Observer-PI scheme for the stabilization and control of high order delayed systems with one or two unstable poles

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REFERENCIAS

Resumen
This paper considers the problem of stabilization and control of linear time invariant high order systems with one or two unstable real poles, n real stable poles plus time-delay. In order to ensure a stable behavior of the closed loop system, necessary and sufficient conditions for the existence of an observer based controller together with a PI compensator are shown explicitly. Numerical simulation on academic examples are provided to illustrate the effectiveness of the proposed control strategy

Keywords: Time-delay, unstable, PI, Observer

1. INTRODUCTION

Time delay is a property of various dynamical systems, a way to define time-delay is "the time interval from the application of a control signal to any observable change in the process variable"[1]. This phenomenon appears in biochemical and chemical engineering process systems, material and information transmission, biological embedded systems, tele-operation, communications systems, among others. We can find the case of unstable time-delay systems in the industrial processes, like continuously stirred tank reactor [2]. Time-delay systems are typically more difficult compared to systems without time delays. In the literature, there are several works reporting different control strategies to solve the problem of time-delay systems: the modified Ziegler-Nichols method [3], classic controllers like Proportional (P), Proportional-Integral (PI), Proportional-derivative (PD) and Proportional-Integral-Derivative (PID) are used to stabilize time-delay systems [4]. Another solution for the control of delayed system is the Smith Predictor (SP), this strategy is an effective way for compensating the dead-time associated with the processes. Despite being one of the most popular dead-time compensating methods, this technique has several limitations, for example, the original SP structure does not have a stabilization step and thus, it can only be used in open-loop stable plants [5].

With others perspectives, several authors proposed modifications to the original SP structure in order to face the limitations of the traditional SP. In [6] a modification of SP to control and to stabilize first order systems with time-delay was presented, [7] proposed a control scheme with two degree-of-freedom for control of unstable delay processes. The case of delayed systems with two unstable poles is addressed in [8], the authors proposed an observer based control scheme to stabilization of high order system with two real unstable poles and one minimum phase zero. Another study has been reported in [9] ,where necessary and sufficient conditions for the stabilization of delayed linear systems with two unstable real poles and n real stable poles by PD/PID controllers are provided

This paper proposes necessary and sufficient conditions to stabilize a kind of time-delay systems with one or two unstable real poles and n stable poles , necessary and sufficient conditions for the existence of the proposed scheme are given. The control scheme is complemented with a PI controller in order to obtain reference tracking and disturbance rejections. One of the proposed innovations in the control scheme with respect to [8], is to add a PI controller in the observer-scheme, this consideration

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provides a more robust control scheme with respect to parametric uncertainties in the process model.

The paper is organized as follows, Section 1 gives a brief introduction to time-delay systems, importance and challenges. In Section 2 we show the problem statement to be discussed in this paper. In Section 3 some preliminaries for the stability analysis are shown. Section 4 presents the proposed control strategy, an academic example with numerical simulation are shown in Section 5 and finally, some conclusions are given in the Section 6.

2. PROBLEM STATEMENT.

Consider the class of linear time invariant system, a Single-Input Single-Output (SISO) with delay in the direct path;

$$\frac{Y(s)}{U(s)} = \frac{N(s)}{D(s)} e^{-\tau s} = G(s) e^{-\tau s} \tag{1}$$

where:

- $Y(s)$ is the output signal
- $U(s)$ is the input signal
- $\tau > 0$ is the time delay
- $N(s)$ and $D(s)$ are polynomials in the complex variable s
- $G(s)$ is the delay-free transfer function

Applying a traditional control strategy based on an output feedback of the form:

$$U(s) = [R(s) - Y(s)]\, C(s) \tag{2}$$

produces a closed-loop system given by:

$$G_1(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)e^{-\tau s}}{1 + C(s)G(s)e^{-\tau s}} \tag{3}$$

where the exponential term $e^{-\tau s}$ located at the characteristic equation of the system (3), leads to a system with an infinite number of poles leading to more difficult stability analysis .

