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Multi-criteria Optimization of PD Controllers for Plants including Integral Action

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Abstract A simple design method for robust PD controllers is presented for systems including integral action. The design method is based on a multi-criteria optimal control formulation, which is easily solved by a few lines of MATLAB code. Most criteria are based on $\mathcal{H}_\infty$ measures, but since the focus is on reference signal tracking and not load disturbance compensation, the settling time is also included as a relevant performance measure. Since a PD controller is equivalent to a lead filter, the optimal PD controller is compared with ordinary text book design rules for lead filters. More specifically, it is shown that the common recommendation to place the mid frequency of the lead filter at the desired gain crossover frequency often gives bad servo performance. The suggested optimal solution, still including robustness and control activity adjustments, is on the other hand a simple and flexible design method for arbitrary plants including integral action.

Keywords: PD control, optimization, lead lag design, $\mathcal{H}_\infty$ control, performance, robustness, education

1. INTRODUCTION

A number of optimal design methods for PI and PID controllers have been presented during the last two decades, see for instance Lennartson and Kristiansson (1997); Åström et al. (1998); Kristiansson and Lennartson (2006a); Lennartson and Kristiansson (2009); Larsson and Hägglund (2011); Garpinger et al. (2014). These methods are mainly based on non-convex multi-criteria optimization, including both performance, mid and high frequency robustness and control activity measures. The goal has been to efficiently compensate load disturbances, still keeping good stability margins and a moderate sensitivity to sensor noise in the control signal.

Very few publications have focused on the special case of optimal PD controllers. One recent exception is an optimal PD controller developed for robot manipulators in Kim et al. (2016). Combined optimal PD and PI controllers also appear, see for instance Alter and Tsao (1996), but they are special parameterizations of general PID controllers.

In Lennartson (2012), an optimal PID design was related to basic text book procedures on loop shaping in the frequency domain, cf. Franklin et al. (2006); Ogata (2002); Glad and Ljung (2006). For plants including integral action, the integral part in the controller is not always included, especially not for servo systems, where the focus is more on reference signal tracking than load disturbance compensation.

As a continuation of Lennartson (2012), the goal of this paper is to focus on the servo behavior of plants controlled by PD controllers. Integral action is assumed to be included in the plant, and the performance criteria are focused on the error signal, i.e. the deviation between the reference and the plant output signal. For this purpose, an $\mathcal{H}_\infty$ measure is considered, as well as the classical integrated absolute error, and the settling time. To obtain a fair optimization strategy, it is also important to include mid- and high frequency robustness and control activity measures in the optimization formulation. This is done by introducing appropriate constraints on suitable $\mathcal{H}_\infty$ criteria, according to Lennartson and Kristiansson (1997); Kristiansson and Lennartson (2006a).

In Glad and Ljung (2006) it is observed that a PD controller including a low pass filter is equivalent to a lead filter. To reduce the complexity of this PD controller/lead filter, the maximum phase lift of the PD controller is recommended to be placed at the desired gain cross over frequency, a standard recommendation for lead filter design, see also Franklin et al. (2006). In this paper, it is shown that this recommendation often is far away from the optimal solution. A typical behavior is a slow set point convergence resulting in up to 100% increase of the settling time t_s compared to the optimal solution.

The main contribution of this paper is the proposed simple optimization procedure that is easily implemented by a few lines of code in MATLAB. A second contribution is a systematic evaluation, by the proposed optimization procedure, of the classical PD-design rule mentioned above. A comparison for a number of typical plant models shows when the old design rule is still relevant and when it should be avoided.

In Section 2, the problem formulation is given, including the suggested PD optimization procedure, and in Section 3 the implementation of this procedure in MATLAB is discussed. In Section 4 the proposed optimization method is used to compare optimal PD controllers with classical PD design. This is followed up by some conclusions in Section 5.

