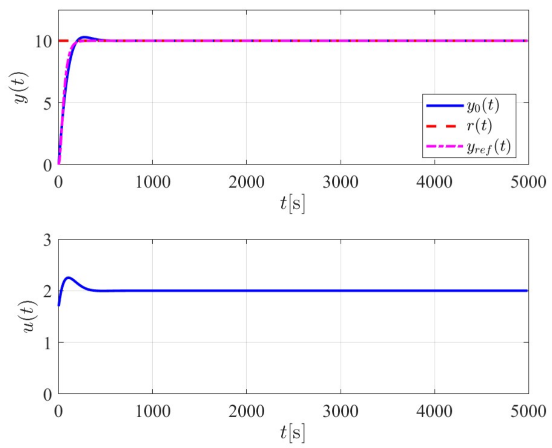

Fig. 4. Initial closed-loop data.

properly adjust the control parameters according to the state of the system.

In this section, the Database-Driven Control method [Yamamoto et al. (2009)] is employed to adopt PID gains calculated in the previous section, irrespective of the occurrence time of system variation. Specifically, the predicted input/output data and PID gains obtained in the previous sections are stored in a database in the format of the information vector $\boldsymbol{\phi}(t)$ as defined below:

$$\boldsymbol{\phi}(t) := [\bar{\boldsymbol{\phi}}(t), \boldsymbol{K}(t)] \tag{34}$$

$$\bar{\boldsymbol{\phi}}(t) := [r(t+1), r(t), \hat{y}(t), \cdots, \hat{y}(t-n_y+1), \hat{u}(t-1), \cdots, \hat{u}(t-n_u+1)] \tag{35}$$

$$\boldsymbol{K}(t) := [K_P(t), K_I(t), K_D(t)], \tag{36}$$

where n_y and n_u represent the degrees of the output and input, respectively. While the details are omitted,

the only difference from the previous study [Yamamoto et al. (2009)] is that the following [Step 1] in which prediction data are stored; otherwise, the same algorithm as in literature [Yamamoto et al. (2009)] is adopted.

[Step 1] Creating a database (in the format of Eq. (34))
[Step 2] Selection of neighboring data
[Step 3] Calculation of PID gains

7. NUMERICAL EXAMPLE

7.1 Controlled Plant and Setting Parameters

In this section, the proposed scheme is applied to a first-order system represented by the following transfer function:

$$G(s) = \frac{K}{1+Ts}, \tag{37}$$

where T and K represent the time constant and system gain, respectively, and are set to $T = 80$ and $K = 1$. The discrete-time system $G(z^{-1})$ corresponding to this continuous-time system is obtained with a sampling time of $T_s = 10$ [s]:

$$G(z^{-1}) = \frac{0.12z^{-1}}{1-0.88z^{-1}}. \tag{38}$$

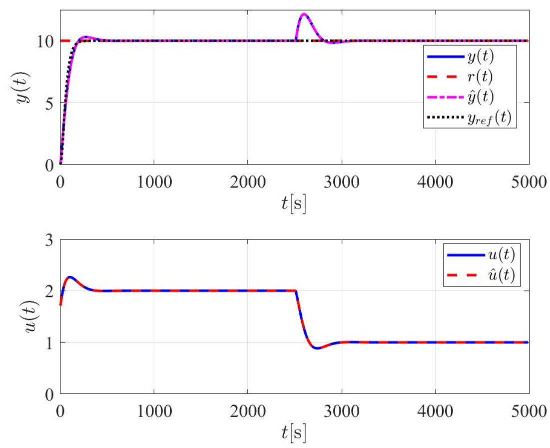

Fig. 5. Predicted data and control result using $G_\Delta(z^{-1})$.

The reference signal is set as $r(t) = 10$, and the parameters related to the reference model $G_m(z^{-1})$ are set to $\sigma = 60$ and $\delta = 0$. Additionally, the initial PID gains for obtaining initial closed-loop data are given by:

$$K_P = 0.14, K_I = 0.03, K_D = 0. \tag{39}$$

Fig. 4 shows the control results utilizing these PID gains. In this section, control results for the system variation are predicted using the data in Fig. 4.

7.2 Predicting Closed-Loop Response Assuming System Variation $G_\Delta(z^{-1})$

By employing the approach outlined in Section 3, the anticipation of the closed-loop response is conducted under the assumption of system variation $G_\Delta(z^{-1})$. The expression for the system variation $G_\Delta(z^{-1})$ is provided by the following equation:

$$G_\Delta(z^{-1}) = 1 + 0.5z^{-251}. \tag{40}$$

In this numerical example, with a sampling time of $T_s = 10$ [s], it is assumed that the system changes at $t = 2510$ [s].

