

# PID control of dead-time processes: robustness, dead-time compensation and constraints handling

Prof. Julio Elias Normey-Rico

Automation and Systems Department  
Federal University of Santa Catarina

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# Dead-time processes

Dead-time processes are common in industry and other areas

Main dead-time (or delay) causes are:

- Transportation dead time (mass, energy)
- Apparent dead time (cascade of low order processes)
- Communication or processing dead time

# Control of dead-time processes

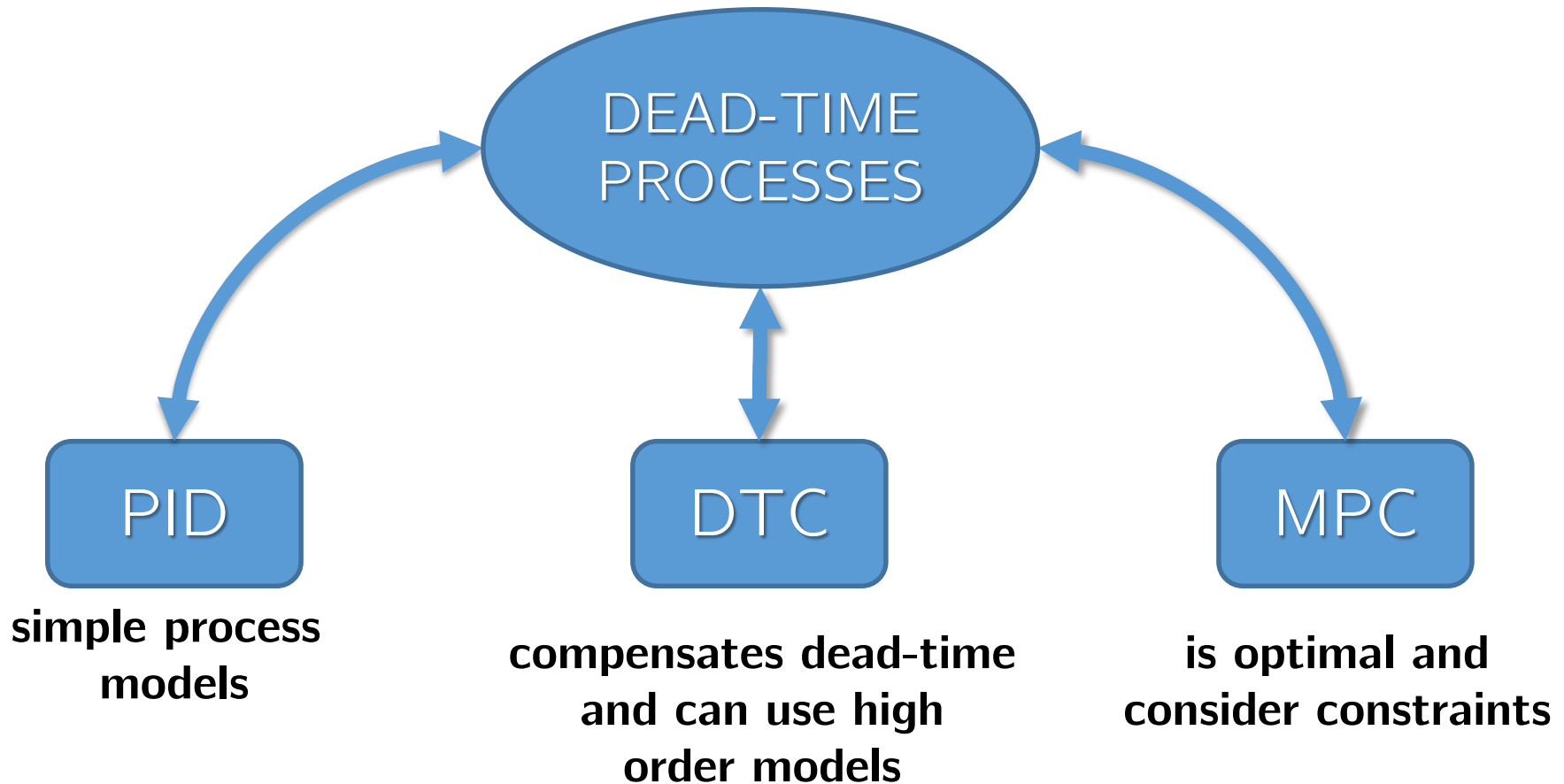
- Dead time makes closed-loop control difficult
- Simplest solution:
  - PID - trade-off robustness and performance
- Basic dead-time compensator - Smith Predictor (SP)
- Improved solutions: Modified SP (ex. FSP)
- Advanced solution: Model Predictive Control - MPC

**Most used in industry PID – DTC – MPC \***

**Industry 4.0 – complex controllers at low level**

\* A Survey on Industry Impact and Challenges Thereof. IEEE CONTROL SYSTEMS MAGAZINE 17

# When to use advanced control?



**Objectives: Analysis of PID, DTC and MPC for dead-time processes**

# Agenda

1. Motivating examples, PID and DTC control.
2. Ideal control of dead-time processes
3. PID tuning using DTC ideas
  1. Unified tuning using FSP (stable and unstable plants)
  2. Trade-off performance-robustness
  3. Comparing PID and DTC
4. MPC, FSP and PID controllers
  1. Unconstrained case
  2. Constrained case – Using anti-windup
5. Conclusions

# Motivating examples

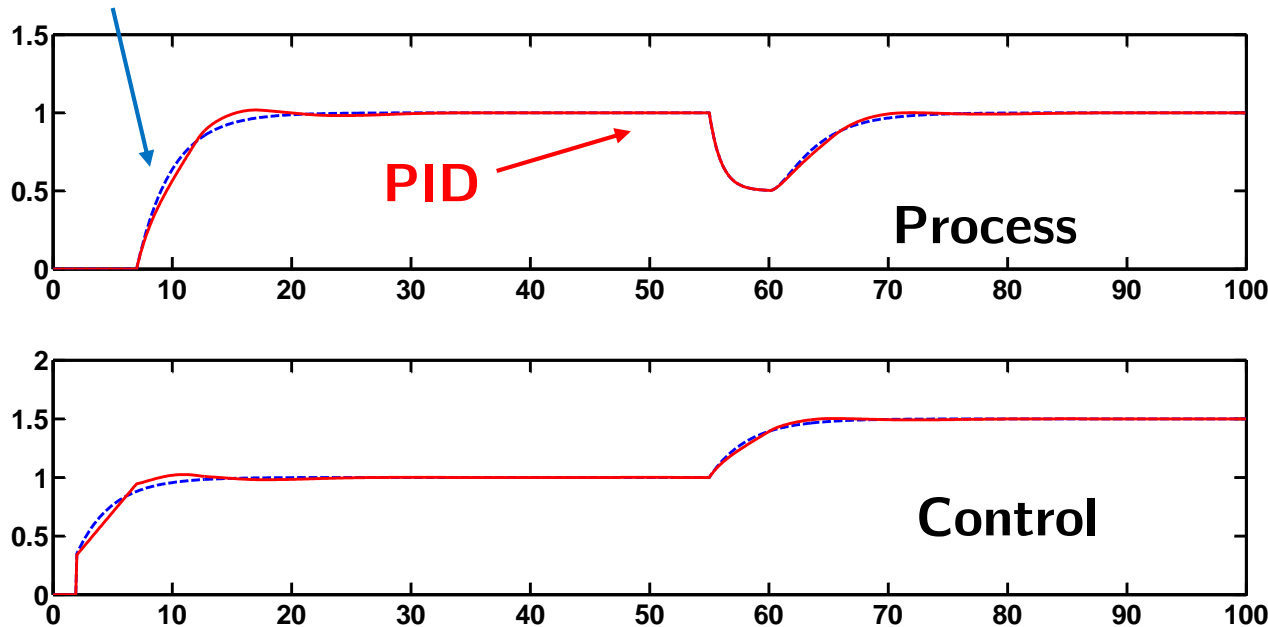
# Simple model – big delay

Simple model with large delay and large modelling error

$$P_n(s) = \frac{e^{-5s}}{s+1}$$

Robust PID and DTC tuning  
(slow response)

DTC



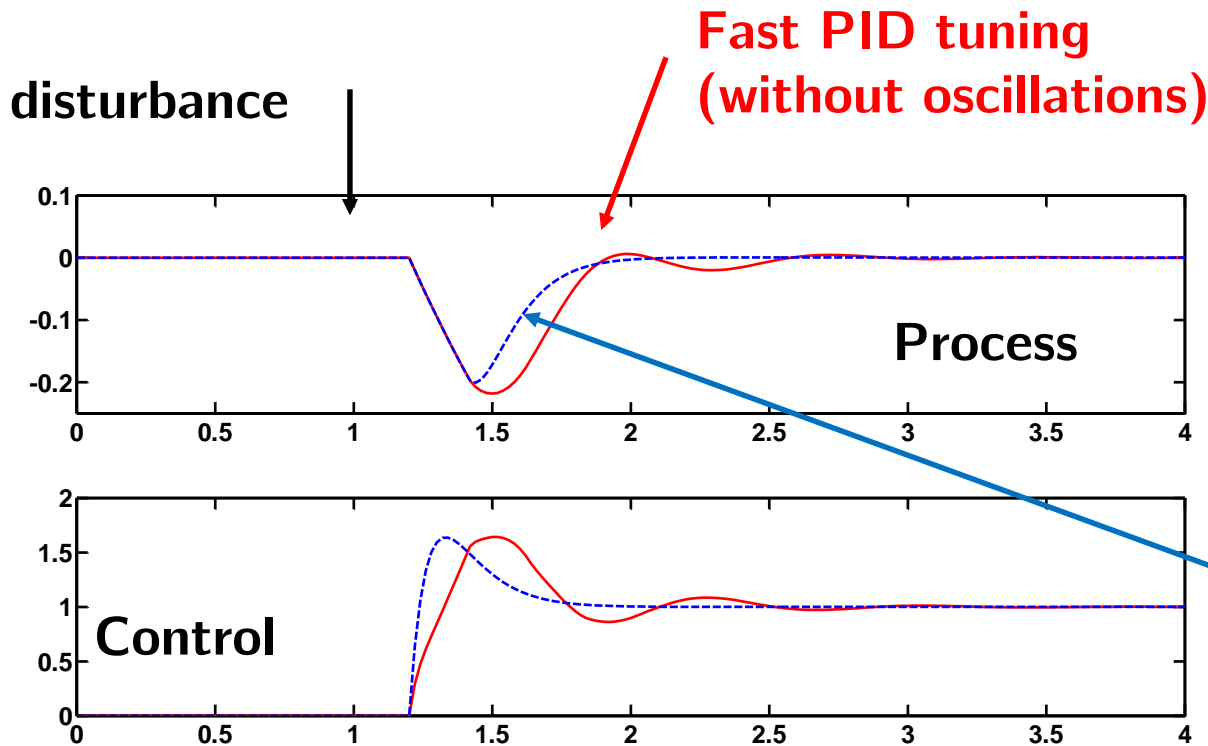
Even for a dominant delay process PID offers a good response

# Fast response – small delay

Simple model with small modelling error

Well known delay (network)

$$P_n(s) = \frac{e^{-0.2s}}{s+1}$$





**To study the advantages of advanced controllers  
for dead-time processes related to:**

- **Process dead-time**
- **Process modeling error (robustness)**
- **Other aspects:**      **Model complexity**



**Constraints handling**

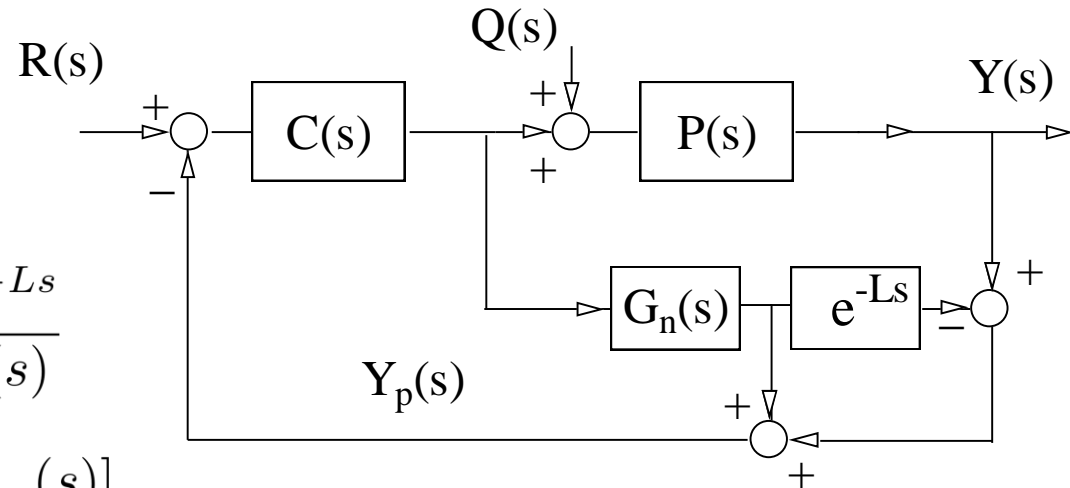
# Ideal control of dead-time processes

# Smith predictor of a pure delay process

$$P(s) = e^{-Ls}$$

$$H_{yr}(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G_n(s)e^{-Ls}}{1 + C(s)G_n(s)}$$