The main conclusion is that it is time to replace classical design methods with optimization formulations that focus on the main issues in feedback control, to understand the conflict between performance, stability margins and high frequency robustness and control activity. Basic courses on feedback control should focus more on how to formulate and solve relevant optimization problems than learning principles that are introduced to be able to solve problems by hand, something that engineers and researchers never do in practice. Let the computer make the calculations and focus more on problem formulations and how to interpret the achieved results from optimized design solutions.

2. MULTIPLE CRITERIA FOR PLANTS INCLUDING INTEGRAL ACTION

A set of important $\mathcal{H}_\infty$ criteria was introduced in Lennartson and Kristiansson (1997); Kristiansson and Lennartson (2006a) for design and evaluation of robust PID controllers. These criteria are briefly presented in this section, where reduction of load disturbances is replaced by tracking error compensation. First, consider a system with a plant $G(s)$ that is controlled by a one-degree of freedom controller $K(s)$, resulting in a loop transfer $L(s) = G(s)K(s)$. Furthermore, the plant is assumed to have m integrators, that is

$$G(s) = \frac{\bar{G}(s)}{s^m},$$

where $m \geq 1$ and $\bar{G}(0)$ is finite.

2.1 Performance measures

A fundamental goal of a feedback system is that the deviation between the reference signal r and the plant output signal y, the error $e = r - y$, is small. The transfer function from r to e is

$$G_{re}(s) = \frac{1}{1 + G(s)K(s)}$$

When the controller has a finite low frequency gain $K(0)$, as the PD controller, and the plant includes m integrators, the low frequency (LF) behavior of this transfer function is $|G_{re}(j\omega)| = \omega^m/(\bar{G}(0)K(0))$. To include a broader frequency range, concerning tracking of the reference signal, we introduce the $\mathcal{H}_\infty$ criterion

$$J_e = \max_\omega \frac{1}{\omega^m}|G_{re}(j\omega)| = ||\frac{1}{s^m}G_{re}(s)||_\infty$$

where the weighting $1/\omega^m$ is motivated by the LF behavior of $|G_{re}(j\omega)|$. A low value of J_e guarantees a small tracking error for lower frequencies up to the middle frequency range.

In MATLAB this criterion is easily computed, assuming that $\bar{G}$ and K are available in terms of transfer functions or state space models. Then the $\mathcal{H}_\infty$ norm is computed as `norm(feedback(1/s^m,G_bar*K),inf)`, where `s=tf('s')`.

An alternative performance criterion, often suggested for evaluation and optimization of PID controllers, see Åström and Hägglund (1995); Garpinger et al. (2014), is the integrated absolute error

$$IAE = \int_0^\infty |e(t)|dt.$$

Since the $\mathcal{H}_\infty$ criterion J_e is easier to compute in MATLAB we will show that the two criteria J_e and IAE give very similar results, and therefore J_e will in this paper be used as the main tracking performance measure.

A complementary measure that is of interest for step responses is the *settling time*. For a response $y(t)$ after a step in the reference signal r at time $t = 0$, the settling time t_s is the minimum time after which

$$\frac{|y(t) - y(\infty)|}{|y(\infty)|} \leq \delta$$

is satisfied for all $t \geq t_s$. Here we assume that $\delta = 0.02$, i.e. a maximum of 2% deviation from the final output value $y(\infty)$ is required for all $t \geq t_s$.

2.2 Control activity

In the high frequency (HF) range, it is important to avoid too much sensor noise in the control signal. Hence, consider the transfer function from the sensor noise $w(t)$ to the control signal $u(t)$

$$G_{wu}(s) = \frac{K(s)}{1 + G(s)K(s)}$$

where the measured plant output is $y_m = y - w$. This transfer function often has its maximum gain at $\omega = \infty$, but peaks may also occur in the middle to high frequency range. In any case, the maximum gain of $G_{wu}(\omega)$ is a simple and suitable measure of the sensitivity to sensor noise in the control signal. Thus, the $\mathcal{H}_\infty$ criterion

$$J_u = \max_\omega |G_{wu}(j\omega)| = ||G_{wu}(s)||_\infty$$

is used as a mid to high frequency measure of the control activity in the feedback system.

