Web-Based Fractional PID Controller Design: www.PIDlab.com

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Abstract: The purpose of this paper is to introduce an advanced virtual tool for fractional PID (FPID) controller design. It implements generic Nyquist plot shaping and/or sensitivity functions shaping capabilities. In this context, one can define e.g. gain and phase margins, sensitivity functions limits or loop bandwidth. The procedure relies on generalized robustness regions method for fractional PID controllers. The technique is best applicable namely for any non-oscillatory or slightly oscillatory linear system even with dead time, both integer and fractional order. The robustness regions can be computed and painted for more system models hence the robust controller design can be done. Here the method is validated on three illustrative examples. The author believes, that the virtual lab will be worthwhile for both researchers and industrial practitioners and will help to boost the employment of fractional order PID controllers.

Keywords: Fractional PID controller, fractional systems, Nyquist plot shaping, loop bandwidth, robustness regions, virtual lab, PIDlab, robust controller design

1. INTRODUCTION

PID controllers are still the most widely utilized "ants" in industrial practice, namely at lower (field) layers of complex control systems, both in process control and robotics Åström and Hägglund (2006). The popularity grew due to the simplicity of control law and necessity to tune only few parameters with clear physical interpretation. Despite this fact, there is no globally accepted, fully reliable method for automatic parameter tuning, either for known or unknown process model. On the contrary, the existing tuning rules are often simple and are based just on few characteristic numbers gained from step of relay test (Liu et al., 2013) hence acceptable for industrial practitioners (Leva, 2001) as elaborated further.

Recently, various extensions of classical PID algorithm have been described. Fractional-order PID controller (FPID) is one of suitable candidates to replace classical PID in cases where the design requirements cannot be fulfilled by PID (Čech and Schlegel, 2013). It has only two additional parameters – order of integration and derivation – with clear physical interpretation preserved (Podlubny, 1999). Moreover, the fragility of controller parameters is satisfactory (Padula and Visioli, 2017) compared to other higher-order controllers [1].

Over the years, a lot PID tuning procedures and simple rules have been developed (Ho et al., 1995, 1996; C.C.Hang et al., 2002; Liu et al., 2013; Padula and Visioli, 2011; Kurokawa et al., 2017). However, only very few tuning procedures are suitable for general linear process model, even non-minimum phase and/or with time delay. One of them, based on D-partition is described in Shafiei and Shenton (1997); Neimark (1948). It is often denoted as *robustness regions* principle and can deal effectively also with model uncertainties, as shown e.g. in Yuan-Jay et al. (2011). The problem is much more complex in case of FPID controller, where usually analytic approaches for a set of design requirements are provided for very simple process models (Luo and Chen, 2009; Hamamci, 2007). However, there are still attempts of graphical FPID tuning based on generalized robustness regions (Wang, 2011; Wang et al., 2017). Unfortunately, none of methods is provided in a compact tool or SW package for routine usage even in well accepted packages for fractional control like CRONE Toolbox, FOMCON [2] or Ninteger (Valerio and S Da Costa (2004)). The most recent interactive tools deal usually with open loop shaping via manipulating poles and zeros (Daz et al., 2017). One exception allowing graphical FPID design is described in Dormido et al. (2012). However, up to the authors knowledge, there is no method dealing with FPID controller with filtered derivative part described in such a general way like in this work. Those are key drivers for development of virtual tool described below.

Earlier, a powerful web tool was developed which allows to use robustness regions method for PID tuning, freely via simple graphical interface with interacting windows (Schlegel and Čech, 2004). This paper presents its substantial re-design and extension, namely the implementation of generalized robustness regions method for FPID controllers (Čech and Schlegel, 2013) and related GUI.

The three illustrative examples show how one can find FPID parameters for different sets of frequency domain requirements rising from various practical control design aspects. Consequently, it is believed that such extended virtual tool will help to spread the applicability of fractional

[1] Remind that FPID controller is always implemented as high order filter approximating ideal FPID on certain frequency band.

[2] http://cronetoolbox.ims-bordeaux.fr/, http://fomcon.net/

PID controller thanks to seamless extension of classical PID. Also point out, that systematic repeating utilization of robustness regions method for certain class of processes may result into simple analytic PID tuning rules (Čech and Schlegel, 2012; Čech and Schlegel, 2011) deployable into compact controllers (Severa and Čech, 2012).

The rest of the paper is organized as follows: In Section 2, the problem formulation is given. Section 3 describes briefly the graphical user interface for FPID design. Illustrative examples are given in Section 4. Conclusions and ideas for future work are summarized in Section 5.