In this work, it is considered two possible types of delayed systems, a system with one unstable pole and n stable poles given by:

$$\frac{Y(s)}{U(s)} = \frac{1}{(s-a)\prod_{i=1}^{n}(s+c_i)} e^{-\tau s}, \tag{4}$$

where $a, c_i, \tau > 0$

and the case that the system has two unstable poles and n stable poles characterized by:

$$\frac{Y(s)}{U(s)} = \frac{1}{(s-a)(s-b)\prod_{i=1}^{n}(s+c_i)} e^{-\tau s} \tag{5}$$

where $a, b, c_i, \tau > 0$ and $a \geqslant b$

The control strategy proposed considers an observer based scheme together with a PI compensator in order to get an estimation of the internal variables of the system to be used as a control signal for the original process.

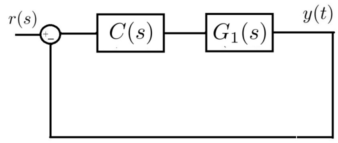

Figura 1. Closed loop controller configuration

The necessary and sufficient stability conditions for the controller and the observer convergence are appointed separately. These conditions allows to establish the closed-loop stability of the proposed control strategy

3. PRELIMINARY RESULTS.

This section presents preliminary results useful to obtain the main results of the present paper.

Now take into consideration the high-order unstable system characterized by:

$$\frac{Y(s)}{R(s)} = \frac{e^{-\tau s}}{(s-\sigma)(s+\rho_1)(s+\rho_2)...(s+\rho_n)}, \tag{6}$$

where $\tau \geq 0$, $\sigma, \rho_1, \rho_2, ...\rho_n > 0$.

Lemma 1. (11). Considering the delayed system (6) is stabilizable by P or PI controller C(s) connected in the configuration shown in Fig. 1 if and only if $\tau < \frac{1}{\sigma} - \sum_{i=1}^{n} \frac{1}{\rho_i}$

Lemma 2. (11). If a process $G(s)$ defined in (1), $N(s) \neq 0$ is stabilizable by a PD controller, so is it by a PID controller. Similarly, stabilizablity by P controller implies stabilizablity by PI controller

Now consider a SISO system with a single input delay

$$H(s) = \frac{P(s)}{Q(s)} e^{-\tau s} \tag{7}$$

Lemma 3. (12). . If $Q(s)$ is a stable polynomial, then the closed-loop system:

$$H_1(s) = \frac{P(s)e^{-\tau s}}{Q(s) + P(s)e^{-\tau s}} \tag{8}$$

is asymplotically stable if and only if:

$$|Q(j\omega)| > |P(j\omega)|\,, \forall \omega \tag{9}$$

4. CONTROL STRATEGY PROPOSED

4.1 CONTROLLER SCHEME

Let us consider the control strategy shown in Fig. 2, with a PI controller of the form,

$$C(s) = K_p + \frac{K_i}{s} \tag{10}$$

with K_p, K_i $\epsilon \mathbb{R}$

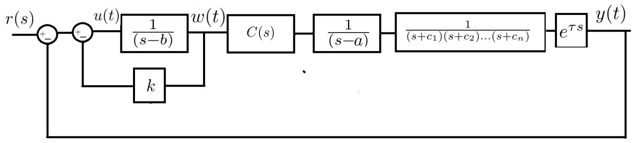

Figura 2. Control scheme

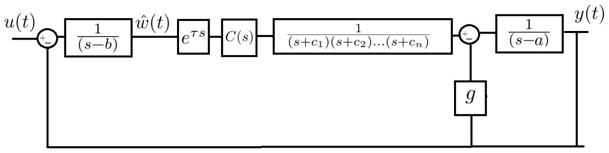

Figura 3. Observer scheme

Lemma 4. Consider the delayed system (5) and the control scheme shown in Fig 2. There exist constant k and a PI controller $C(s)$ such that the close-loop system is stable if and only if $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$

Proof.*Sufficiency.* Let us consider $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$. Then, $\tau = \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i} - \beta$ for some $\beta > 0$. Therefore, there exists k such that $\beta > \frac{1}{k-b} > 0$ then we can determine

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i} - \frac{1}{k-b} \tag{11}$$

Selecting a correct gain k, we have a system with one unstable pole and n stable poles. We can conclude from Lemma 1, where $\sigma = a$, $\rho_n = c_i$ and $\rho_{m+1} = k - b$, there exists a PI controller $C(s)$ such that the plant behavior is stable if $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$.