2.3 Stability margin

In the pass band, robustness is achieved by ensuring good stability margins. Generally, the loop transfer $G(j\omega)K(j\omega)$ must be kept at an acceptable distance from the critical point (-1,0) in the Nyquist plot. To ensure this, the shortest distance to the point (-1,0), $\min_\omega |1 + G(j\omega)K(j\omega)|$, has been introduced as a stability measure. Consider the sensitivity function

$$S(s) = \frac{1}{1 + G(s)K(s)}$$

and its maximum gain

$$M_S = \max_\omega |S(j\omega)| = ||S(s)||_\infty$$

Obviously, this $\mathcal{H}_\infty$ criterion is the inverse of the shortest distance to the point (-1,0) in the Nyquist plot, and hence a lower value of M_S means a larger stability margin. For unstable plants, including those with at least two integrators in the loop transfer $L(s)$, it is also important to consider the complementary sensitivity function

$$T(s) = 1 - S(s) = \frac{G(s)K(s)}{1 + G(s)K(s)}$$

and its maximum gain

$$M_T = \max_\omega T(j\omega)| = ||T(s)||_\infty$$

A restriction on M_T also controls the damping of the system, without reducing M_S too much.

2.4 Controller design by multi-criteria optimization

In all kinds of controller design, a set of tuning parameters, here included in a vector x, has to be adjusted to obtain a desired closed loop behavior. In our case the behavior is measured by the $\mathcal{H}_\infty$ criteria suggested in this section, and the tuning parameters are the PD controller parameters to be optimized.

An objective method to evaluate different design methods is to minimize one criterion with respect to the tuning parameters in x, while constraints are introduced on the other criteria. In this paper the performance measure J_e is most often minimized, while constraints are included on J_u, M_S and M_T. Thus, controllers are designed by solving the following constrained optimization problem:

$$\begin{aligned} &\min_x J_e(x) \\ &M_S(x) \le M_{S_{max}},\ M_T(x) \le M_{T_{max}},\ J_u(x) \le J_{u_{max}}, \end{aligned} \tag{1}$$

which also can be considered as a multi-criteria optimization problem. In this paper, the default demands on M_S and M_T are $M_S \le M_{S_{max}} = 1.6$ and $M_T \le M_{T_{max}} = 1.3$. The constant $J_{u_{max}}$ depends on the plant model $\bar{G}(s)$. It is chosen to give a reasonable control activity, while only marginally deteriorating J_e compared to a high gain solution. In some evaluations J_e will be replaced by IAE and t_s.

3. OPTIMAL PD CONTROLLER IMPLEMENTATION

A PD controller is often formulated as

$$K_{PD}(s) = K_p\left(1 + \frac{sT_d}{1+sT_f}\right)$$

By introducing the filter time constant $\tau = T_d + T_f$ and the ratio $b = \tau/T_f$, the PD controller can be reformulated as

$$K_{PD}(s) = K_p\frac{1+s\tau}{1+s\tau/b}$$

This is a lead filter with a lead ratio b, where the maximum phase lift

$$\varphi_{max} = \arcsin\frac{b-1}{b+1}$$

occurs at the mid frequency $\omega_m = \sqrt{b}/\tau$. In the literature, see e.g. Franklin et al. (2006), it is recommended to choose the lead filter, here the PD controller, such that the gain cross over frequency ω_c ($|L(j\omega_c)| = 1$) is equal to, or at least in the region of, the mid frequency ω_m. Thus, the restriction

$$\omega_c = \omega_m = \frac{\sqrt{b}}{\tau}$$

will be introduced as an optional equally constraint, to be able to evaluate this classical lead filter and PD controller design rule.