2. PROBLEM FORMULATION

The virtual tool described further serves to solve broad range of control problems, but still a generic design task directly solvable using the presented tool can be formulated: Consider an arbitrary linear, time-invariant SISO system (integer or fractional order) with a known transfer function $P(s)$. Further, consider $P(s)$ in a feedback loop with FPI and FPID controller in the forms [3]

$$C(s) = K + \frac{K_I}{s^\alpha}, \tag{1}$$

$$C(s) = K\left(1 + \frac{1}{T_I s^\alpha} + \frac{T_D s^\beta}{\frac{T_D}{N}s + 1}\right), \tag{2}$$

where $\alpha, \beta \in \mathbf{R}^+$ are orders of integration and derivation, respectively. Consequently, let us define an open loop transfer function $L(s) = C(s)P(s)$.

Assumption 1. The derivative filter parameter N is fixed during the design procedure according to the noise level in process variable. The typical value is $N \in (2, 20)$.

Assumption 2. For time domain implementation of FPID controller, consider a restricted range $\alpha, \beta \in (0, 2)$ which is, apparently wide enough for most of considered applications.

Remark 3. The transfer function (2) could be seen for $\beta > 1$ as not-proper as the order of numerator exceeds the denominator order. However, the term $s^{(\beta-1)}$ is always implemented as a higher-order proper transfer function linked in series to a standard PID derivative term.

Definition 1. Further, consider $\mathcal{X}$ being a subset of following design specifications:

- Gain margin (GM), phase margin (PM)
- General shaping point $X = u + \mathrm{j}v$
- Sensitivity function ($\mathrm{SF} = 1/(1 + L(s))$ upper limit M_S
- Complementary sensitivity function ($\mathrm{CSF} = L(s)/(1+L(s))$ upper limit M_T
- Low frequency disturbance damping $[\epsilon_S, \omega_S]$
- Loop bandwidth $[\epsilon_T, \omega_T]$

Remark 4. All of these requirements can be in case of stable open loop viewed as shaping conditions for a Nyquist plot $L(\mathrm{j}\omega) = C(\mathrm{j}\omega)P(\mathrm{j}\omega)$ or SF/CSF limits, see Fig. 1 and 2, respectively.

2.1 General shaping point X and FPI controller

Firstly, let us clarify the idea of computing robust stability regions for general shaping point $X = u + \mathrm{j}v$ in the Nyquist plot complex plain. Our aim is to find all possible pairs of parameters K, K_I of the FPI controller (1) for which the point X lies on the left side of the Nyquist curve. For this purpose the equation

$$L(\mathrm{j}\omega) = \left(K + \frac{K_I}{(\mathrm{j}\omega)^\alpha}\right)(a(\omega) + \mathrm{j}b(\omega)) = u + \mathrm{j}v \tag{3}$$

where $a(\omega) = \mathrm{Re}\{P(\mathrm{j}\omega)\}$ and $b(\omega) = \mathrm{Im}\{P(\mathrm{j}\omega)\}$ must be solved for unknown K, K_I and fixed α. The solutions (described in Čech and Schlegel (2013)) is for $\omega \in (0, \infty)$ the parametric curve in $K - K_I$ plane which together with K and K_I axis [4] splits the plane into several regions. Any points inside certain region lead to similar relative location of the Nyquist plot $L(\mathrm{j}\omega)$ and a shaping point X (i.e. left/right side).

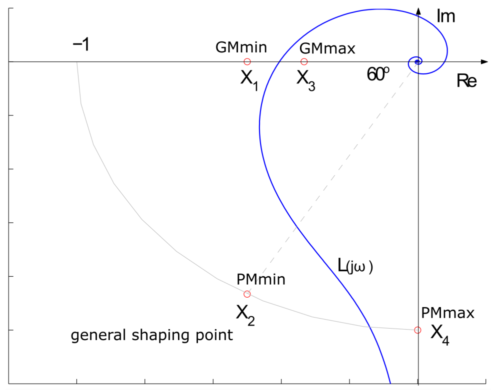

Fig. 1. General shaping points defining e.g. GM and PM bounds of Nyquist plot $L(\mathrm{j}\omega) = C(\mathrm{j}\omega)P(\mathrm{j}\omega)$ in the virtual tool

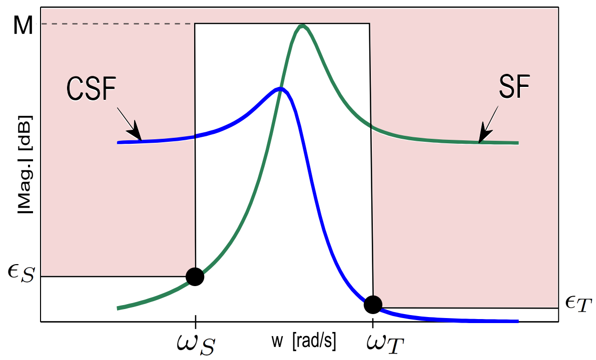

Fig. 2. Design specifications $\mathcal{X}$ supported in the virtual tool: Sensitivity (SF) and complementary sensitivity (CSF) function limits.

2.2 Generalized robustness regions for FPID controller

Although the procedure is much more complex, the robustness regions can be computed for FPID controller in

[3] There are various forms of FPID controller, the form (2) was selected as a seamless extension of standardized ISA PID form acceptable in industry

[4] Only the positive values resulting into stable controller are considered.