Necessity. Consider the delayed system (5) and the state feedback controller shown in Fig. 4, with a constant gain k such that the process is stable. The closed loop transfer function of the system can be written as follows:

$$\frac{Y(s)}{R(s)} = \frac{e^{-\tau s}}{(s-a)(s+c_1)...(s+c_n)(s+\phi) + e^{-\tau s}} \tag{12}$$

with $\phi = k - b$, note that ϕ is a free parameter function of k, with $\phi > 0$ the system only has one unstable pole. Its know that $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$ is also necessary condition to stabilize the auxiliary system (12) by PI controller [9]

4.2 OBSERVER SCHEME

In most of the practical processes, some of the state variables of the system cannot be measured. In this way the authors propose an observer based on an output injection strategy, let us take into consideration the static output injection scheme shown in Fig. 3, the stability of the observer can be addressed as follows.

Lemma 5. Consider the delayed system (6), and the static output injection scheme shown in Fig. 3. There exist constant g and a PI controller $C(s)$ such that the closed-loop system is stable if and only if $\tau < \frac{1}{b} - \sum_{i=1}^{n} \frac{1}{c_i}$

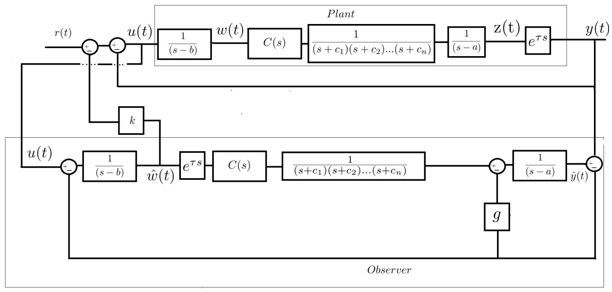

Figura 4. Control Strategy Proposed.

Proof. The proof can be easily derived from a dual procedure of the previous result.

4.3 Observer-Based Controller.

The main result of this paper is presented. The authors propose an observer based controller shown in the Fig. 4. Using the previous results the following lemma can be formulated

Lemma 6. Consider the delayed system (5) and the Predictor-Controller scheme shown in Fig. 4. There exist gains k, g and a PI controller C(s) such that the corresponding closed loop system is stable if and only if:

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$$

Proof For guarantee a correct estimation of the state variables we demonstrate that the behavior of the error signal converges asymptotically to zero if and only if the condition of Lemma 7 is satisfied

Consider a state space representation of the system (5) with a PI controller $C(s)$ characterized by:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) + Bu(t) \tag{13}$$

$$y(t) = Cx(t)$$

with $x(t) = [\omega(t)\ x_1(t)\ x_2(t)\ \cdots\ x_m(t)\ 0\ z(t)]^T$ where the states $x_i(t)$ represent the stable poles of the system, $\omega(t)$ and $z(t)$ represent the unstable part of the open loop system. So we have:

$$A_0 = \begin{bmatrix} a & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -c_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -c_2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -c_n & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & b \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 & \frac{k_i}{k_p}+1 \end{bmatrix} \quad (14)$$

The state space representation characterized by (14) can be returned to its transfer function representation by mean of:

$$\frac{Y(s)}{U(s)} = C(sI - (A_0 + A_1 e^{-\tau s}))^{-1} B \quad (15)$$

with this step we return to the delayed transfer function. The dynamics of the predictor shown in Fig 4 can be described as follows:

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + A_1\hat{x}(t-\tau) + Bu(t) - G(C\hat{x}(t) - y(t)) \quad (16)$$

Where $\hat{x}(t)$ is the estimated state of $x(t)$ and the proportional gains vectors are defined by $K = [k\ 0\ 0\ \cdots\ 0\ ]$ and $G = [1\ 0\ 0\ \cdots\ g\ ]^T$. We define the predictor error as $e(t) = x(t) - \hat{x}(t)$

The error dynamics can be written as:

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) = (A_0 - GC)e(t) + A_1 e(t-\tau) \quad (17)$$

and the controlled system:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau) - BKC\hat{x}(t) \quad (18)$$