In the optimization, the tuning parameters in the vector x are the parameters in the lead filter formulation of the PD controller. Thus,

$$x = [\,K_p\ \tau\ b\,]$$

The constrained optimization problem (1) is solved by the routine `fmincon` in Matlab's Optimization Toolbox. A plant model, for instance $G(s) = \frac{1}{s(s+1)(1+0.5s)}$, plus criteria constraints, are in MATLAB coded as

```
s=tf('s'); G_bar=1/(1+s)/(1+0.5*s); m=1; G=G_bar/s;
MS_max=1.6; MT_max=1.3; Ju_max=10;
```

The routine `fmincon` requires the following MATLAB code, which implements the objective function

```
function Je=objfun(x,G_bar,m,s)
   Kp=x(1); tau=x(2); b=x(3);
   K=tf(Kp*[tau 1],[tau/beta 1]);
   Je=norm(feedback(1/s^m,G_bar*K),inf);
return
```

The next code, which is also required by `fmincon`, implements the actual constraints, in this code also including the optional equality constraint $\omega_c = \omega_m$.

```
function [C,Ceq]=confun(x,G,MS_max,MT_max,Ju_max)
   Kp=x(1); tau=x(2); b=x(3);
   K=tf(Kp*[tau 1],[tau/beta 1]);
   MS=norm(feedback(1,G*K),inf);
   MT=norm(feedback(G*K,1),inf);
   stab=norm(feedback(G*K,1));
   if stab<inf, C=[MS-MS_max; MT-MT_max; Ju-Ju_max];
   else C=[1; 1; 1]; end
   [Am,phim,wpi,wc]=margin(G*K);
   Ceq=[wc-sqrt(beta)/tau];
return
```

In the final implementation of this optimization routine, lower and upper limits on the three control parameters in x are also required. Generally, it is recommended to start with wider control parameter intervals, and then tighten when the optimal parameter region has been identified. Table 1 gives some guidelines for typical PD control parameter values to start with. Note that all plant models below are normalized such that the gain $|\bar{G}(0)| = 1$.

4. EVALUATION OF OPTIMAL PD CONTROLLERS

Optimal PD controllers for the following plant models are evaluated in this paper.

$$G_1(s) = \frac{1}{s(1+s)(1+0.1s)}$$

$$G_2(s) = \frac{1}{s(1+s)^2}$$

$$G_3(s) = \frac{1}{s(1+s)(1+0.7s)(1+0.7^2s)(1+0.7^3s)}$$

$$G_4(s) = \frac{1}{s(1+0.6s+s^2)(1+0.1s)}$$

$$G_5(s) = \frac{1}{s^2(1+0.1s)}$$

$$G_6(s) = \frac{1}{s(s-1)}$$

For the plant models $G_1(s)$-$G_6(s)$, resulting $\mathcal{H}_\infty$ criteria, based on the constrained optimization (1), are given in Table 1, including optimal PD parameters. The first plant $G_1(s)$ has simple dynamics, which means that high gain always improves the performance. This fact also gives room for an increased stability margin by reducing M_S to 1.4.

Lag filter for resonant plant The plant model $G_4(s)$ has a resonance with a damping factor 0.3. The optimal PD controller

Table 1. $\mathcal{H}_\infty$ criteria and optimal PD parameters for different plant models when J_e is minimized according to (1).

Model	J_e	J_u	M_S	M_T	K_p	τ	β
$G_1(s)$	0.476	15	1.4	1.00	2.10	1.03	7.14
$G_2(s)$	1.16	10	1.6	1.09	0.862	2.08	11.6
$G_3(s)$	2.43	5	1.6	1.00	0.413	1.44	12.1
$G_4(s)$	3.52	0.303	1.6	1.01	0.300	1.20	0.486
$G_5(s)$	2.13	5	1.53	1.3	0.470	3.05	10.6
$G_6(s)$	1.64	15	1.6	1.8	0.610	4.48	24.6

Table 2. A comparison between J_e and IAE when each of them is optimized.

Model	Opt. crit.	J_e	IAE
$G_2(s)$	J_e	1.16	1.42
	IAE	1.16	1.42
$G_3(s)$	J_e	2.43	2.83
	IAE	2.57	2.66
$G_4(s)$	J_e	3.52	3.78
	IAE	3.53	3.76

for this model is indeed a lag filter, since $\beta < 1$. The reason is that the PD controller cannot actively damp the resonance due to lack of complex zeros. The best it can do is to reduce the gain at the resonance frequency and above, but at the same time increase the relative gain at lower frequencies below the resonance. This is achieved by the lag filter.