$$H_{yq}(s) = \frac{Y(s)}{Q(s)} = P_n(s) [1 - H_{yr}(s)]$$



$$G_n(s) = 1$$

$$C(s) = K_c$$

Ideal case  $K_c \rightarrow \infty$

$$H_{yr}(s) = e^{-Ls}$$

$$H_{yq}(s) = e^{-Ls} [1 - e^{-Ls}]$$

**1 delay**

**2 delays**

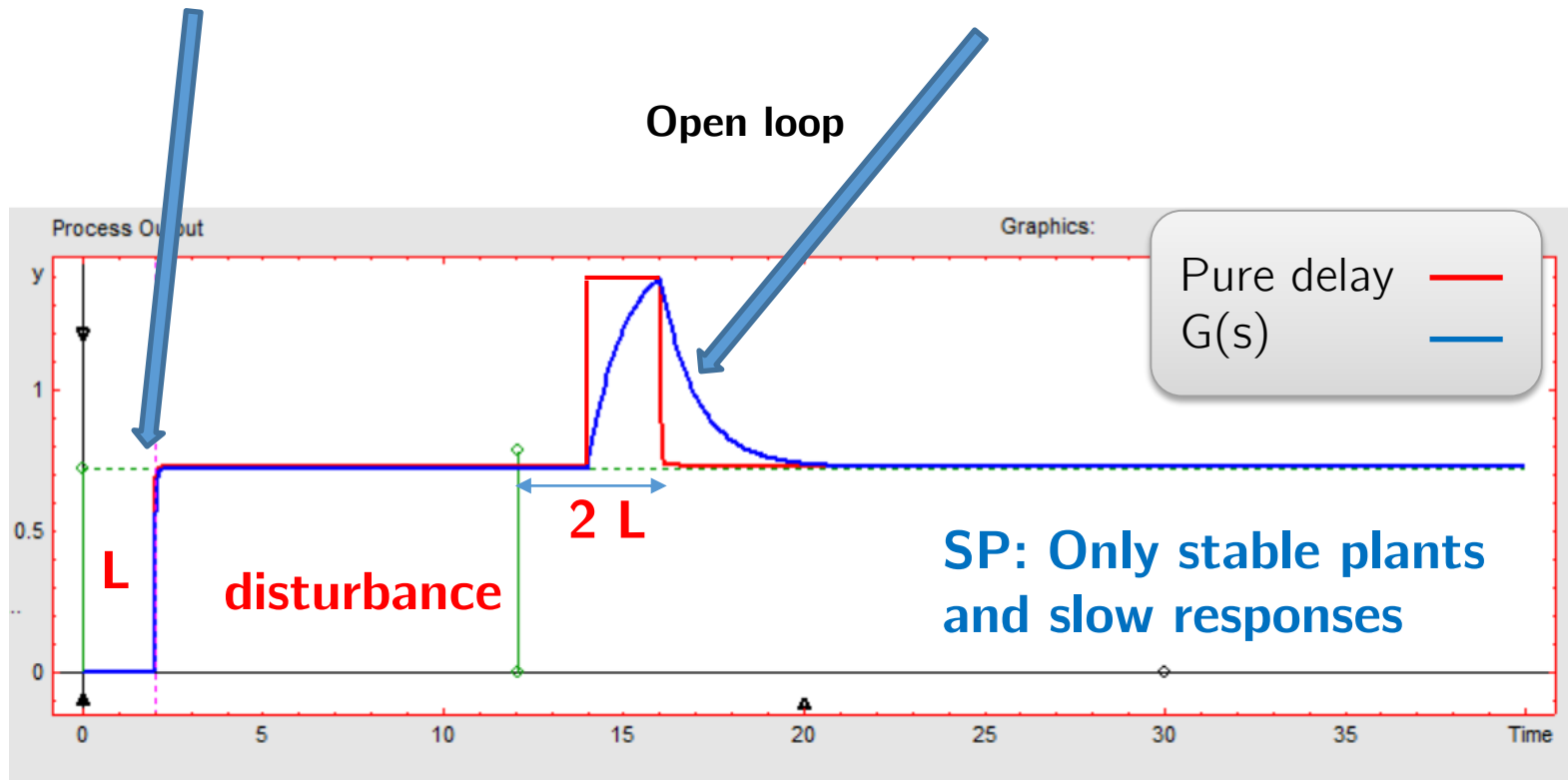
# Smith predictor of a FOPDT process

$$P(s) = \frac{K_e}{1+sT} e^{-Ls}$$

Using  $C(s) = K_c$  and ideal case  $K_c \rightarrow \infty$

$$H_{yr}(s) = e^{-Ls}$$

$$H_{yq}(s) = \frac{K_e}{1+sT} e^{-Ls} [1 - e^{-Ls}]$$

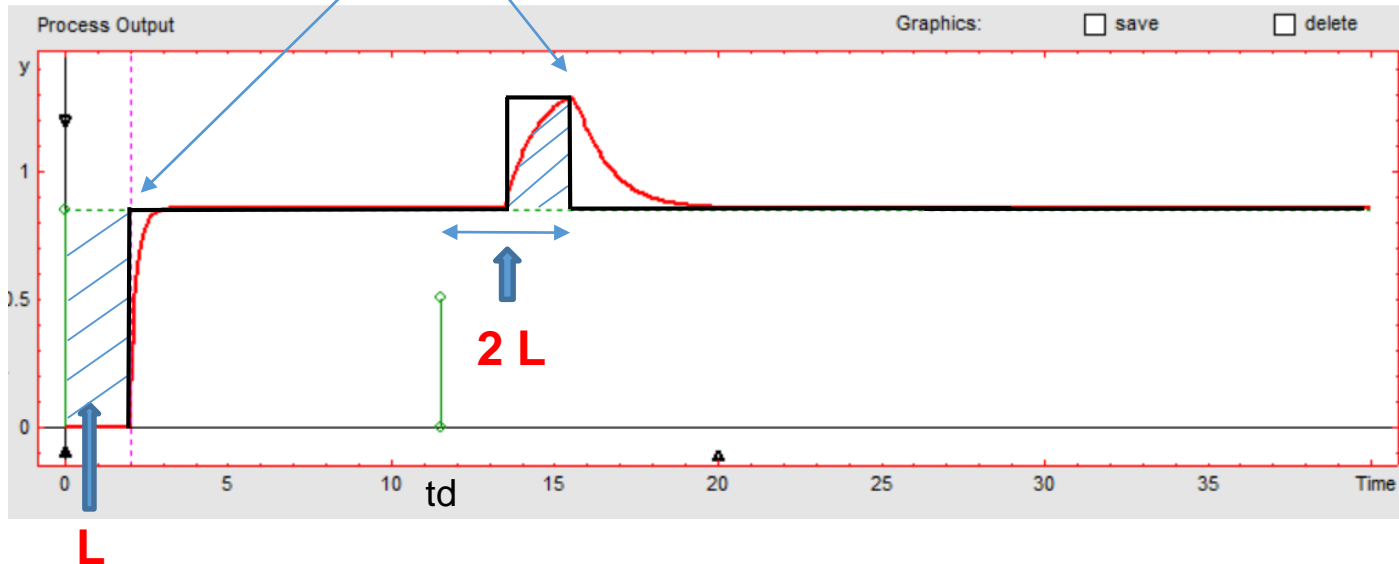


# Ideal Control – Achievable Performance

Normal index  $e(t) = r(t) - y(t)$

$$J = \int_0^{\infty} |e(t)| dt$$

**No controller  
can act before**



# Ideal Control – Achievable Performance

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~~$J = \int_0^\infty |e(t)| dt$~~

SETPOINT

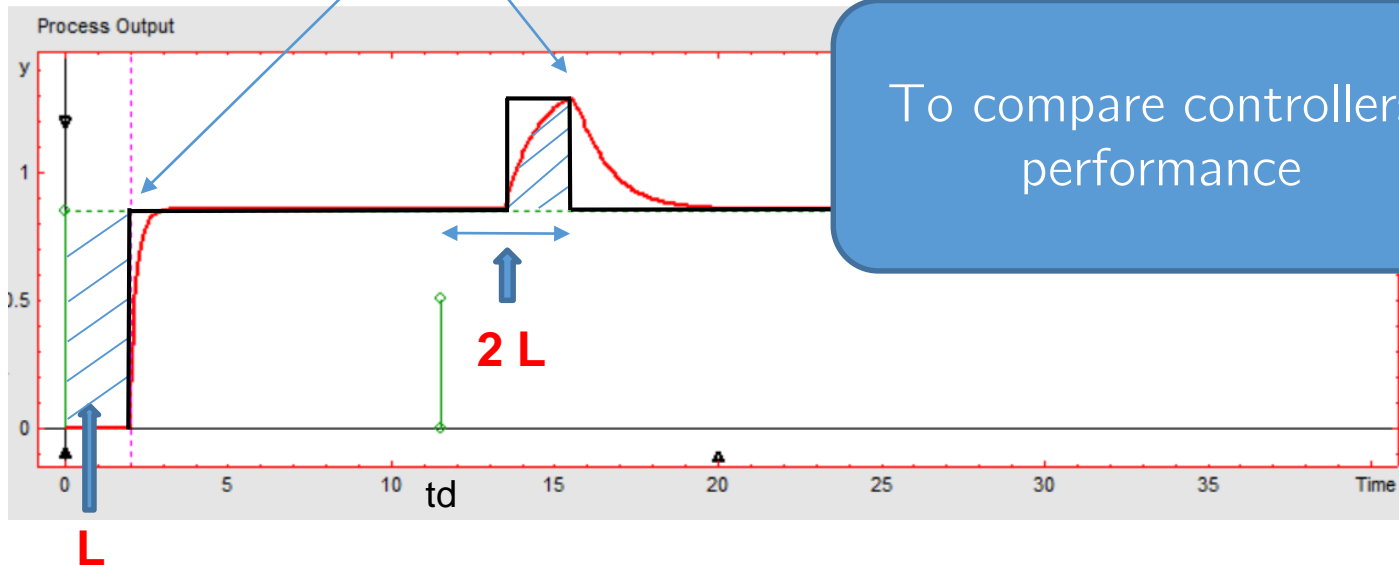
$J_{sp} = \int_L^\infty |e(t)| dt$

DISTURBANCE

$J_{dr} = \int_{t_d+2L}^\infty |e(t)| dt$

**No controller can act before**

To compare controllers' performance



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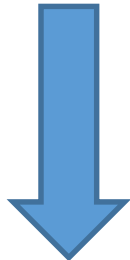
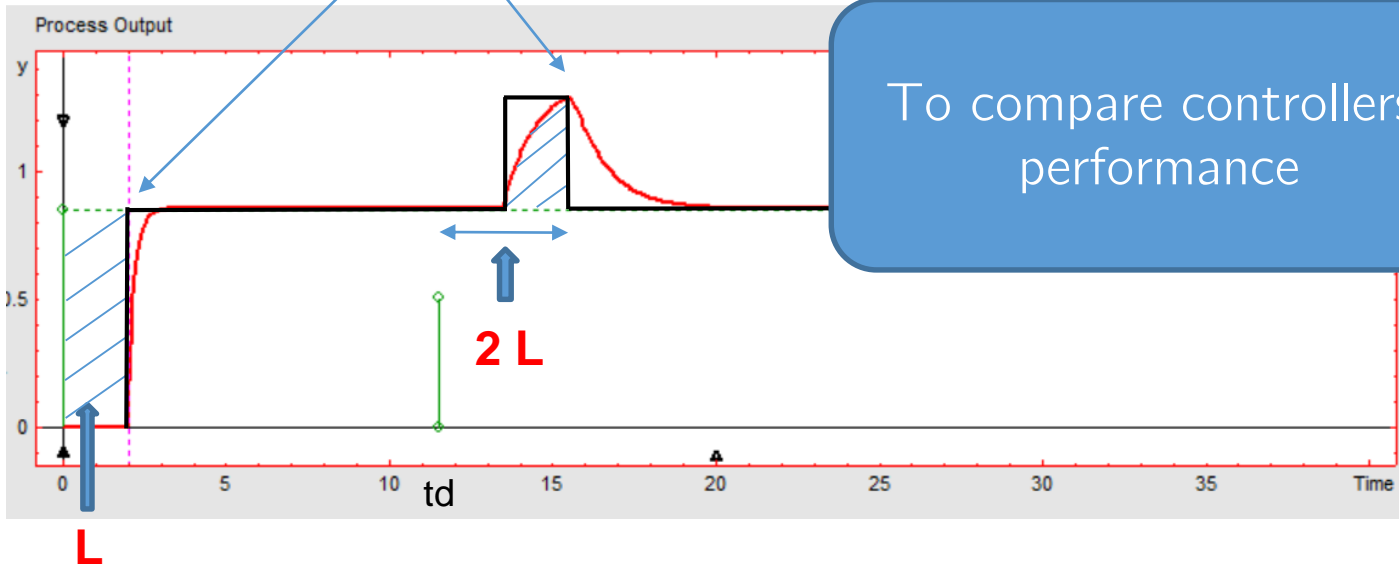
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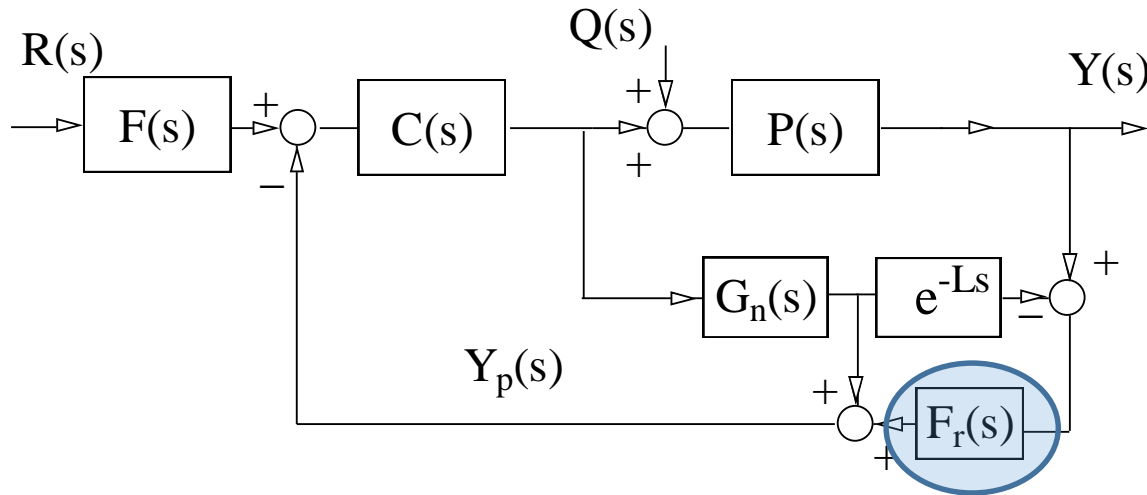
**“Ideal”**  
 $J_{min} = 0$

# How to achieve ideal response?

Is it ideally possible to achieve  $J_{min} = 0$ ?



**Filtered Smith Predictor**



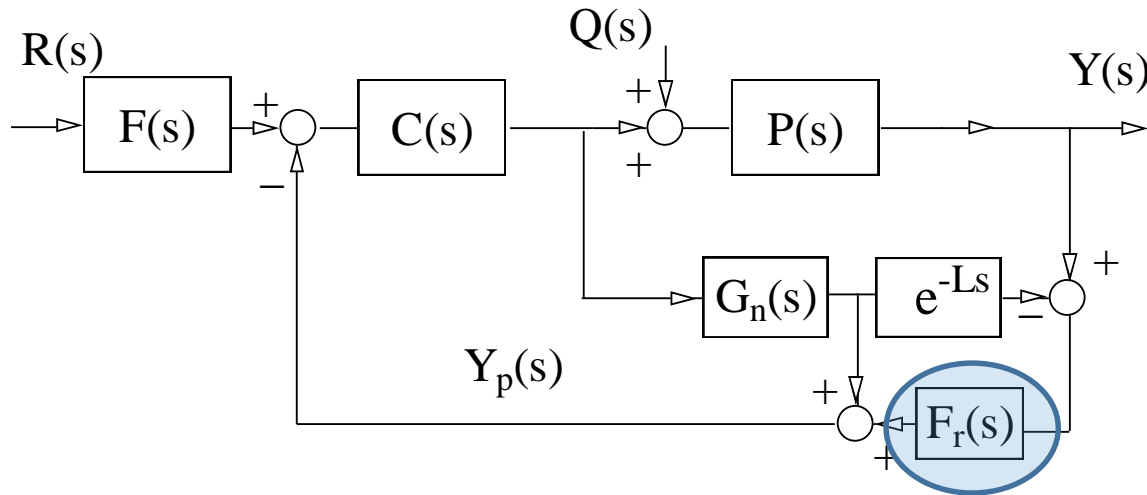


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**Filtered Smith Predictor**



The same  $H_{yr}$  as SP

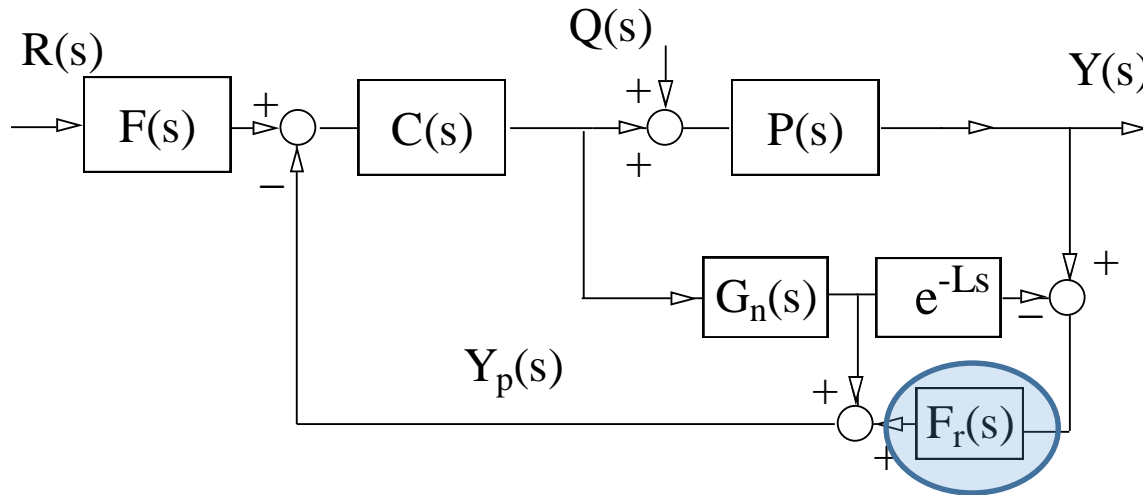
$$H_{yr}(s) = \frac{C(s)G_n(s)e^{-Ls}}{1 + C(s)G_n(s)} F(s)$$

# How to achieve ideal response?

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**Filtered Smith Predictor**



The filter  $F_r(s)$  allows:

- Eliminates the open-loop dynamics from the input disturbance response
- FSP for unstable plants
- FSP for ramp and other disturbances
- Robustness-Performance trade-off

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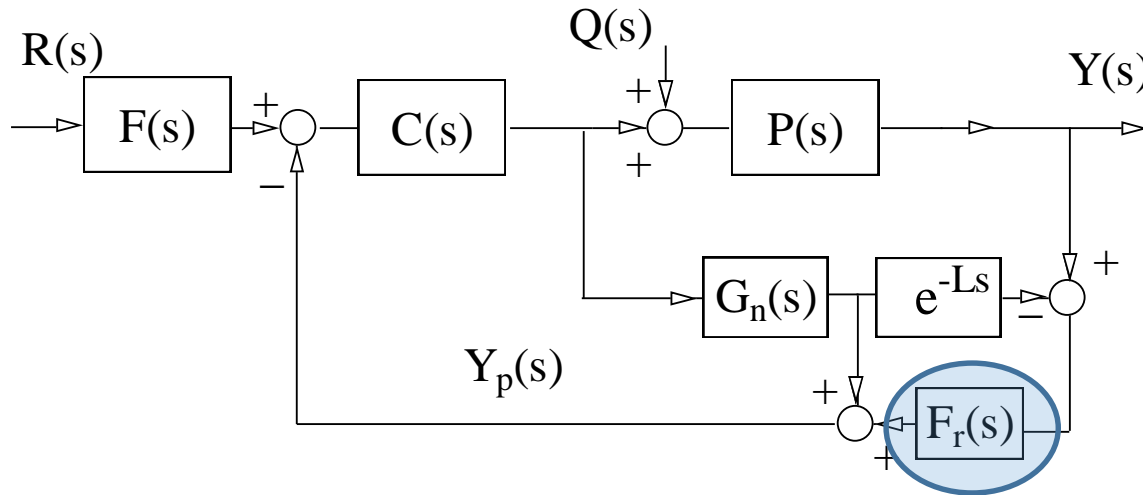
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$$H_{yq}(s) = P_n(s) [1 - H_{yr}(s) F_r(s)]$$

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$$P(s) = \frac{K_e}{1+sT} e^{-Ls}$$



$$H_{yr}(s) = e^{-Ls}$$

$$H_{yq}(s) = e^{-Ls} [1 - e^{-Ls}]$$



$$J_{min}=0!$$



Many FSP successful applications in practice:\*

Termo-solar systems, Compression systems, Neonatal Care Unit.