Fig. 5 shows the control result of using the technique in Section 3. The good agreement between $y(t)$ and $\hat{y}(t)$ confirms the effective prediction during system variation by the proposed scheme.

7.3 Design of an Ideal Controller $C^*_{FIR}(z^{-1})$

A controller $C^*_{FIR}(z^{-1})$ is designed using the technique in Section 4 to improve the control performance in Fig. 5. Fig. 6 shows the control result when this is completed. According to the results in Fig. 6, compared to Fig. 5, the control performance is improved around $t = 2510$ [s].

7.4 Design of Controller $C_{PID}(z^{-1})$ and Application of DD-PID

To address the high order of $C^*_{FIR}(z^{-1})$ in Fig. 6, the methodology in Section 5 is applied to design a PID controller $C_{PID}(z^{-1})$ mimicking $C^*_{FIR}(z^{-1})$. In this case, since the system can be divided into two sections around

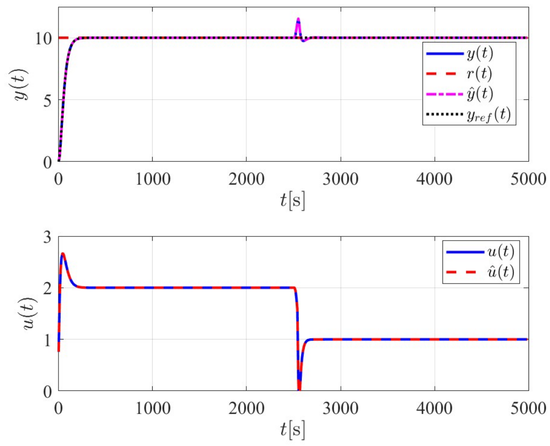

Fig. 6. Predicted data and control result using $G_\Delta(z^{-1})$ and $C^*_{FIR}(z^{-1})$.

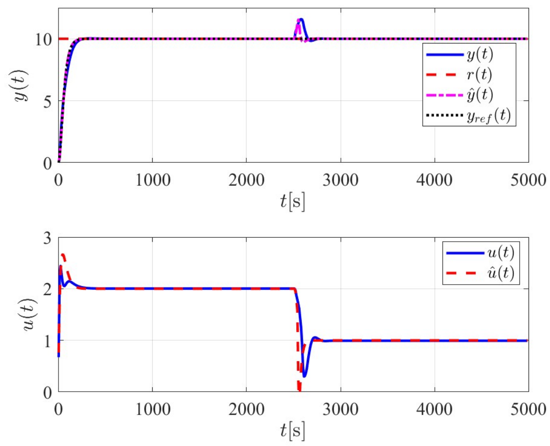

Fig. 7. Control result of the proposed DD-PID.

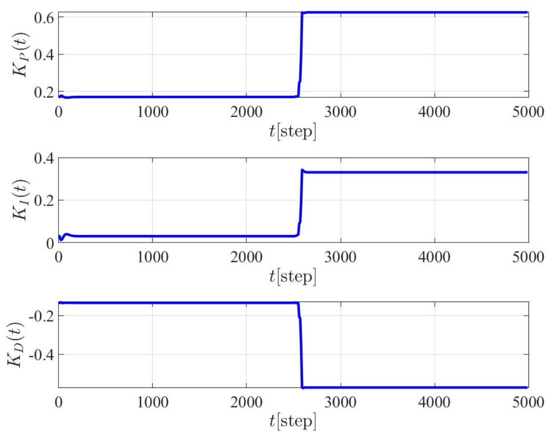

Fig. 8. Trajectories of PID gains corresponding to Fig. 7.

$t = 2510$ [s], the least squares method is applied to calculate PID gains suitable for each section.

Subsequently, in line with the procedure in Section 6, pairs of $\hat{u}(t), \hat{y}(t)$ and PID gains from Fig. 6 are stored into a database. By utilizing this database, the control results of

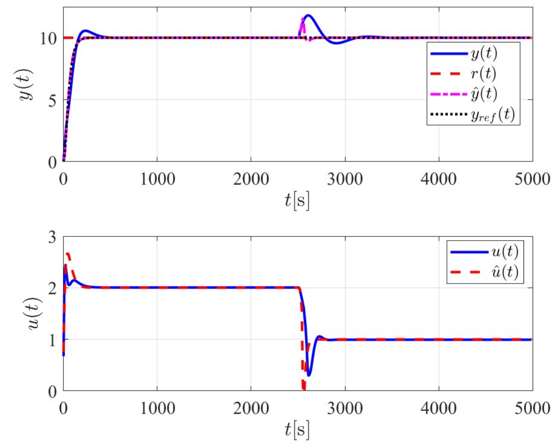

Fig. 9. Control result of the conventional DD-PID.