Comparison between J_e and IAE In Table 2 the minimization of J_e in (1) is compared with the minimization where J_e is replaced by IAE. The resulting optimal controllers generate similar results, independent of which of the criteria that is minimized. Thus, we choose to continue with the minimization of J_e, since it is easier to compute in MATLAB.

Plant complexity The complexity of a plant model can be characterized by its κ number, $\kappa = |G(j\omega_{180_G})|/G(0)$, cf. Hang et al. (1991); Åström and Hägglund (1995), where ω_{180_G} is the frequency where the plant has a phase lag of $-180°$. For plants with integral action, the κ number is modified as

$$\kappa = \frac{\omega_{180_G}^m |G(j\omega_{180_G})|}{\bar{G}(0)}.$$

Evaluation of the tuning rule $\omega_c = \omega_m$ In Table 3 the optimal solution according to (1) is compared with the restriction to force $\omega_c = \omega_m$. This means that ω_c is placed at the maximum phase lift for the PD controller. First we see that the optimal relation for ω_c/ω_m decreases with increased plant complexity based on its κ number. This is valid for $G_1(s) - G_3(s)$, but not for $G_4(s)$. The reason for the special behavior of the resonant plant is that the optimal controller is a lag filter. For the other models this means that J_e also deteriorates more when the κ number increases when $\omega_c = \omega_m$. In the same way the settling time also increases when $\omega_c = \omega_m$. One exception is $G_1(s)$ for $M_S = 1.5$, in which case the optimal relation ω_c/ω_m is close to one. On the other hand it is enough to reduce M_S to 1.4 to achieve a nearly duplication of the settling time when ω_c/ω_m is forced to be equal one.

Table 3. κ number, $\mathcal{H}_\infty$ criteria and settling time t_s when J_e is minimized according to (1). A free relation between ω_c and ω_m is compared with the tuning rule $\omega_c = \omega_m$.

Model	κ	ω_c/ω_m	M_S	J_e	t_s
$G_1(s)$	0.287	0.779	1.4	0.476	1.74
		1	1.4	0.624	3.50
		1.02	1.5	0.450	2.42
		1	1.5	0.451	2.39
$G_2(s)$	0.500	0.598	1.6	1.16	6.52
		1	1.6	1.88	13.4
$G_3(s)$	0.697	0.171	1.6	2.42	7.99
		1	1.6	4.20	13.9
$G_4(s)$	1.70	0.478	1.6	3.52	14.8
		1	1.6	3.77	20.2

Table 4. A comparison between J_e and t_s when each of them is optimized. A free relation between ω_c and ω_m is compared with the rule $\omega_c = \omega_m$.

Model	Opt. crit.	ω_c/ω_m	J_e	t_s
$G_2(s)$	J_e	0.598	1.16	6.52
	t_s	0.281	1.55	3.11
	J_e	**1**	1.88	13.4
	t_s	**1**	1.88	13.4
$G_3(s)$	J_e	0.171	2.42	7.99
	t_s	0.175	2.71	4.84
	J_e	**1**	4.20	13.9
	t_s	**1**	4.48	13.4
$G_4(s)$	J_e	0.478	3.52	14.8
	t_s	0.268	3.91	10.5
	J_e	**1**	3.77	20.2
	t_s	**1**	4.03	10.8
$G_5(s)$	J_e	1.27	2.13	7.24
	t_s	1.06	2.23	7.05
	J_e	**1**	2.31	7.08
	t_s	**1**	2.31	7.08
$G_6(s)$	J_e	2.10	1.64	9.42
	t_s	2.10	1.64	9.42