FSP autotuning for simple process\*\*

**Idea: To derive a PID tuning for dead-time processes using the FSP approach**

**PID is a low frequency approximation of the FSP.**

$$C(s) = \frac{K_c(1+sT_i)(1+sT_d)}{sT_i(1+s\alpha T_d)}$$

\*Torrico, Cavalcante, Braga, Normey-Rico, Albuquerque, I&EC Res. 2013.

\*Flesch, Normey-Rico, Control Eng. Practice, 2017

\*\*Normey-Rico, Sartori, Veronesi, Visioli. Control Eng. Practice, 2014

\* Roca, Guzman, Normey-Rico, Berenguel, Yebra, Solar Energy, 2011

# PID tuning using FSP

# Tuning procedure

- **Process models: FOPDT, IPDT, UFOPDT**

$$G_n(s) = \frac{K_p}{1+sT}$$

$$G_n(s) = \frac{K_p}{s}$$

$$G_n(s) = \frac{K_p}{sT-1}$$

- **PI primary controller (only P for the IPDT)**  $C(s) = K \frac{1+s\tau_i}{s\tau_i}$

- **FO predictor filter**  $F_r(s) = \frac{1+sT_1}{1+sT_2}$  **(tuning for step disturbances)**

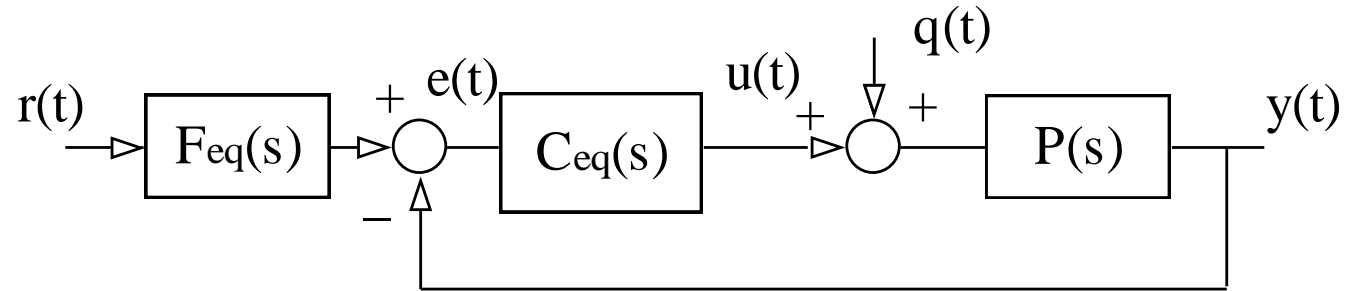
- **Tuning for a delay-free-closed-loop system with pole (double pole) in  $s=-1/T_0$**

- **$T_0$  is the **only tuning** parameter for a **trade-off robustness-performance****



# Tuning procedure

## Equivalent 2DOF controller



$$C_{eq}(s) = \frac{C(s)F_r(s)}{1 + C(s)G_n(s)(1 - e^{-L_n s}F_r(s))}, \quad F_{eq}(s) = \frac{F(s)}{F_r(s)},$$

$$e^{-L_n s} \rightarrow \frac{1 - 0.5L_n s}{1 + 0.5L_n s}$$



**2DOF PID**

- $C_{eq}$  avoids pole-zero cancellation
- $T_o$  free tuning parameter



## Tuning advantages of the predictor-PID

- ❑ Unified approach for FOPDT, IPDT and UFOPDT ( $L < 2T$ )
- ❑ It has only one tuning parameter  $T_0^*$
- ❑ Has **similar performance** than well known methods\*
- ❑ It is a **low frequency approximation of the ideal solution** for first order dead-time models

**Interesting PID tuning method to use in comparisons with dead-time compensators and predictive controllers**

Next: To compare PID and FSP

\* Normey-Rico and Guzmán. Ind. & Eng. Chem. Res., 2013

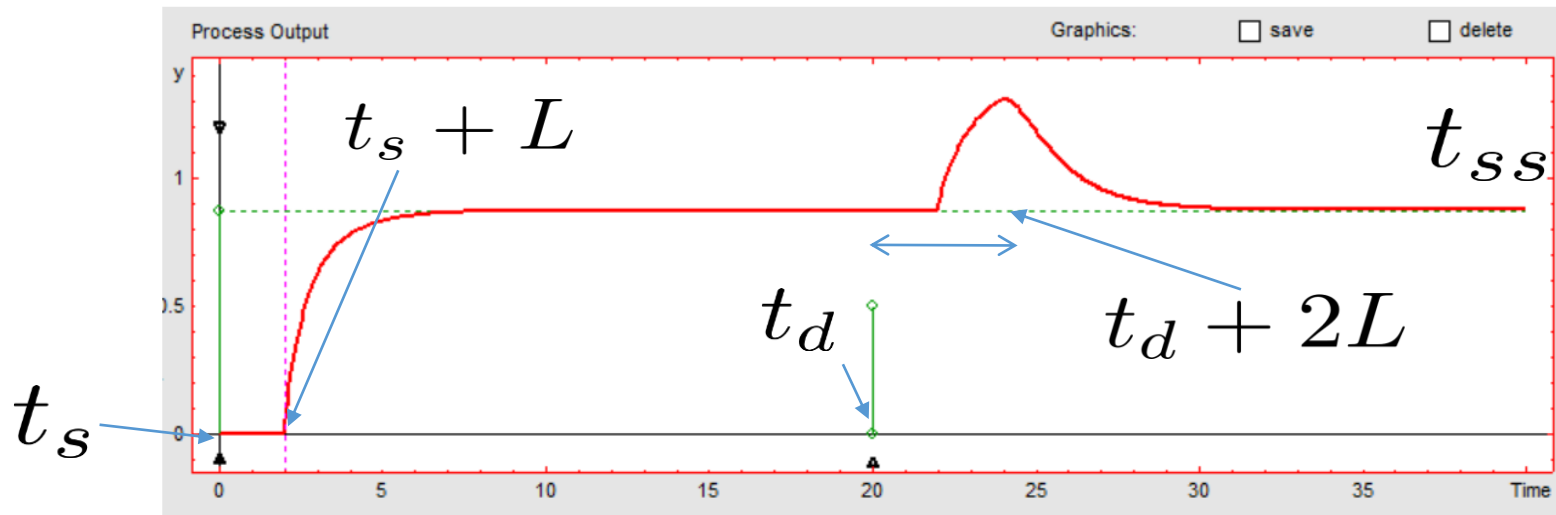
\* Astrom and Hagglund, Research Triangle Park, 2006

## Performance Index

$$J = \lambda \int_{t=t_s+L}^{t_d} |r(t) - y(t)| + (1 - \lambda) \int_{t=t_d+2L}^{t_{ss}} |r(t) - y(t)|$$

$$\lambda \in [0, 1]$$

$\lambda = 0.5$  in this work

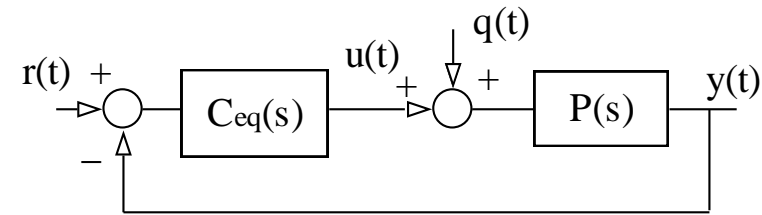
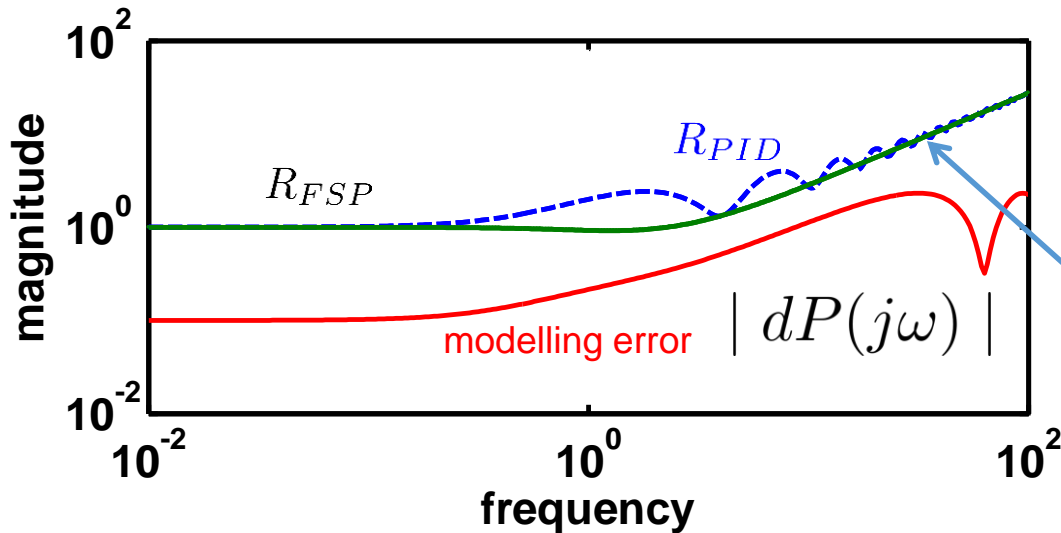


# FSP-PID comparative analysis

## Robustness

$$P(j\omega) = P_n(j\omega)[1 + dP(j\omega)]$$

$C_{eq}(s)$  stabilizes  $P_n(s)$



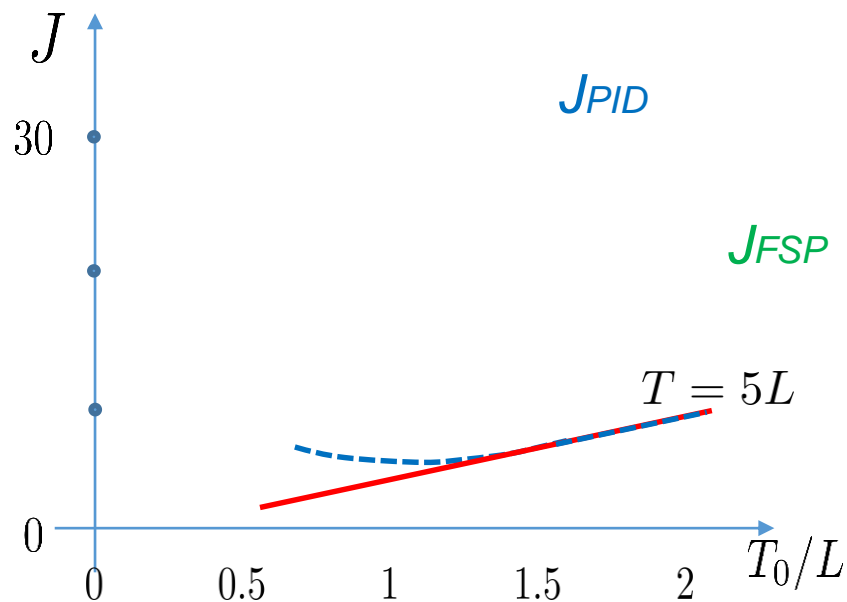
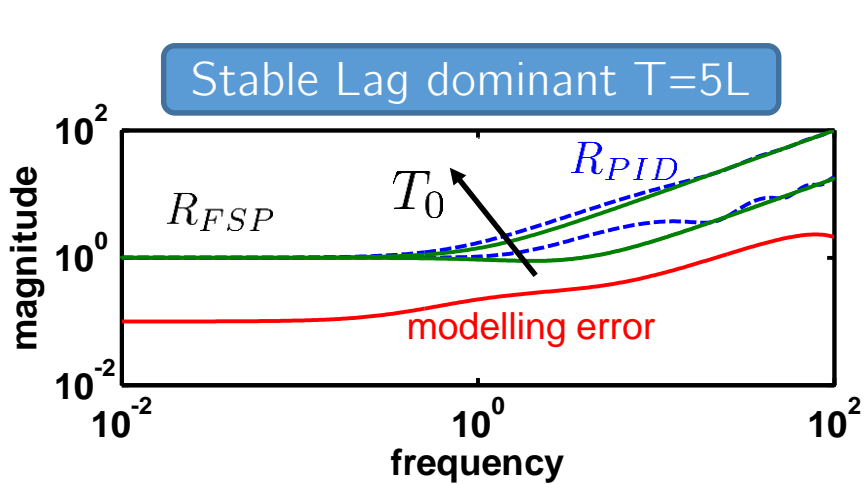
$$R(\omega) := \frac{|1 + C_{eq}(j\omega)P_n(j\omega)|}{|C_{eq}(j\omega)P_n(j\omega)|}$$

Robust condition  $R(\omega) > \overline{dP}(\omega) \geq |dP(j\omega)| \quad \forall \omega > 0$

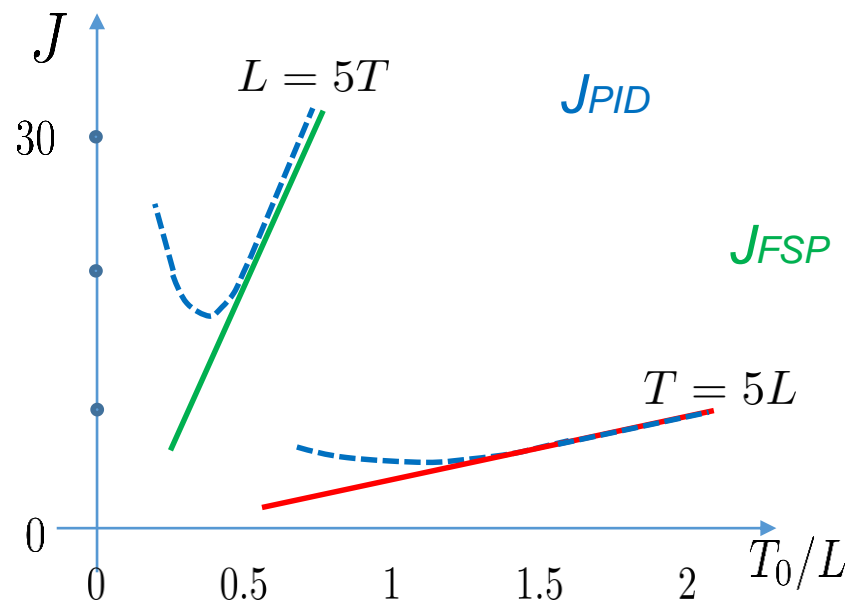
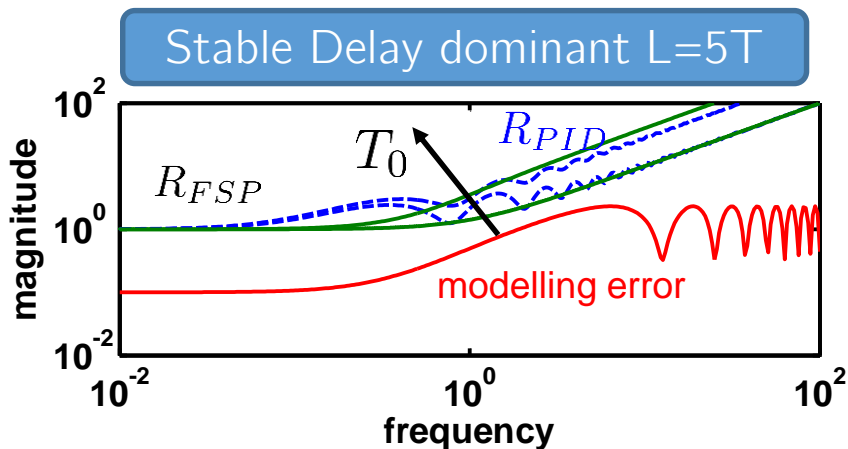
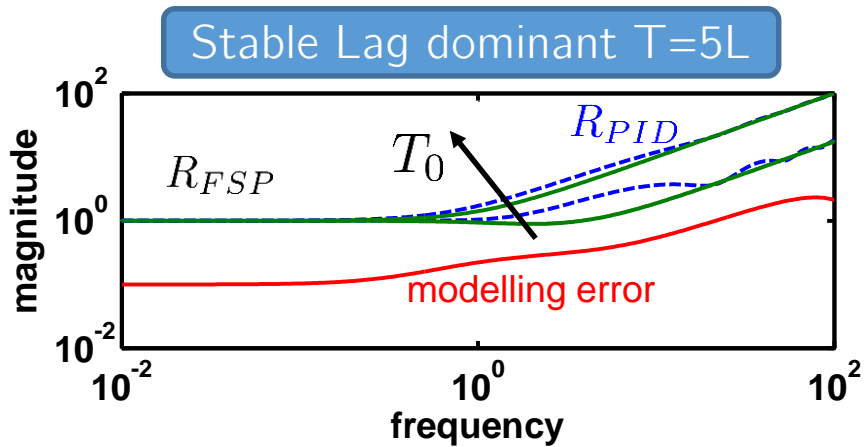
**Conservatism can be avoided separating dead-time uncertainties\***