We can write the closed loop system with the observer and the controller proposed in the Fig 4 as:

$$\dot{x}_e(t) = \begin{bmatrix} A_0 - BKC & BKC \\ 0 & A_0 - GC \end{bmatrix} x_e(t) + \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} x_e(t-\tau)$$

$$y(t) = [C\ 0]\, x_e(t) \quad (19)$$

The observer based controller proposed satisfies the separation principle. Then, the stability of the observer scheme is enough to assure an adequate error convergence, that is there exist proportional gain g , such that $\lim_{t\to\infty} [\hat{\omega}(t) - \omega(t)] = 0$ if and only if

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$$

Reminding the stability conditions stated previously in Lemma 5 and Lemma 6, is clear that the controller stability condition is more restrictive than the observer stability condition that is to say:

$$\frac{1}{b} - \sum_{i=1}^{n} \frac{1}{c_i} < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$$

Therefore, there exist a proportional gains k, and g such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$$

4.4 PI controller

The control scheme is complemented with a PI controller in order to obtain reference tracking and disturbance rejections. One of the proposed innovations in the control scheme is to add a PI controller in the observer-scheme, this addition provides a more robust system. In order to obtain a correct estimation of the system states, the gains of PI for the controller scheme, must be equal than the gains of PI for the observer scheme. For high-order time delay systems with one unstable pole and n stable poles, a PI controller design is proposed in [12].

in order to guarantee that the same PI controller stabilize the observer and controller scheme, the next methodology is proposed:

The values of the gains g and k should be selected using the following equation

$$-a + g = -b + k > 0 \quad (20)$$

If the condition of Lemma 7 and equation (20) are satisfied, we can ensure the existence of PI controller $C(s)$ such that stabilize the observer and controller scheme.

Remark 1. . It is important to note that although from the separation principle, the PI for the observer and controller scheme can be designed separately, the observer scheme stability range for the gains k_i and k_p are the more restrictive. This property allows using the gains of the observer as the tuning parameters for both schemes.

This is, from the angle condition, is easy to proof that $a > b$, $arctan(\frac{\omega}{a}) < arctan(\frac{\omega}{b})$. Therefore, it is easy to note that all stabilizing k_i values of the controller scheme are included in the stabilizing parameters of the predictor scheme, such that, if

$$arctan(\frac{\omega}{a}) > \omega\tau - arctan(\frac{k_i}{\omega}) - \sum_{i=1}^{n} \frac{\omega}{c_i} \quad (21)$$

$$max(\angle Q(j\omega))\ |_{\omega>0} > -\pi \quad (22)$$

For $\omega > 0$

4.5 Systems with one unstable pole.

Using the results shown in the Lemma 5, Lemma 6 and Lemma 7, it is easy to deduce the stability conditions of a system with only one unstable pole and n stable poles using the control scheme proposed in this paper.

Controller Scheme.

Corollary 7. Consider the delayed system (4) and the control scheme shown in Fig 2.There is a constant k and a PI controller $C(s)$ such that the close-loop system is stable for any delay value

Proof. Consider the delayed system (4) and the state feedback controller shown in Fig. 4, with constant gain k such that the process is stable. The closed loop transfer function of the system can be written as follows:

$$\frac{Y(s)}{R(s)} = \frac{e^{-\tau s}}{(s+\phi)(s+c_1)...(s+c_n)+e^{-\tau s}} \tag{23}$$

with $\phi = k - a$, note that ϕ is a free parameter function of k, with $\phi > 0$ the system only has stable poles. We can conclude from Lemma 4, there exist k and P controller such that the plant behavior is stable if and only if $|U(j\omega)| > |Y(j\omega)|, \forall\omega$. The PI case follows from Lemma 3

Remark 2. The system (4) does not have zero dynamics,thus, the condition $|U(j\omega)| > |Y(j\omega)|, \forall\omega$ always true

Observer Scheme.

Corollary 8. Consider the delayed system (4), and the static output injection scheme shown in Fig 3. There exist constant gains g and a PI controller $C(s)$ such that the closed- loop system is stable if and only if $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$

Proof. The proof can be easily derived from a dual procedure of the previous result

Observer-Based Controller.

Corollary 9. Consider the delayed system (4) and the Predictor-Controller scheme shown in Fig 4. There exist gains k, g and a PI controller $C(s)$ such that the corresponding closed loop system is stable if and only if: $\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$

Proof. Reminding the stability conditions stated previously in Corollary 8 and Corollary 9, is clear that the observer stability condition is more restrictive than the controller stability condition.