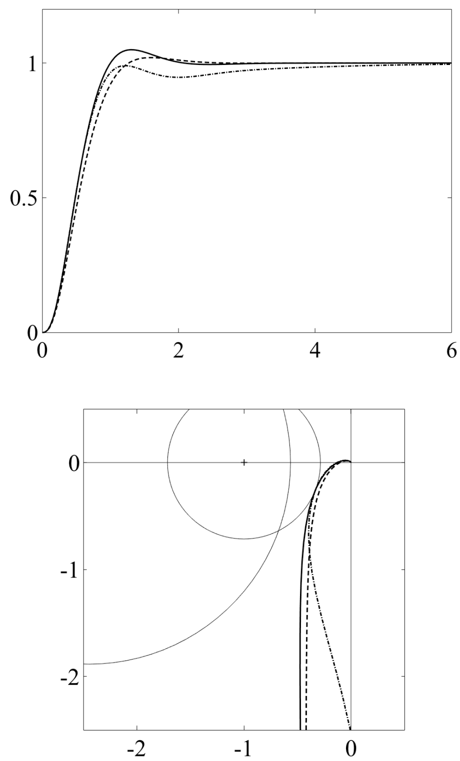

Figure 1. Step responses and Nyquist curves when $G_1(s)$ is controlled by an optimal PD controller. Solid curve optimal J_e, dashed curve optimal t_s, dashed dotted curve optimal J_e as well as optimal t_s with $\omega_c = \omega_m$.

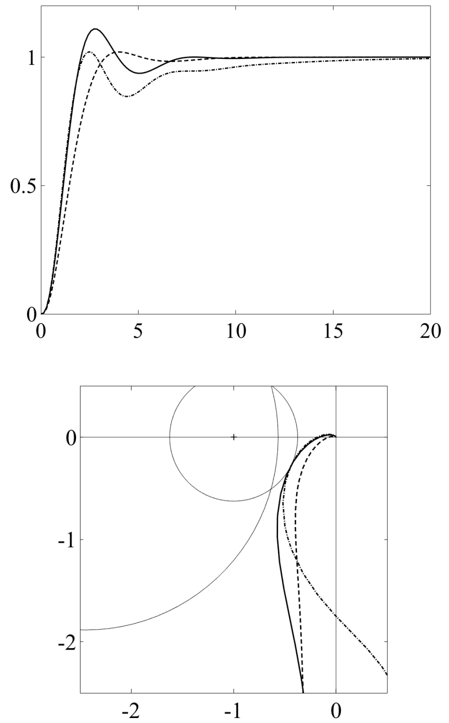

Figure 2. Step responses and Nyquist curves when $G_2(s)$ is controlled by an optimal PD controller. Solid curve optimal J_e, dashed curve optimal t_s, dashed dotted curve optimal J_e as well as optimal t_s with $\omega_c = \omega_m$.

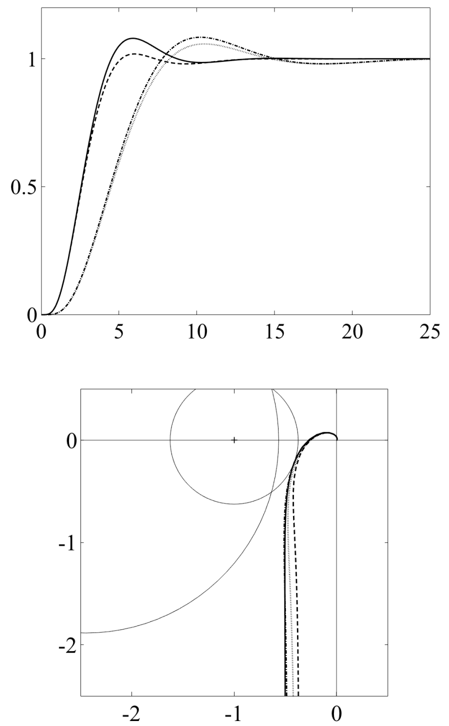

Figure 3. Step responses and Nyquist curves when $G_3(s)$ is controlled by an optimal PD controller. Solid curve optimal J_e, dashed curve optimal t_s, dashed dotted curve optimal J_e with $\omega_c = \omega_m$, and dotted curve optimal t_s with $\omega_c = \omega_m$.