\*Larsson and Hagglund (2009), ECC 2008

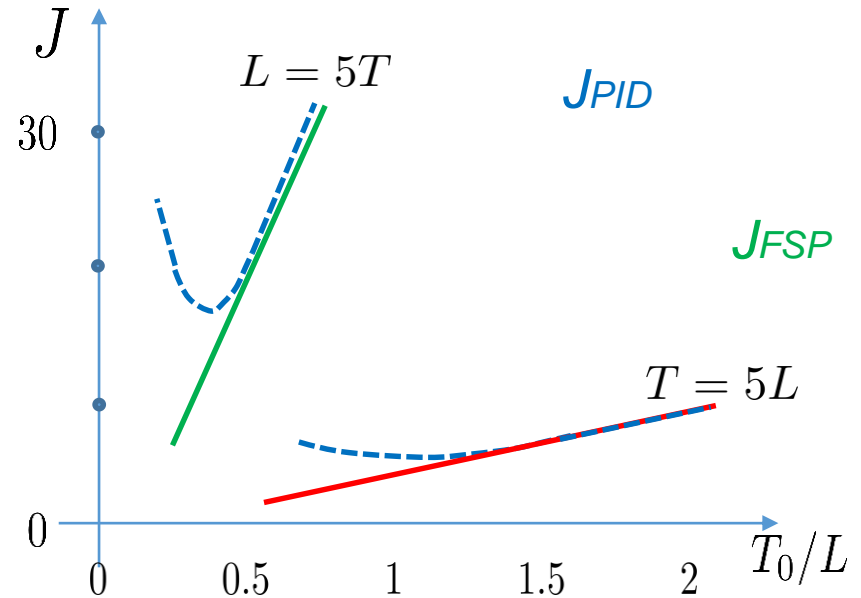
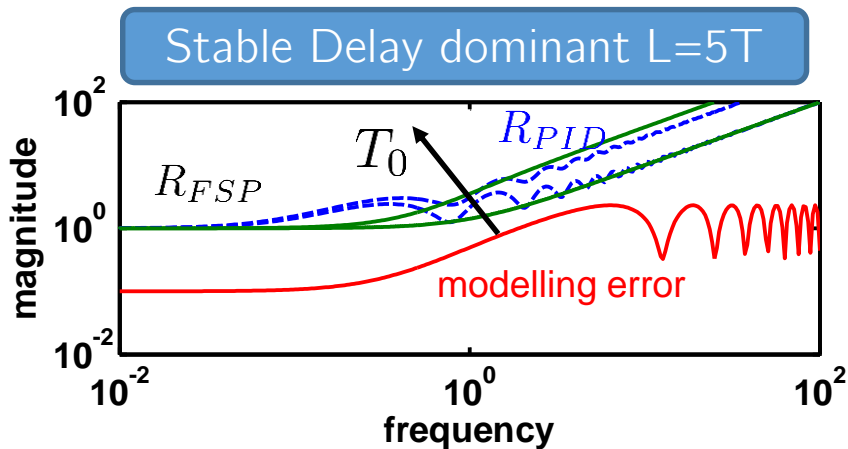
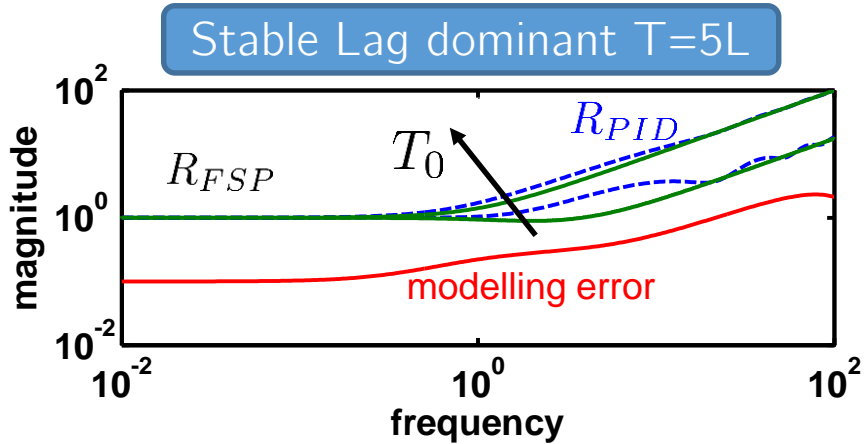
# FSP-PID comparative analysis



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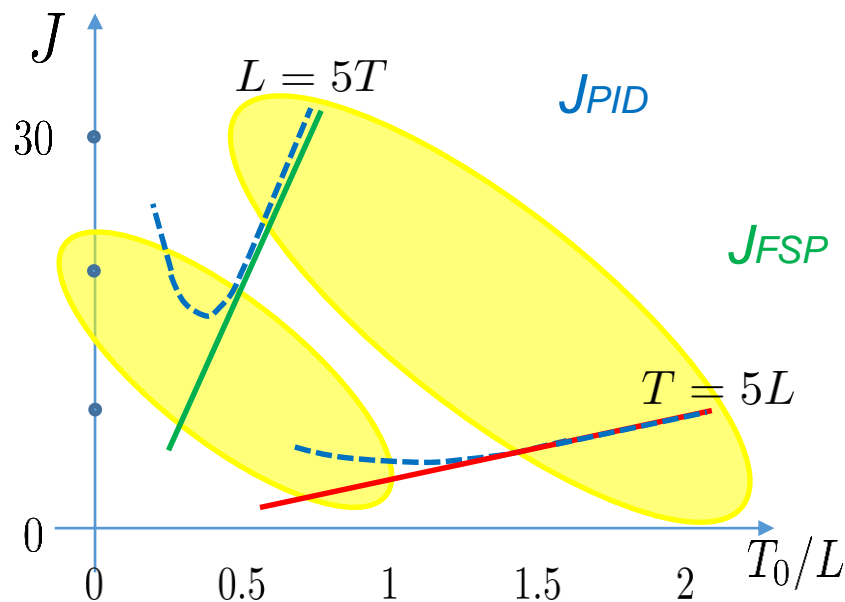
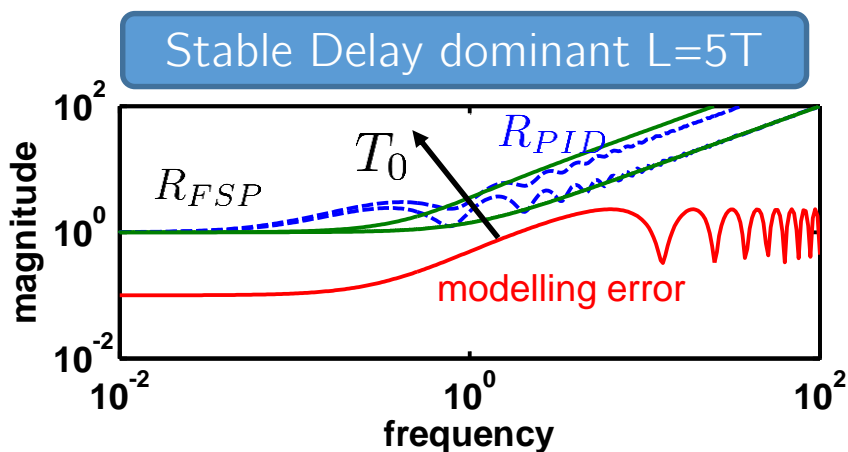
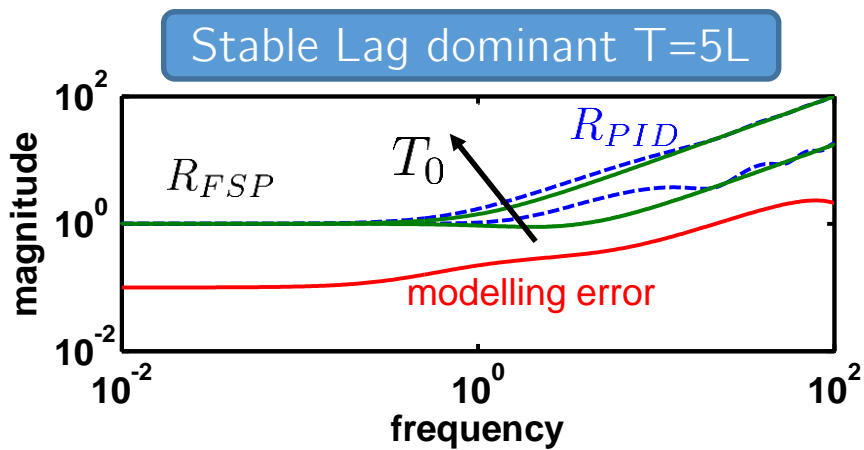


# FSP-PID comparative analysis



- Robust tuning  $J_{FSP} \approx J_{PID}$
- Fast tuning  $J_{FSP} < J_{PID}$

# FSP-PID comparative analysis



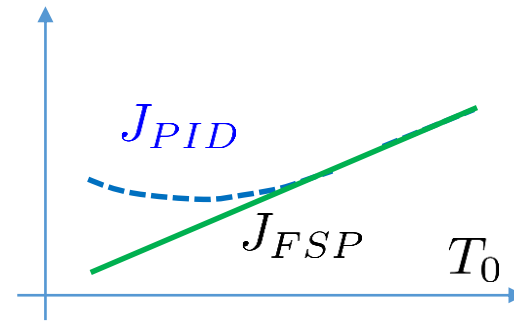
- Robust tuning  $J_{FSP} \approx J_{PID}$
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*PID* for robust solutions  
*FSP* has advantages with good models

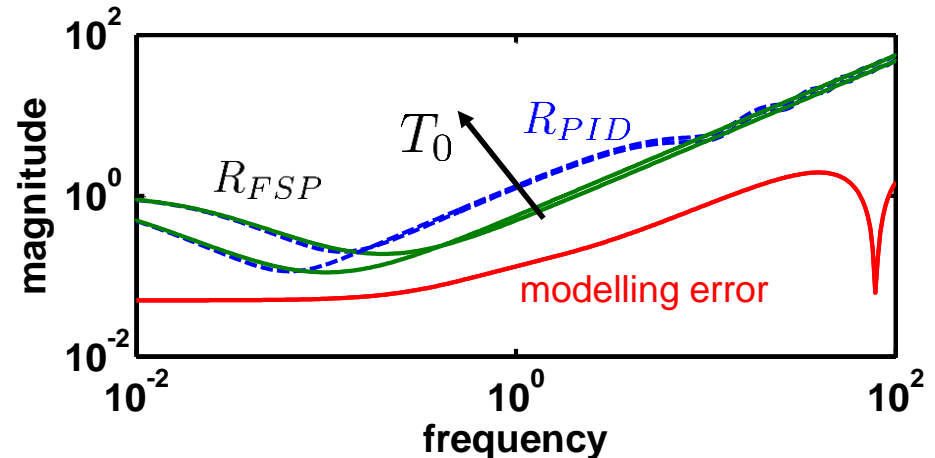
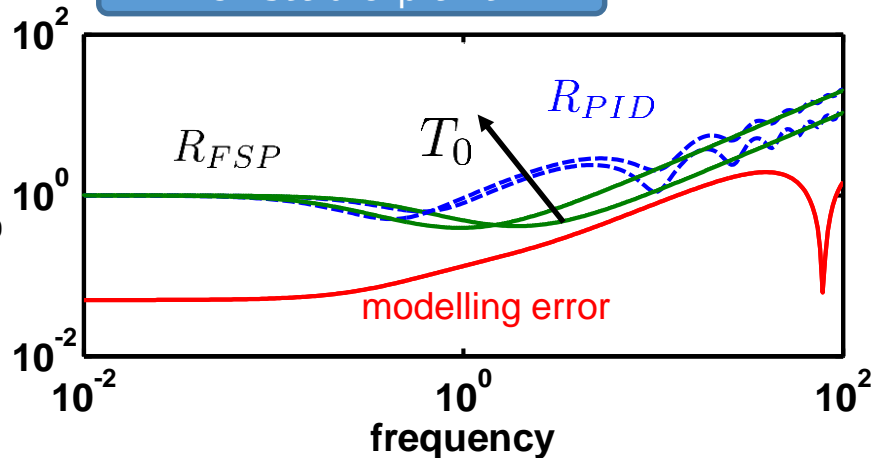
# FSP-PID comparative analysis

## Integrative plant

- Similar to Lag-dominant plants



## Unstable plant



- Same conclusions as in FOPDT
- UFOPDT Robustness has a limit increasing  $T_0$  \*

\* Normey-Rico and Camacho, 2007, Springer



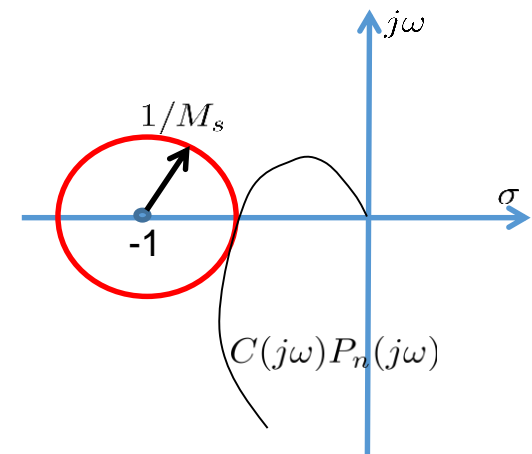
## Tuning: Trade-off Robustness-Performance

- Minimise  $J$  for robust stability for a given modelling error

**Particular tuning using:**  $R(\omega) > \overline{dP}(\omega) \quad \forall \omega > 0$

- Minimise  $J$  for robust stability for a given  $R_m = \min_{\omega} R(\omega)$

**General tuning using**  $R_m$  (or  $M_S$ )



\* Grimholt and Skogestad 2012, IFAC PID 2012.

## Tuning: Trade-off Robustness-Performance

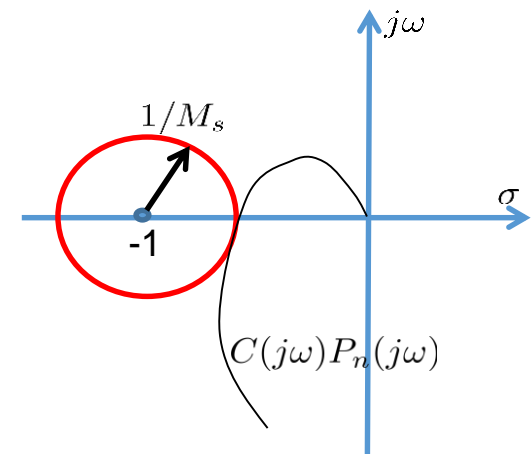
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Control effort (total variation) and noise attenuation are directly related to robustness indexes as  $R_m$  (or  $M_S$ )\*



\* Grimholt and Skogestad 2012, IFAC PID 2012.

## Conclusions

- **Case 1: poor model information (large modelling error)**
  - Simple model is used for tuning
  - High robustness is mandatory
  - Step disturbances

**PID will be the best solution, even for dead-time dominant systems**

- **Case 2: good model is available (small modelling error)**
  - Fast responses are required
  - Low robustness is enough
  - Complex models or disturbances

**FSP will be better (even for lag-dominant systems) because of the PID nominal limitations**

## Conclusions

**Concerning dead-time: dead-time value is less important than dead-time modelling error.**

## Implementation issues:

- FSP is implemented as a 2DOF discrete controller
- FSP is a complex algorithm (**delay order** (in samples) + **model order**)
- PID is simple to implement

General problems in industry:  
large modelling error, noise,  
simple models and solutions



Use a well tuned  
PID for dead-time  
processes

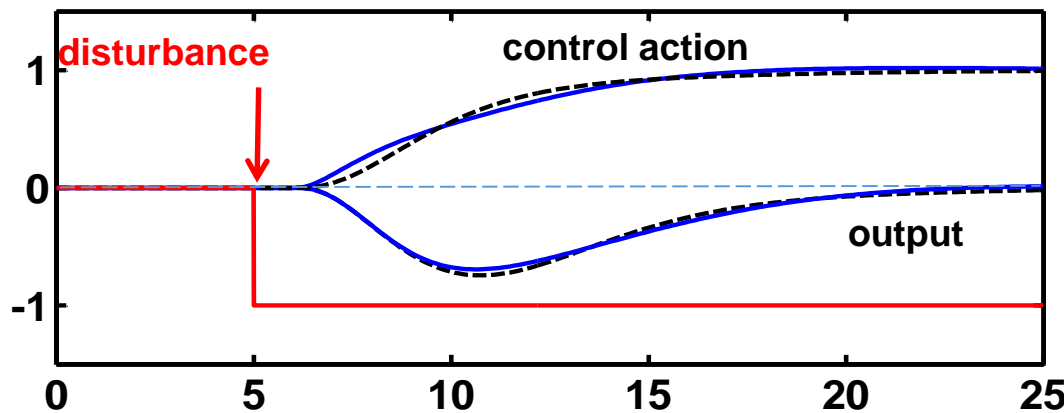
# Example 1: High-order system

$$P(s) = \frac{e^{-s}}{(s+1)^3}$$

$$P_n(s) = \frac{e^{-2s}}{(2s+1)}$$

Prediction Model for FSP

Robust tuning for  $M_s=1.2$



PID tuning  
using SWORD \* tool

**FSP and PID have the same performance**

\*\*Garpinger, O. and T. Hägglund (2015), Journal of Process Control.