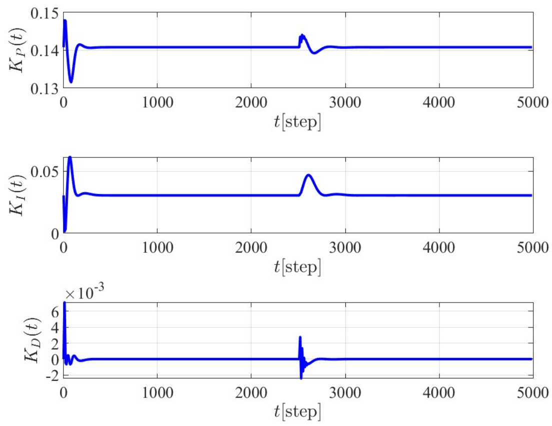

Fig. 10. Trajectories of PID gains corresponding to Fig. 9.

DD-PID and the corresponding trajectories of PID gains are presented in Fig. 7, 8. For comparison, the results of conventional DD-PID, where only the initial input/output data from Fig. 4 is stored, are shown in Fig. 9, 10. From the results in Fig. 7 and Fig. 9, it is evident that the proposed scheme quickly returns to the reference signal during system variation, demonstrating its effectiveness.

7.5 Control Results with Changed System Variation Time

In the previous numerical examples, the time of system variation was assumed to be $t = 2510$ [s] in advance, and predictive input/output data were calculated accordingly. However, in reality, the time of system variation is often unknown beforehand. In Fig. 11, the control results of DD-PID are presented when the assumed system variation time is different, specifically set to $t = 1250$ [s]. For comparison, the control results using $C^*_{FIR}(z^{-1})$ from Fig. 6 are shown in Fig. 12.

Fig. 11 demonstrates that the algorithm of the database-driven control collects neighboring data and calculates PID gains at every time step, resulting in good control performance even when the system variation time is uncertain. On the other hand, Fig. 12 shows a deterioration in control performance not only at the originally assumed

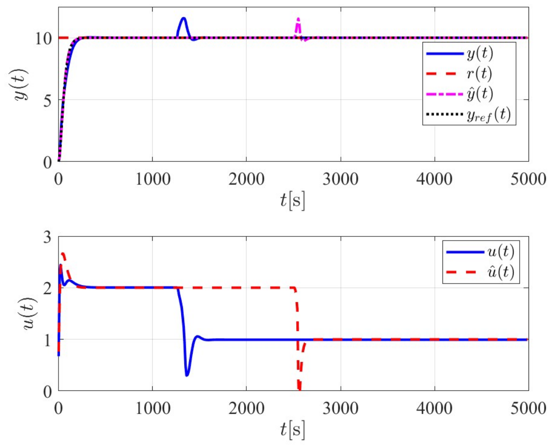

Fig. 11. Control result of the proposed DD-PID with a different system variation time.

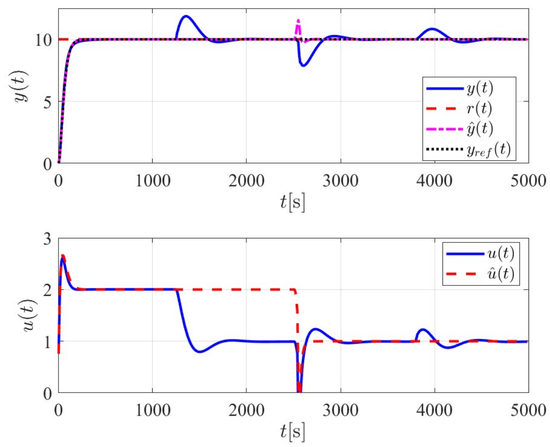

Fig. 12. Control result using $C^*_{FIR}(z^{-1})$ with a different system variation time.

system variation time but also at $t = 2510$ [s]. This degradation is attributed to the fact that a FIR-type controller specialized for the assumed system variation was designed in advance. These results confirm the effectiveness of the proposed scheme in predicting input/output data in advance and combining it with DD-PID, even when the system variation time is unknown.

8. CONCLUSION

In this study, an input/output data prediction method was proposed for when a linear time-varying system is mimicked with the FIR-type system variation, and system variation is generated. In addition, it was confirmed that by designing a controller to improve the control performance degraded by the system variation and applying it to the database-driven control, good control results can be obtained even if the system variation time is unknown.

Future work are to continue discussing the impacts of time-delay and noise, and pursue the study of data prediction assuming a nonlinear system.

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