Therefore, there exist a proportional gains k,g such that the closed-loop system is stable if and only if

$$\tau < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i}$$

5. EXAMPLES

The performance of the control strategy proposed is evaluated through comparative examples taken from the literature.

Example 1. The next example is taken from [9]. Consider the second-order delayed system with two unstable poles characterized with the following transfer function:

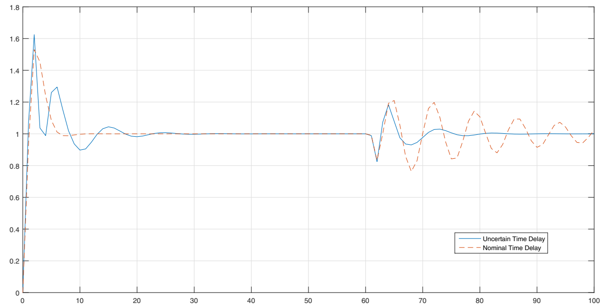

Figura 5. Closed-loop behavior of the example 1 with a delay uncertainty of 50 percent

$$G(s) = \frac{1}{(s-1)(s-0.3)} e^{-(0.3\pm\alpha)s} \tag{24}$$

Where α is a delay uncertainty. From Lemma 7 we can conclude that the range of delays under which the system described in (24) can be stabilized by static output feedback are $0 < \tau < 1$. It is clear that the stability condition given in Lemma 7 is satisfied , therefore there exists an observer based structure with proportional gains k, and g such that the resulting closed-loop system is stable due to:

$$\tau = 0.3 < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i} = 1$$

the selected gains for the example are shown in Table 1

g	40.3
k	41
k_p	50
k_i	18

Cuadro 1. Observer Based Controller Gains: Example 1.

Dynamic behavior of the Closed-loop system is show in Fig. 5 with a delay uncertainty of 50 percent, $\alpha = -0.15$, then $\tau = 0.15$ in the observer scheme. A step disturbance with magnitude $d = 0.01$ is applied at $t = 60$.

The performances of the proposed strategy controller give a satisfactory simulation results.

Example 2. Let us consider the fourth-order delayed process with one unstable poles, three stable poles given by the transfer function:

$$G(s) = \frac{1}{(s-1\pm\delta)(s+1.5)(s+2)(s+2.5)} e^{-0.4s} \tag{25}$$

With δ is an uncertainty in the poles of the system It is clear that the stability condition given in Corollary 10 is satisfied , therefore there exists an observer based structure with proportional gains k and g such that the resulting closed-loop system is stable due to:

$$\tau = 0.4 < \frac{1}{a} - \sum_{i=1}^{n} \frac{1}{c_i} = 1$$

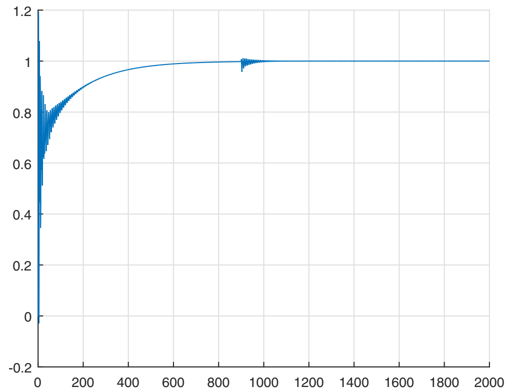

Figura 6. Closed-loop behavior of the example 2 with an uncertainty of 8 percent in the poles

the selected gains for the example 2 are shown in Table 2

g	81
k	32
k_p	500
k_i	4

Cuadro 2. Observer Based Controller Gains: Example 2.

Fig. 6 illustrates the simulated performance of the observer-based controller for a unit step reference with an uncertainty of 8 percent in the poles of the system $\delta = -0.08$. A step negative disturbance is presented in $t = 900$

6. CONCLUSIONS

This paper present necessary and sufficient conditions that ensure the existence of an observer based controller in order to stabilize and control high order systems with one or two unstable poles, n stable poles and time-delay. The control scheme is complemented with a PI controller to solve the problem of regulation and step disturbance rejection. Academic examples and numerical simulations are provided to show the effectiveness of the proposed strategy controller

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