\*\* SWORD Matlab software tool.

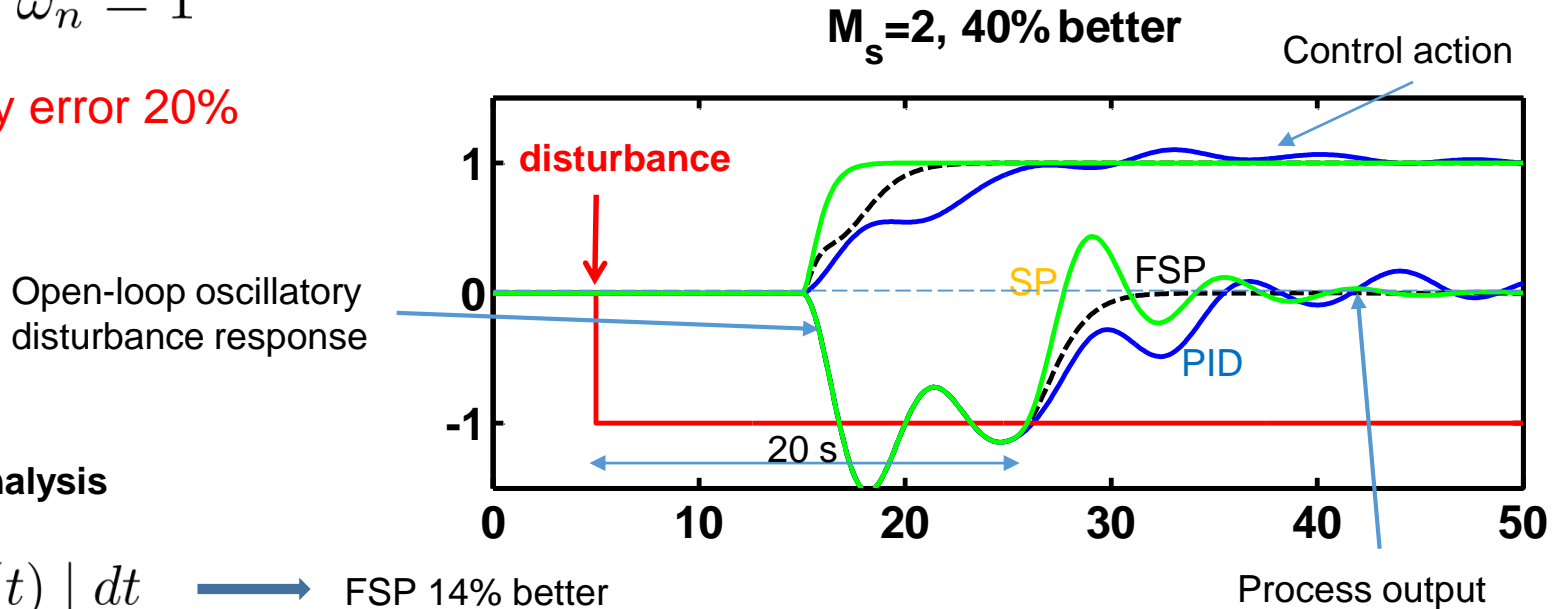
# Example 2: PID, SP and FSP

$$P(s) = \frac{e^{-10s}}{1 + \frac{2\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

$$\xi = 0.2, \omega_n = 1$$

Max. delay error 20%

- SP and FSP with the same primary PID controller
- PID tuning for min IAE for  $M_s=2$  (using sword tool)



## Performance Analysis

$$J = \int_0^{\infty} |e(t)| dt \quad \longrightarrow \quad \text{FSP 14\% better}$$

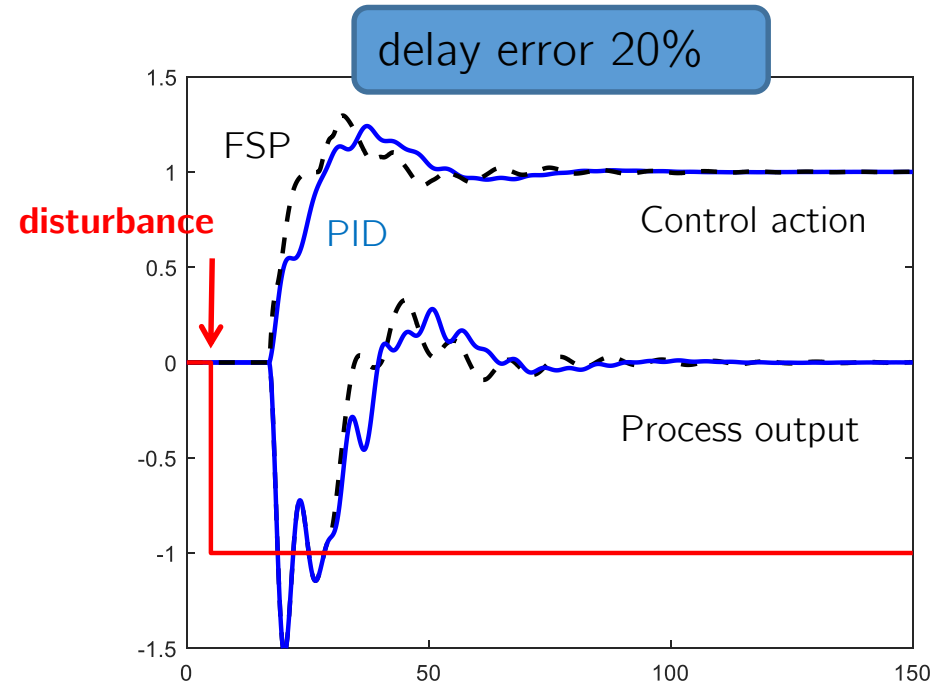
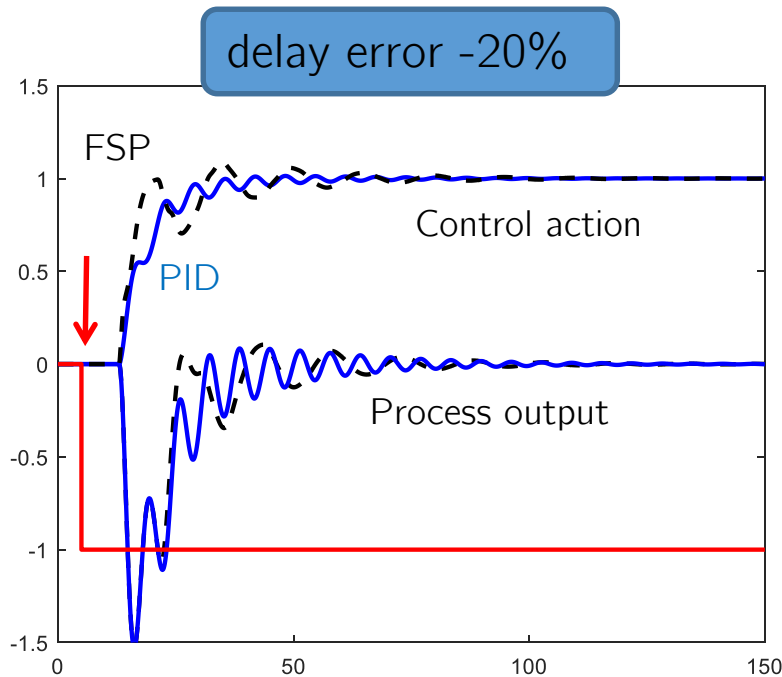
$$J_{dr} = \int_{t_d+2L}^{\infty} |e(t)| dt \quad \longrightarrow \quad \text{FSP 40\% better}$$

**Robustness : FSP stable up to -35% or +35% delay error, SP unstable for 20% delay error**

# Example 2: PID, SP and FSP

$$P(s) = \frac{e^{-10s}}{1 + \frac{2\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

$$\xi = 0.2, \omega_n = 1$$



- SP unstable for this case
- PID and FSP similar responses

- In real process control action is limited, as well as slew rate
- Also, process output should be between limits
- Anti-windup (AW) can be used to mitigate the effect of the saturation in the integral action in PID and FSP
- MPC appears as a **direct solution** to implement optimal control under system constraints

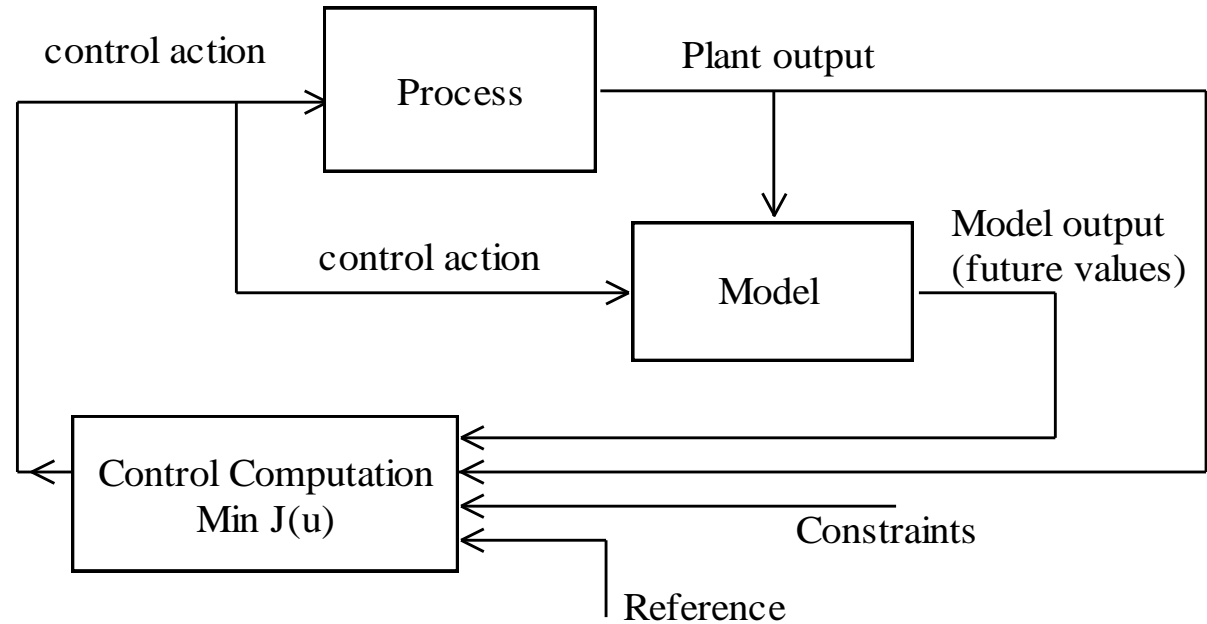
**When is MPC a better choice?**



# MPC, FSP and PID

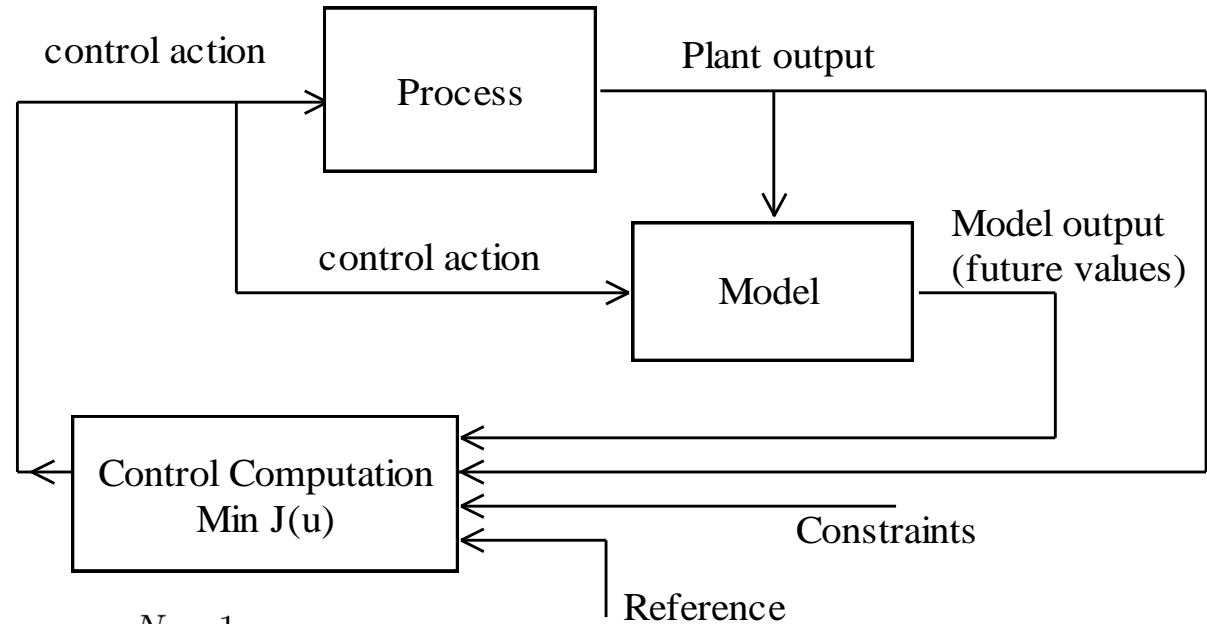
GPC – Generalized predictive controller

## General MPC idea



# GPC analysis for Dead-time Processes

## General MPC idea



## GPC cost

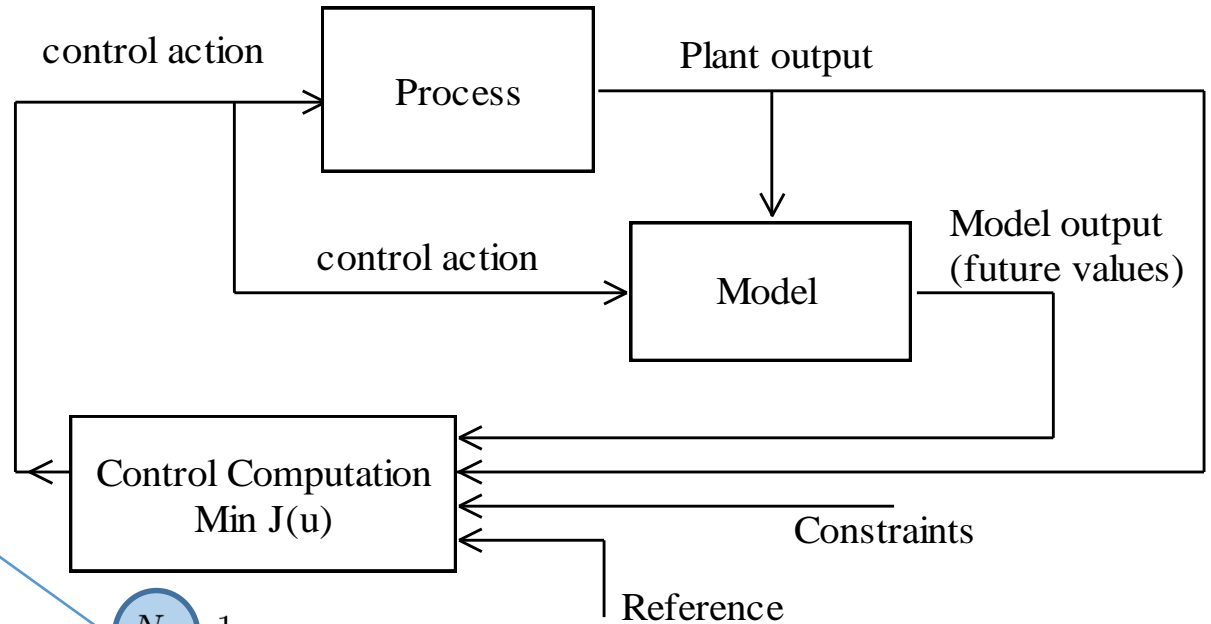
$$J = \sum_{j=d+1}^{d+N_y} [y(k+j|k) - w(k+j)]^2 + \sum_{j=0}^{N_u-1} \lambda [\Delta u(k+j)]^2,$$

## GPC Model

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k-1) + \frac{e(t)}{\Delta} \quad L = dT_s$$