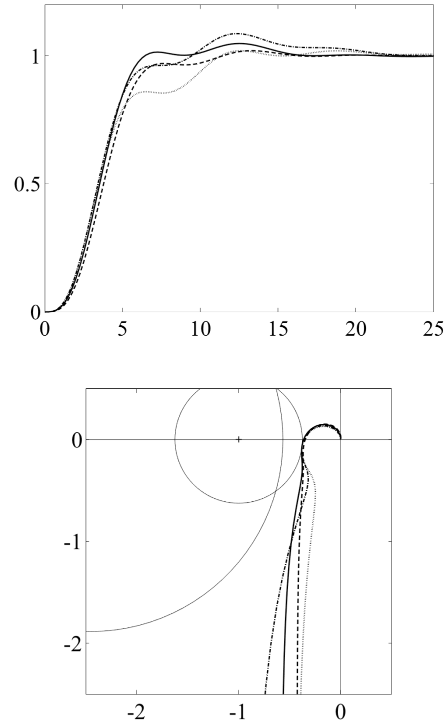

Figure 4. Step responses and Nyquist curves when $G_4(s)$ is controlled by an optimal PD controller. Solid curve optimal J_e, dashed curve optimal t_s, dashed dotted curve optimal J_e with $\omega_c = \omega_m$, and dotted curve optimal t_s with $\omega_c = \omega_m$.

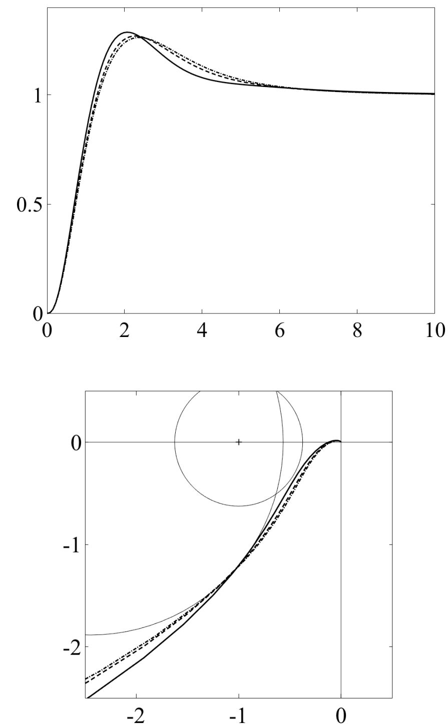

Figure 5. Step responses and Nyquist curves when $G_5(s)$ is controlled by an optimal PD controller. Solid curve optimal J_e, dashed curve optimal t_s, dashed dotted curve optimal J_e as well as optimal t_s with $\omega_c = \omega_m$.

Settling time minimization In Table 4 the optimization of J_e is compared with the optimization of t_s, for the models $G_2(s)-G_5(s)$ including the restriction $\omega_c = \omega_m$. The results show that $\omega_c = \omega_m$ increases t_s significantly up to 100%. We also see that avoiding this restriction makes it possible to reduce the settling time even further by minimizing t_s. This is confirmed by the step responses for the different plants in Fig. 1 - Fig. 5.

The related Nyquist curves show that the restriction of M_S is the active constraint for the first four models G_1 - G_4. For lower frequencies the gain is smaller, especially for G_1 and G_2 when $\omega_c = \omega_m$. This results in a slower steady state convergence in the step responses, a price that needs to be paid to keep the desired stability margin M_S also when the non-optimal relation between ω_c and ω_m is chosen. Finally, observe that no feasible solution at all is obtained for the unstable plant G_6 when $\omega_c = \omega_m$.

5. CONCLUSIONS

For plants with integral action, a simple optimization procedure is presented for design of PD controllers. This optimal design is compared with a standard tuning rule, which recommends that the maximum phase lift of a PD controller should be placed at the desired gain cross over frequency, The evaluation in this paper shows that this tuning rule cannot be recommended, especially not for systems with more complex dynamics. The main conclusion is that old tuning rules should be replaced by simple optimization procedures. Let the computer make the calculations and focus more on problem formulations and how to interpret the achieved results from optimized design solutions.

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