# GPC analysis for Dead-time Processes

## General MPC idea



## GPC cost

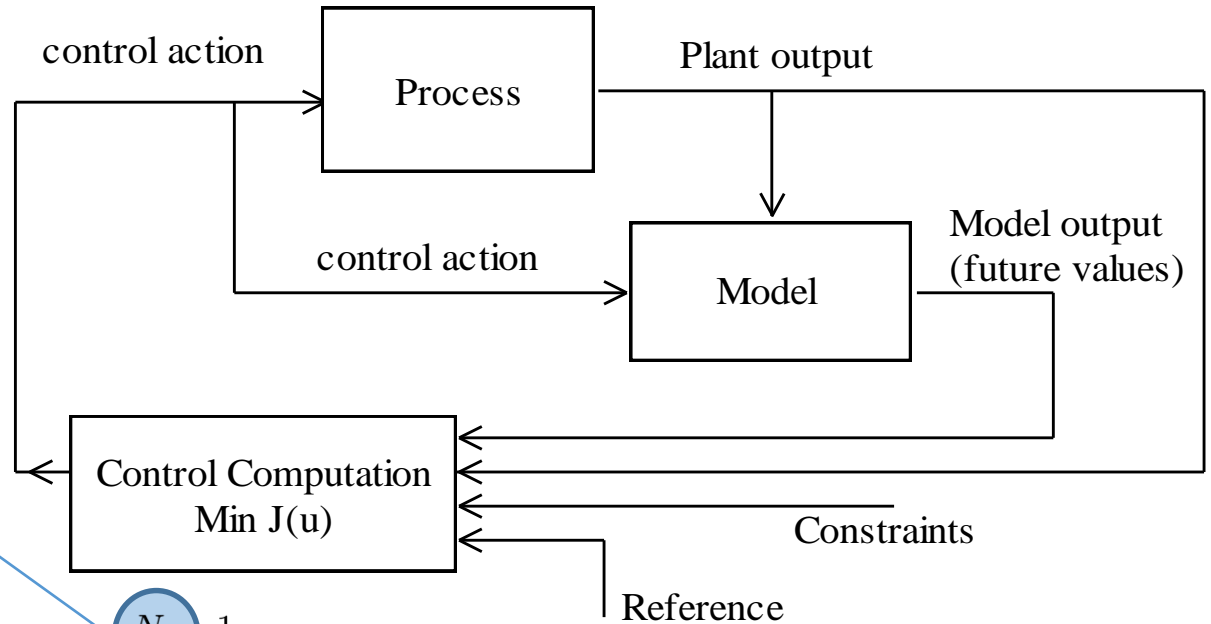
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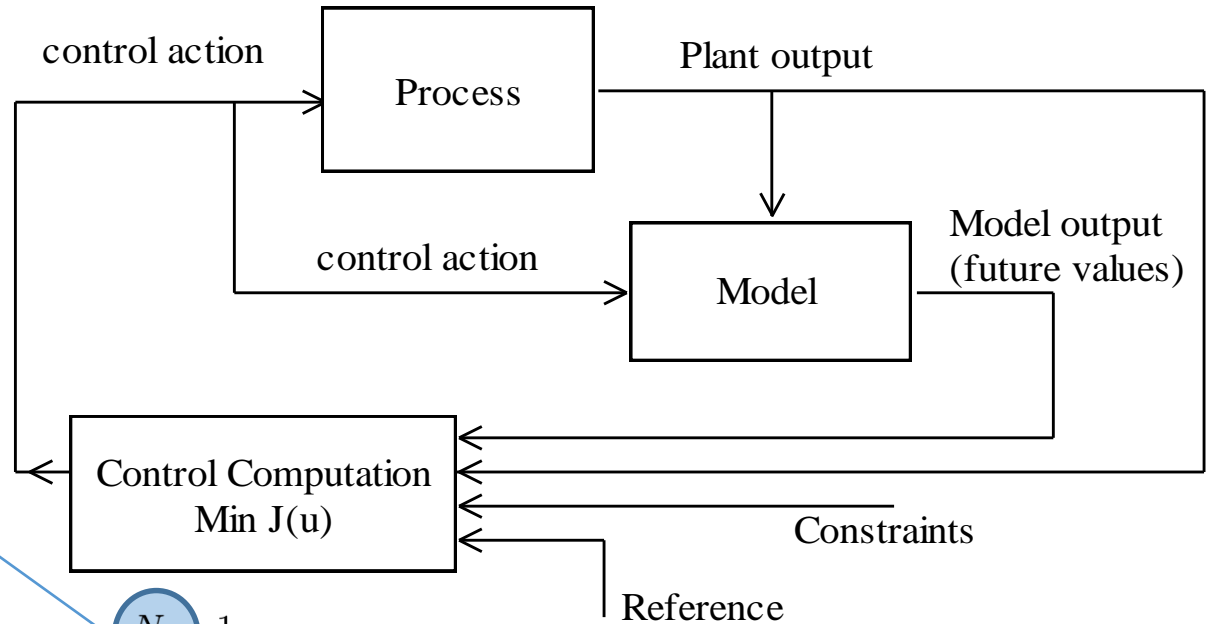
## GPC Model

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k-1) + \frac{e(t)}{\Delta} \quad L = dT_s$$

$$y(k+d+j/k) = f(u_{fut}, y_{past}, u_{past})$$

# GPC analysis for Dead-time Processes

## General MPC idea



## GPC cost

$$J = \sum_{j=d+1}^{d+N_y} [y(k+j|k) - w(k+j)]^2 + \sum_{j=0}^{N_u-1} \lambda [\Delta u(k+j)]^2,$$

## GPC Model

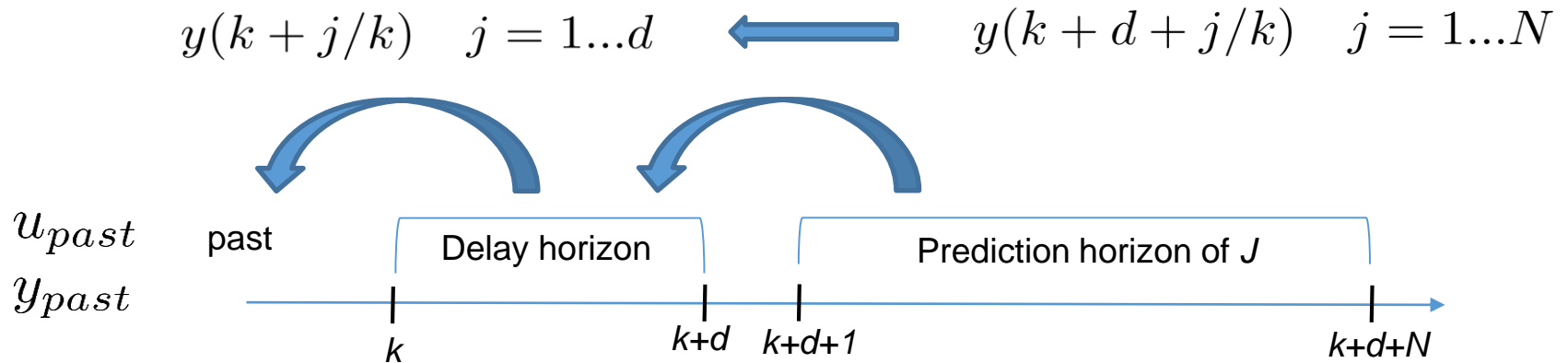
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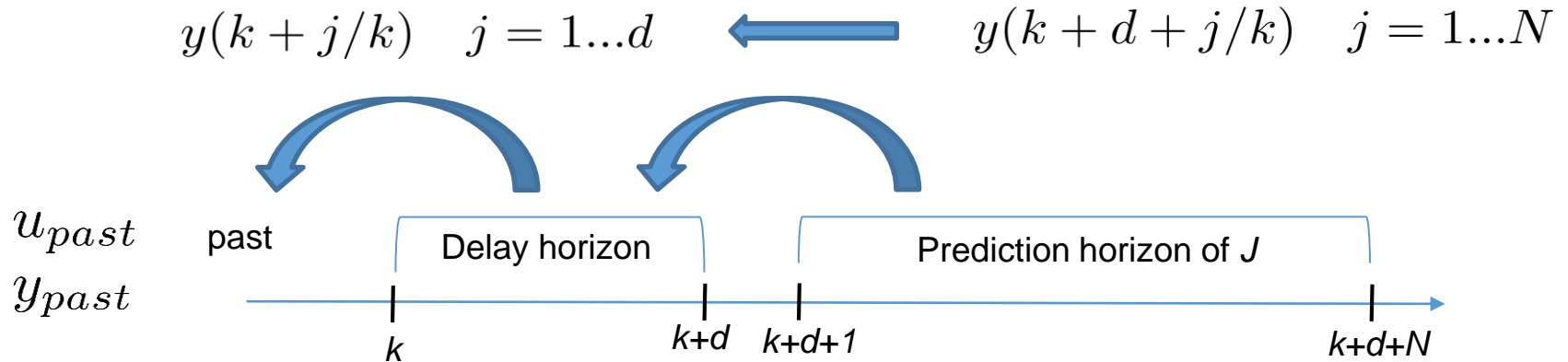
$$J = J(u_{fut})$$

# GPC analysis for Dead-time Processes

## Prediction computation



## Prediction computation

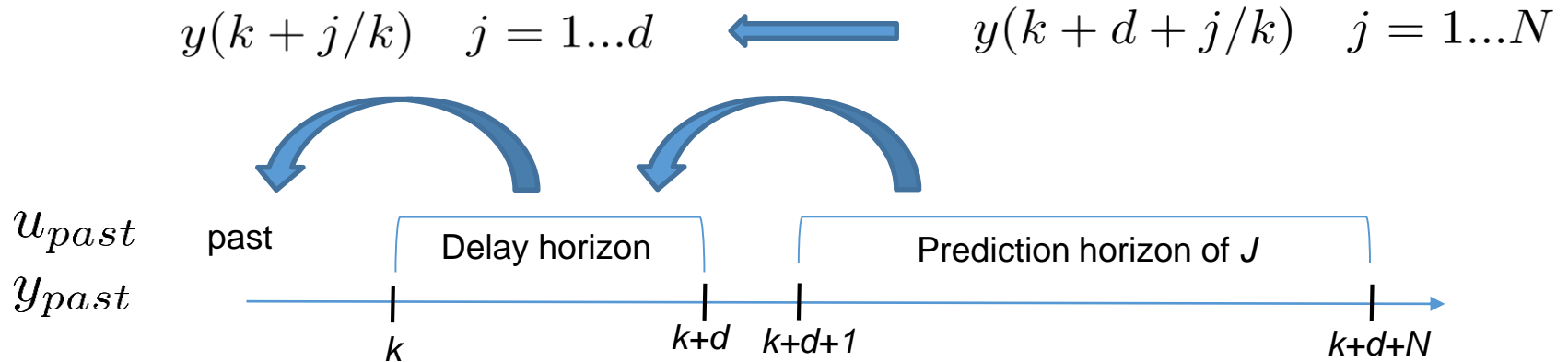


GPC structure ?

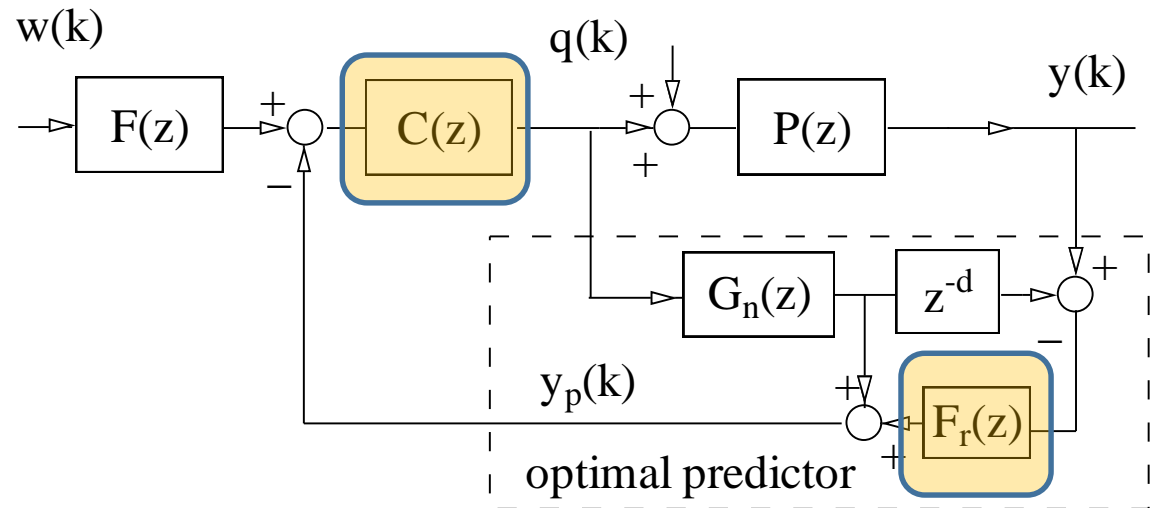


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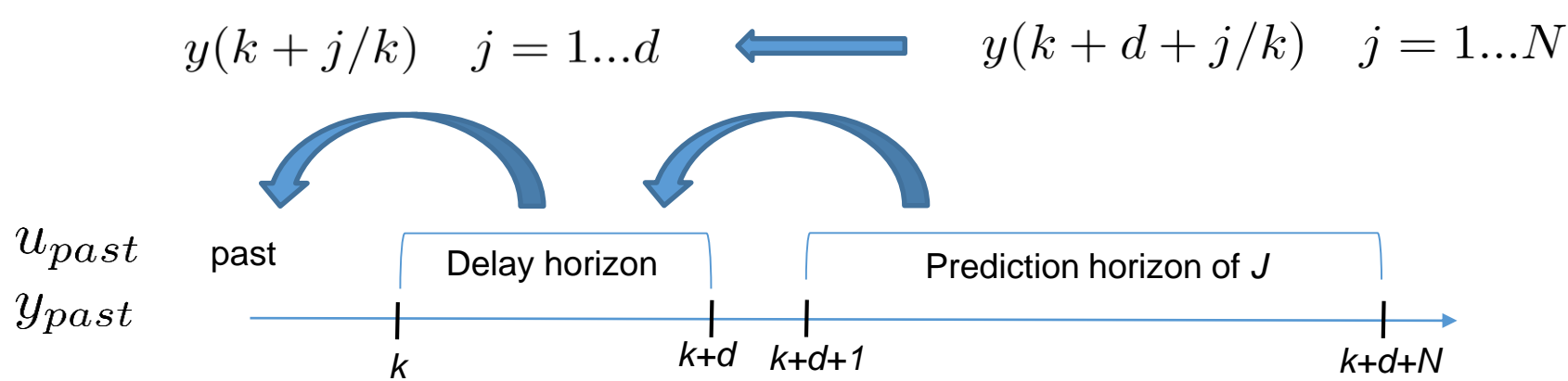


## GPC structure ? (unconstrained)



# GPC analysis for Dead-time Processes

## Prediction computation



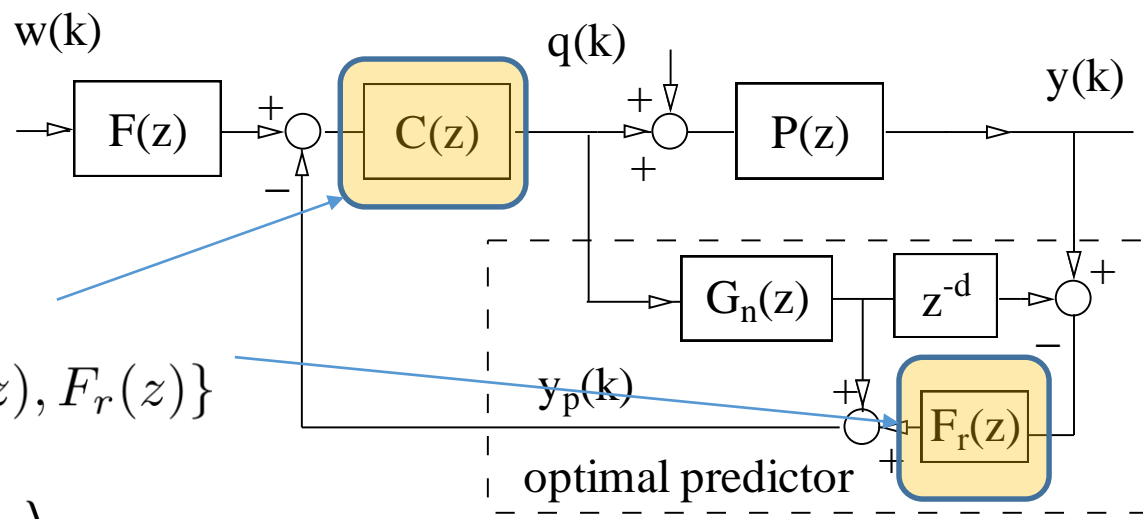
## GPC structure ?

(unconstrained)

$C(z)$  integral action

$order\{G_n(z)\} \rightarrow order\{C(z), F_r(z)\}$

coefficients related to  $N, N_u, \lambda$



## Unconstrained GPC structure

- GPC is equivalent to a discrete FSP
- FSP can be tuned using GPC method (exactly the same solution)
- FSP-MPC can be used (for robust controllers and easy tuning)\*
- For 1<sup>st</sup> order models → GPC → 2DOF FSP (PI primary controller)

**Comparison FSP-PID is valid for GPC-PID for 1<sup>st</sup> order models**

**Is valid for other linear MPC (simply a model rearrangement)**

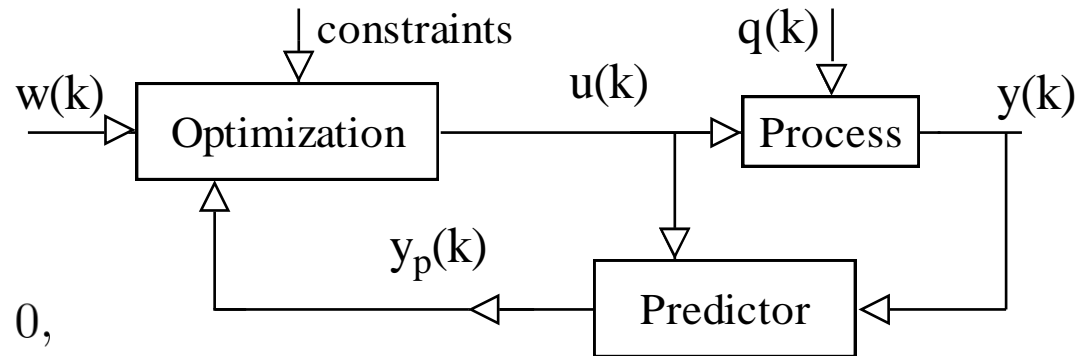
**Constrained case?**

\* Normey-Rico and Camacho, 2007, Springer

\* Lima, Santos and Normey-Rico, 2015, ISA Transactions

## Constrained GPC

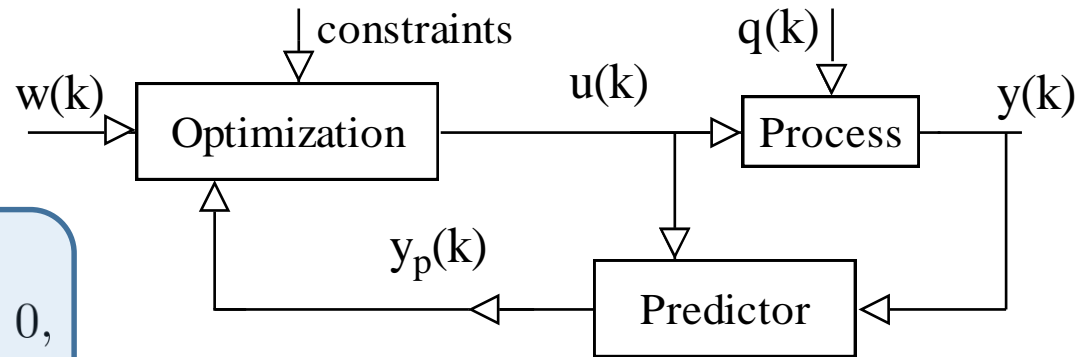
$$\begin{aligned} \underline{U} &\leq u(k) \leq \bar{U} \quad \forall k \geq 0, \\ \underline{u} &\leq u(k) - u(k-1) \leq \bar{u} \quad \forall t \geq 0, \\ \underline{y} &\leq y(k) \leq \bar{y} \quad \forall k \geq 0. \end{aligned}$$



$$\mathbf{u} = [\Delta u(k) \dots \Delta u(k + N_u - 1)]$$

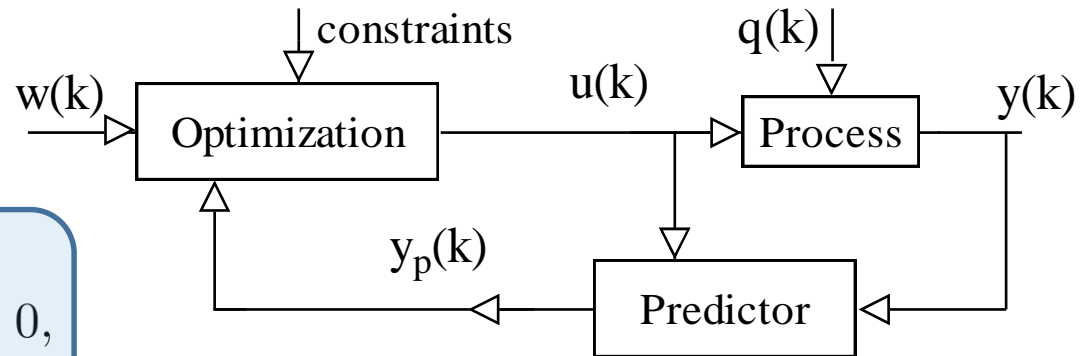
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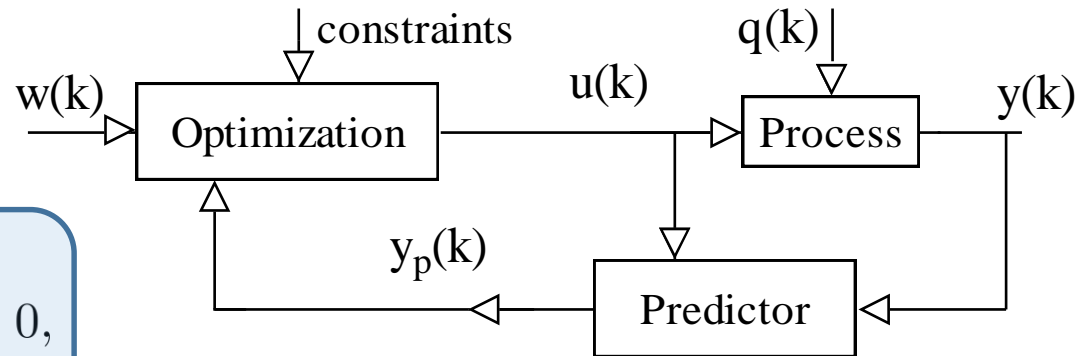
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**All constraints are written as a linear inequality on  $\mathbf{u}$**

## Constrained GPC



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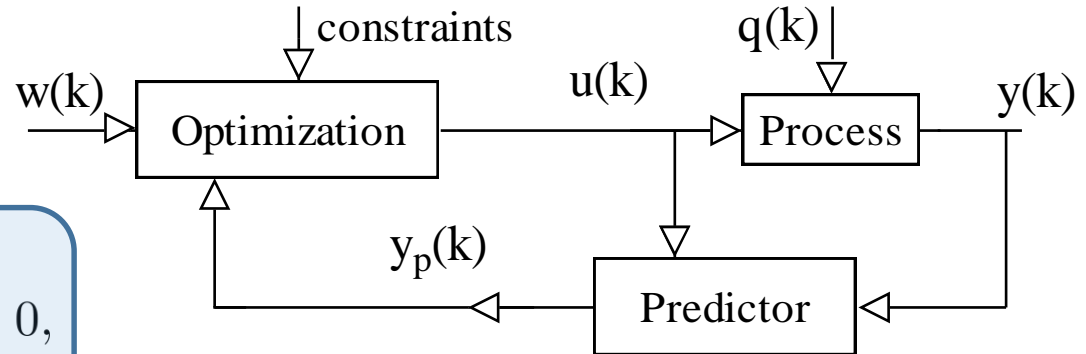
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**All constraints are written as a linear inequality on u**

- **QP solved at each sample time**
- **Only  $u(k)$  is applied**
- **The horizon window is displaced**

# GPC for dead-time processes

## Constrained GPC



$$\begin{aligned} \underline{U} &\leq u(k) \leq \bar{U} \quad \forall k \geq 0, \\ \underline{u} &\leq u(k) - u(k-1) \leq \bar{u} \quad \forall t \geq 0, \\ \underline{y} &\leq y(k) \leq \bar{y} \quad \forall k \geq 0. \end{aligned}$$

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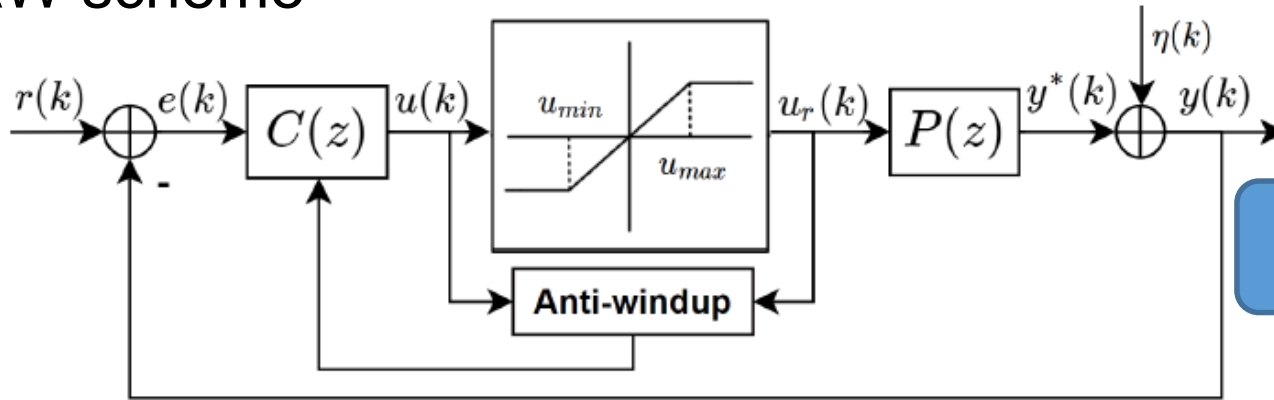
All constraints are written as a linear inequality on  $\mathbf{u}$

GPC gives goods results with small  $N_u$  (in many applications  $N_u=1$  is enough\*)

\* De Keyser and Ionescu, IEEE CCA 2003



## AW scheme



Valid for PID and FSP

$u_i(k)$  has the integral action of PID or FSP

$$u(k) = u_i(k) + u_d(k)$$

$u_d(k)$  has the rest of the control action of PID or FSP

**AW originally derived for control action constraints**

**Several AW strategies in literature**

# AWP with error recalculation (ER)

Recalculation of the error signal at every sample

Objective: to maintain the consistence between  $u(k)$  (computed) and  $u_r(k)$  (applied)

\* Flesch and Normey-Rico, Control Eng. Practice, 2017

\*Silva, Flesch and Normey-Rico, IFAC PID 18

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**PID case**  $\left[ \begin{array}{l} u(k) = u(k-1) + n_0 e(k) + n_1 e(k-1) + n_2 e(k-2) \\ u(k) > u_{max} \rightarrow u_r(k) = u_{max} \end{array} \right.$

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Consider:  $u_r(k) = u(k-1) + n_0 e^*(k) + n_1 e(k-1) + n_2 e(k-2)$   
?

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$\rightarrow e^*(k) = e(k) + \frac{u_r(k) - u(k)}{n_0}$

Used in the code to  
 update the error:  
 $e(k-1) = e^*(k)$

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**ER\* better results, principally in noise environment**

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\*Silva, Flesch and Normey-Rico, IFAC PID 18

# AW for dead-time processes

Including several constraints in AW scheme

$$u(k) < U_{max}$$

$$\Delta u(k) < \Delta u_{max}$$

$$y(k) < y_{max}$$

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Direct



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Direct

$$\Delta u(k) = u(k) - u(k-1) < \Delta u_{max}$$

$$u(k) < \Delta u_{max} + u(k-1)$$

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Direct

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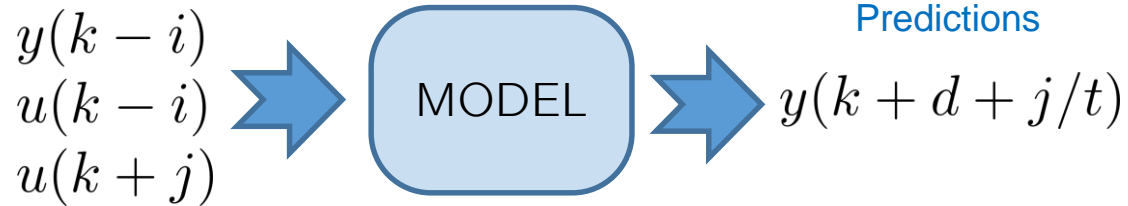
$$y(k) < y_{max}$$

$$\Delta u(k) = u(k) - u(k-1) < \Delta u_{max}$$

Using prediction ideas

$$u(k) < \Delta u_{max} + u(k-1)$$

$$y(k+j) < y_{max} \quad \forall j = 1 \dots N_y$$



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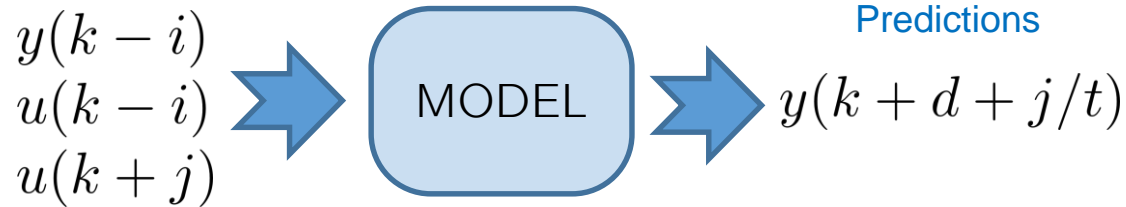
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$$y(k+j) < y_{max} \quad \forall j = 1 \dots N_y$$

$$u(k) < \Delta u_{max} + u(k-1)$$



Assuming  $N_u = 1$   
 $u(k+j) = u(k) \quad \forall j$



$$y(k+d+j/k) = f(u(k), y(k-i), u(k-i))$$

# AW for dead-time processes

**SIMPLE CASE: FOPDT**

$$y(k) = ay(k - 1) + bu(k - d - 1)$$

# AW for dead-time processes

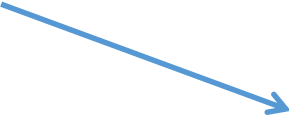
**SIMPLE CASE: FOPDT**       $y(k) = ay(k - 1) + bu(k - d - 1)$

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$$y(k + d + j) = a^j y(k + d) + \underbrace{(a^{j-1} + a^{j-2} + \dots + 1)b}_{K_j} u(k)$$

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$$y(k + d + j) < y_{max} \quad \Rightarrow \quad u(k) < \frac{y_{max} - a^j y(k + d)}{K_j}$$

$$u(k) < \min\{U_{max}; \Delta u_{max} + u(k - 1); \frac{y_{max} - a^j y(k + d)}{K_j}\}$$



# GPC or FSP(PID) ER-AW?

- Constrained GPC or FSP-ER-AW
  - Good tuned FSP with ER-AWP **equivalent** to GPC ( $Nu=1$ )
  - On-line **optimization is avoided** with FSP
  - FSP filter tuning is **easy** in practice

Several successful applications in solar systems and refrigeration plants \*

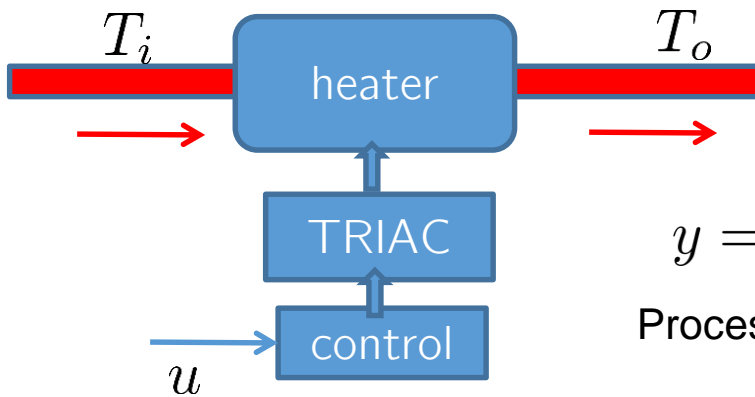
- In robust industrial solutions → PID-ER-AW
  - **Simple models** are used
  - **Robust tuning** (low  $M_s$  or high  $R_m$  values)

\* Roca, Guzman, Normey-Rico, Berenguel and Yebra, Solar Energy, 2011

\* Flesch and Normey-Rico, Control Eng. Practice, 2017

# Water temperature control

Experiments: Electrical water heater



$$y = T_o - T_i$$

Process variable

Normalized Control variable  
 (number pulses)

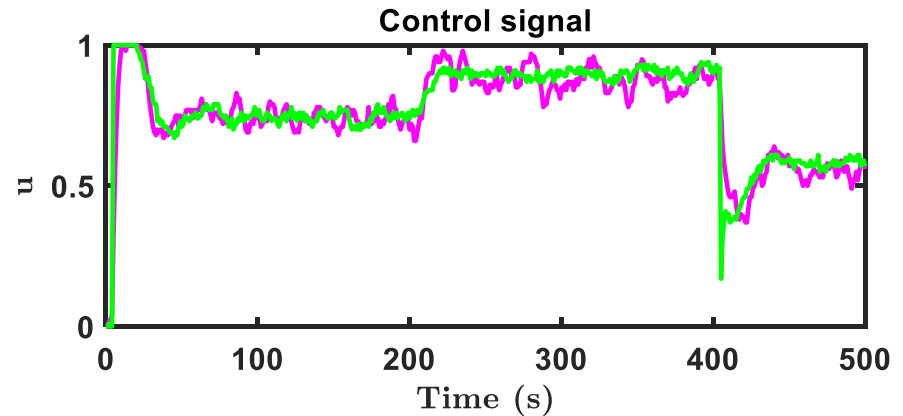
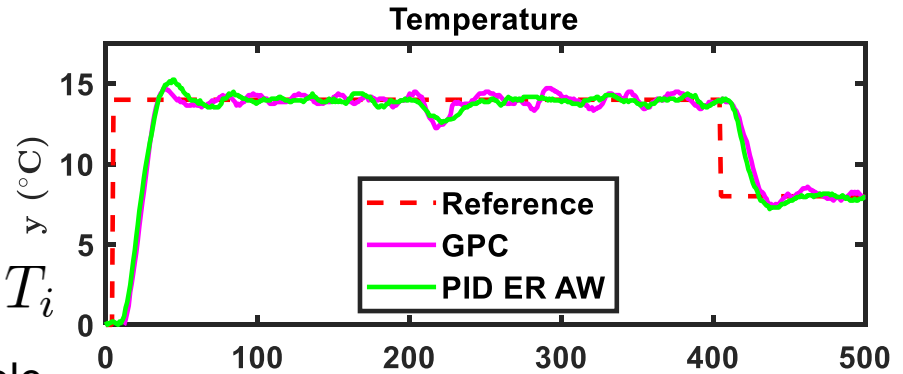
$$u_{max} = 1$$

$$u_{min} = 0$$

Model identification:  
 step test

$$P(s) = \frac{18.7e^{-8s}}{13.1s+1}$$

GPC -  $N = 60, N_u = 10, \lambda_n = 1$   
 PID -  $T_0 = 8$



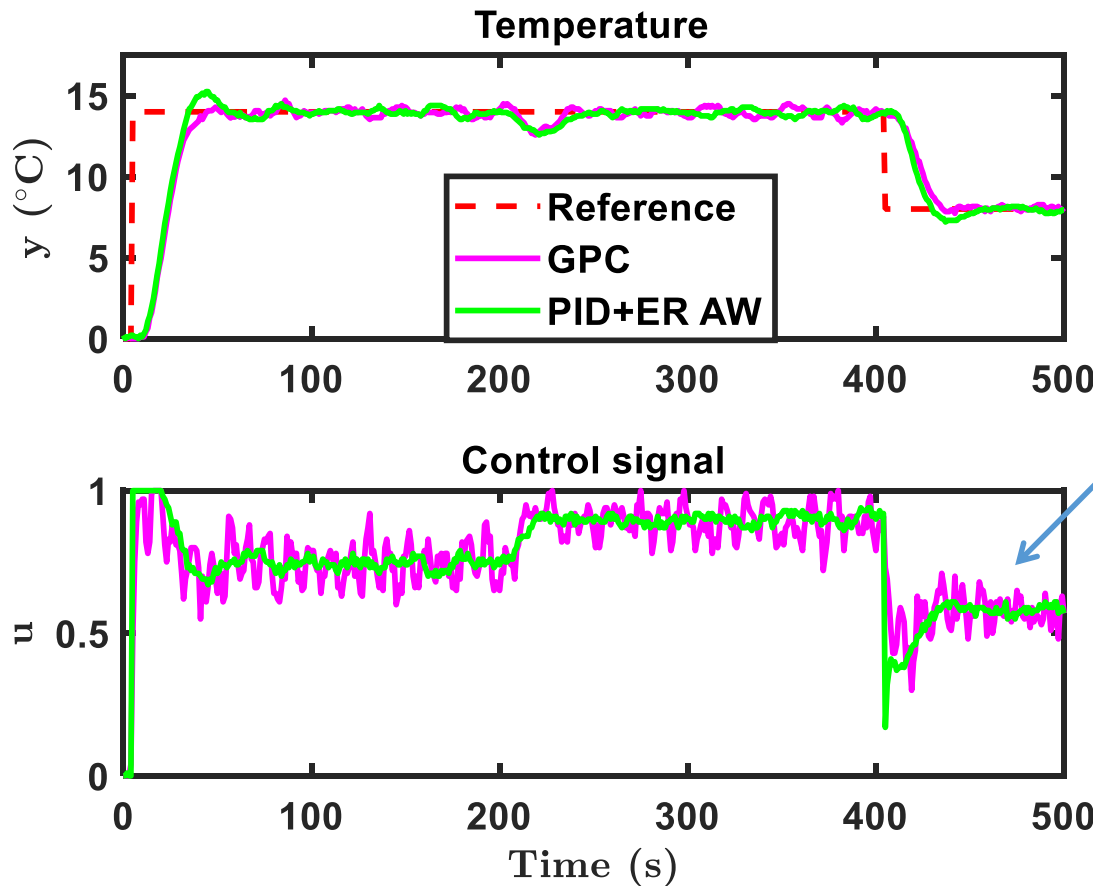
Same IAE performance  
 PID smoother control action

# Temperature control

PID -  $T_0 = 8$

New GPC tuning to accelerate the responses

GPC -  $N = 60, N_u = 10, \lambda_n = 0.3$



Problems:

- Small performance improvement
- Lower robustness
- Lower noise attenuation

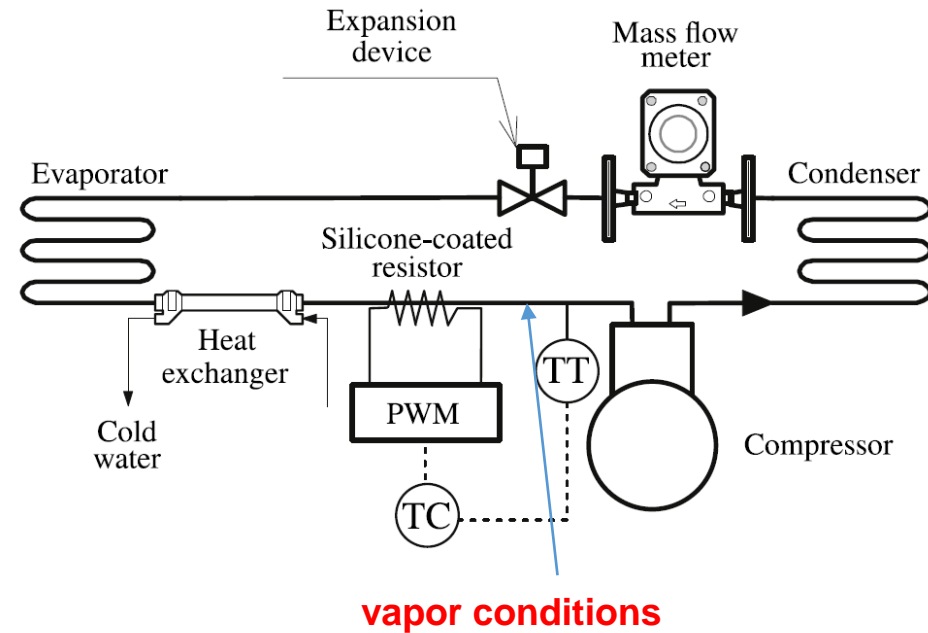
**PID is simpler and better**

# Compressor-test system

$$\frac{T(s)}{U(s)} = \frac{0.76}{304.7s+1} e^{-108s}$$

$$u_{max} = 95\%$$

$$u_{min} = 5\%$$

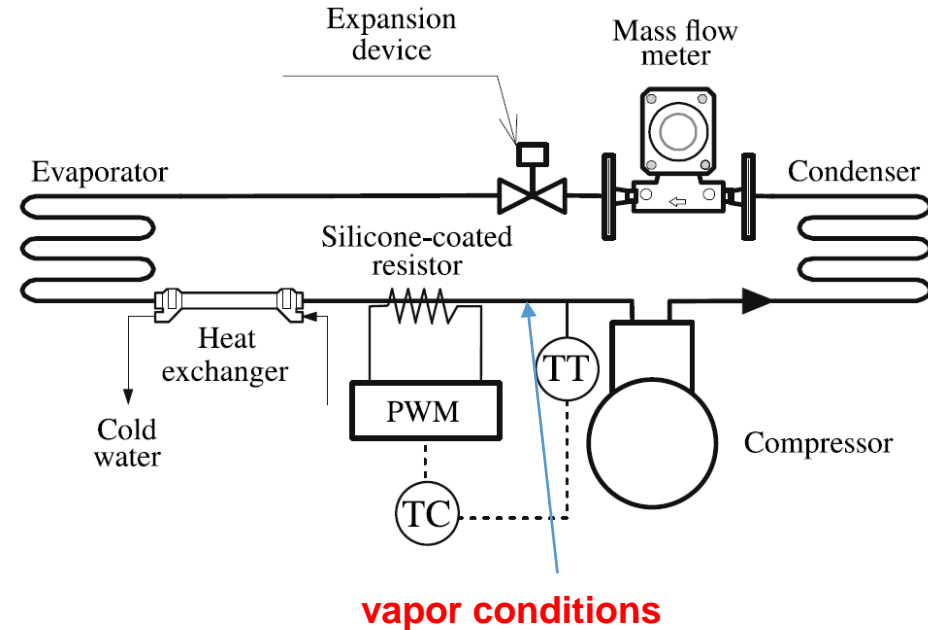


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## Important

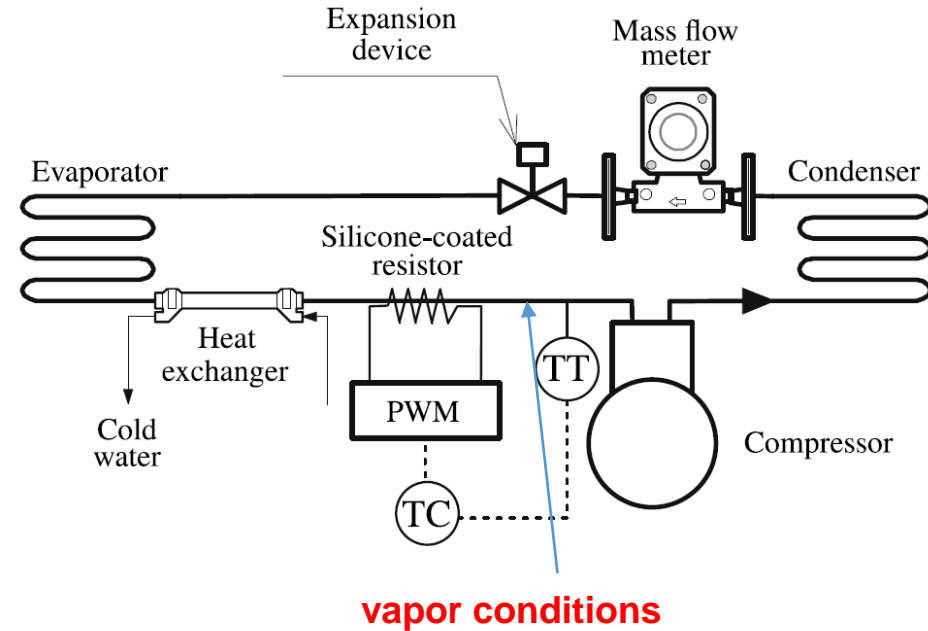
- To maintain Inlet temperature
- Fast set-point response
- Fast disturbance rejection
- Delay error well estimated

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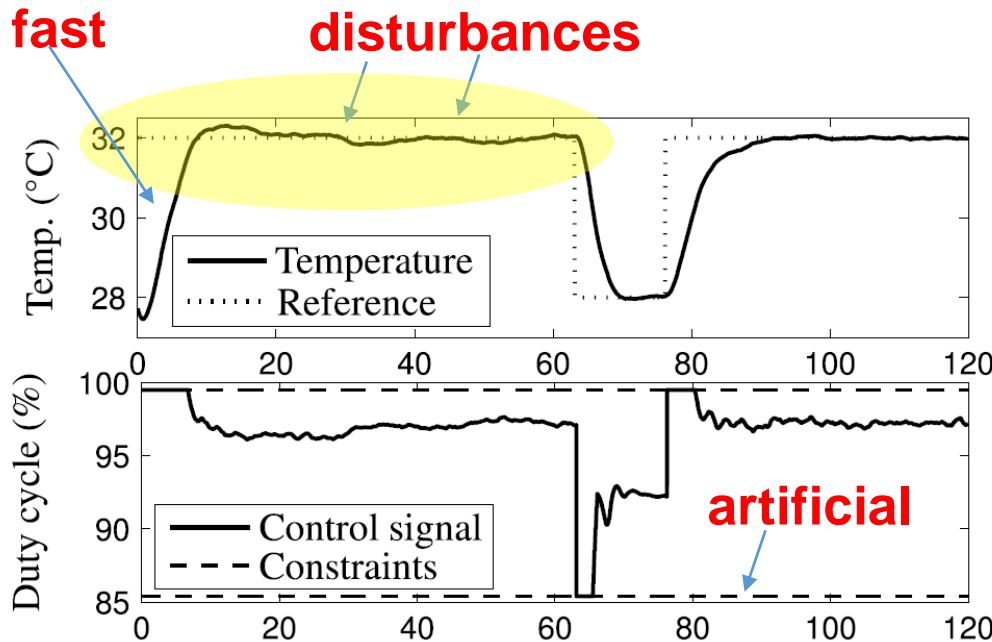
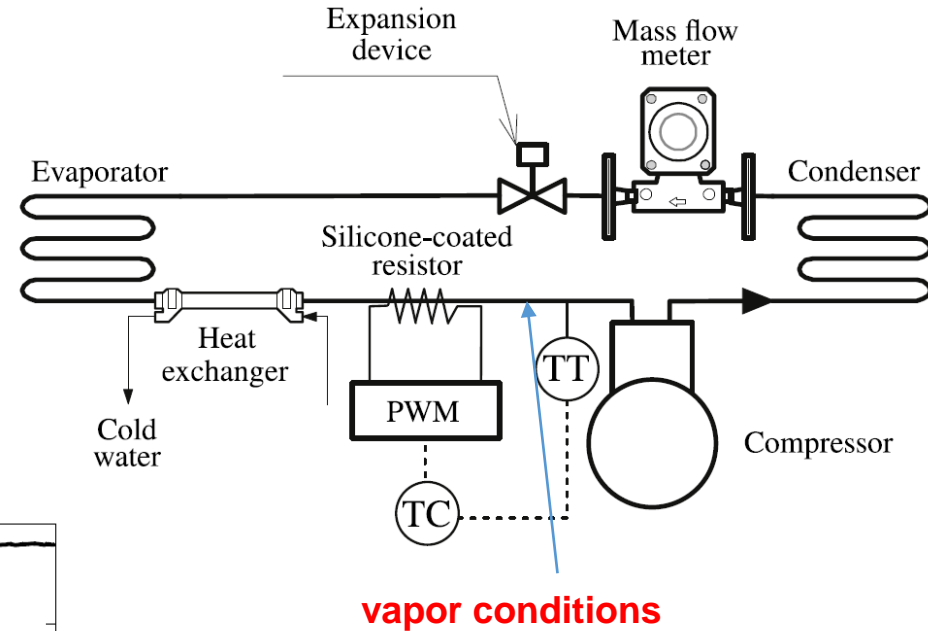
**FSP ER-AWP**

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**FSP ER-AWP**

# Conclusions



# Conclusions

- When controlling dead-time processes....
  - Performance measurement after the dead-time
  - Ideal solution can be achieved by FSP (or other improved DTC)
  - Dead-time estimation error is very important
  - Constrained case: ER AW FSP can be equivalent to MPC
- PID for dead-time processes
  - Can be tuned as a low order approximation of FSP
  - Performance improvement is limited in complex cases
  - For high robust solutions PID is equivalent to FSP (even for high L)
  - ER AW PID sub-optimal solution with good results.

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Low-order-process models  
Large modelling error  
Noise environment  
Typical constraints



**Well tuned robust PID  
with AW is the best  
option**

# Conclusions

- PID still has an important figure in process industry
- DTC strategies with PI or PID primary controllers can be considered as extensions of simple PID control and used in particular cases
- Improved AW PID algorithms (or FSP AW) can be the solution in modern real-time distributed control systems for simple constrained systems
- MPC solutions are important in complex well modeled systems and at second level control

# Thanks!

For your  
attention

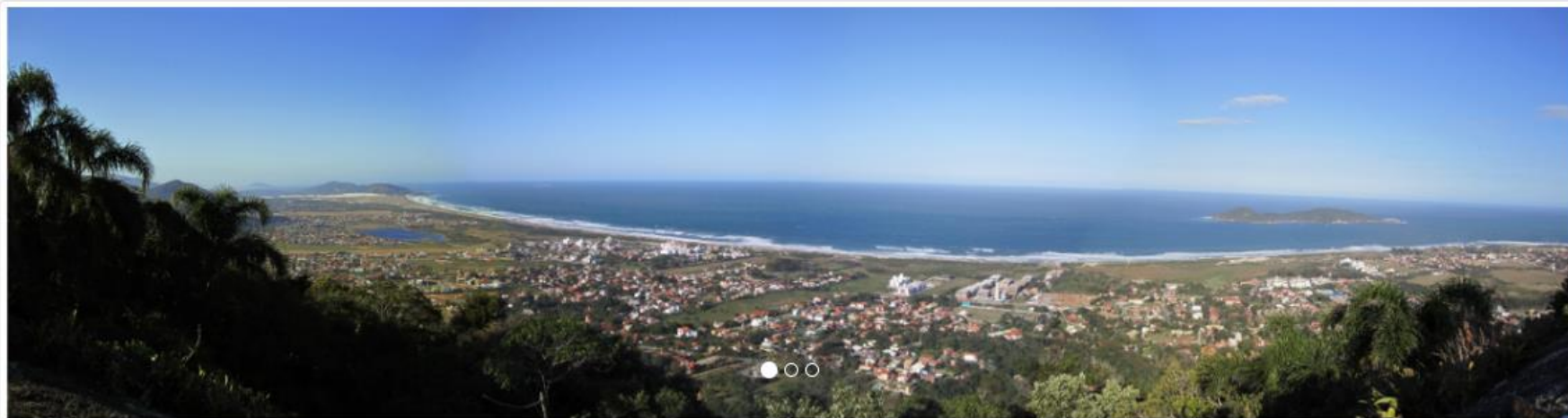
PID18  
organizers



UFSC



julio.normey@ufsc.br



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DYCOPS 2019

[dycops.cab2019@gmail.com](mailto:dycops.cab2019@gmail.com)

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