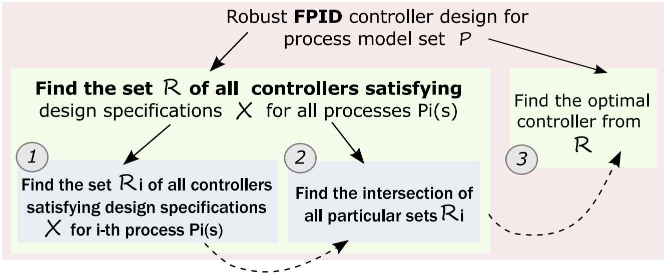

Fig. 3. Robust FPID controller design for a model set $\mathcal{P} = \{P_i(s), i = 1, 2, \ldots n\}$ using virtual laboratory

the form (2) for any design specification $\mathcal{X}$ according to Definition 1.

When a certain set of processes $\mathcal{P} = \{P_i(s), i = 1, 2, \ldots n\}$ enters a design procedure one can apparently speak about robust controller design [5]. Robust design procedure relies on computing intersection of all suitable regions for all processes from $\mathcal{P}$ as summarized in Fig. 3.

Remark 5. Steps 1 and 2 in the Figure aim to find set of all controllers satisfying frequency domain requirements. Consequently, in step 3, time domain optimization on the set is obviously done. For example, choosing controller with maximum K_I means minimization of criterion $J = \int_0^\infty e(t)\mathrm{d}t$.

3. GUI DESCRIPTION

The virtual lab starts with initial 4-window layout shown in Fig. 4. It allows to follow the design procedure described in Section 2 by passing through following tabs and windows:

3.1 Process tab

Firstly, define process(es) here in various forms of transfer function (polynomial coeffs, zero/poles, time constants), even with time-delay. For every process added, one can check its frequency response in complex plane in the bottom window.

3.2 Controller tab

After switching to controller tab, one can chose its form (PI, PID, FPID, ...). In case of FPID controller, the "FPID plane" window can be activated after clicking on related icon. In this window the controller 'order cross' can be dragged continuously. In such a way the order of integration α and derivation β can be changed interactively affecting directly all other windows and figures.

Design specifications The general Nyquist plot shaping point X_i can be added by clicking directly in Nyquist plot frame (bottom-left). SF and CSF shaping 'points' $[\epsilon_S, \omega_S]$, $[\epsilon_T, \omega_T]$ can be added in similar way by clicking into proper tab in control loop performance frame (bottom-right).

3.3 Nyquist plot window

In this window one can check fulfillment of all design specifications for all processes in the set. The defined shaping points can be changed by mouse dragging, in the same way, one can modify M-circles interactively in order to define upper limit of SF and CSF.

3.4 Robustness regions window

In upper-right window one can see all robustness regions for all processes and all design specifications. It is the base for initial selection of controller parameters in $K - K_I$ plane in the appropriate intersection of all regions.

3.5 Loop performance window

In the bottom-right window on can check the closed loop performance in time domain including evaluation of various critarions (ISE, ITAE) and evaluate a closed loop robustness through four well known sensitivity functions (see 'Gang of Four' in Åström and Hägglund (2006)).

4. ILLUSTRATIVE EXAMPLES

4.1 Example 1: Simple process

Consider a nominal integer order SOPDT (second-order plus dead-time) process model

$$P(s) = \frac{1}{(0.2s + 1)(s + 1)} \mathrm{e}^{-0.05s} \tag{4}$$

and following set of design specifications:

$$\mathcal{X} : PM = 1.1, M_s = 1.2. \tag{5}$$

Further, one wants to ensure loop robustness for large gain variations, often referred as to 'iso-damping' property. It is well known that such behavior can be ensured by fractional integrator $1/s^m,\ m \in \mathbf{R}^+$ as a reference model for a Nyquist plot [6]. Consequently, three general shaping points have been defined:

$$\begin{aligned} &X_1 = (-0.25, -0.5j), X_2 = (-0.5, -1j), \\ &X_3 = (-0.75, -1.5j). \end{aligned} \tag{6}$$

It is difficult to design PID controller leading to Nyquist plot passing through points X_1, X_2, X_3. FPID controller with embedded fractional integrator has more flexible structure to follow design specifications (6). The satisfactory controller exists for order $\alpha = 1.08, \beta = 1.15$. It was found by changing controller order via mouse dragging in the interactive 'FPID plane' window (see Fig. 5). The remaining controller parameters are $K = 1.05, T_i = 0.68, T_d = 0.17, N = 6$ leading to closed loop with required performance as shown in Fig. 6 and 7.

4.2 Example 2

Consider a fractional order process model

$$P(s) = \frac{1}{(0.6s + 1)^{1.666}} \tag{7}$$

and a set of five design specifications defining sensitivity and complementary sensitivity function regions according to Fig. 2:

$$\mathcal{X} : M_S = 2, \epsilon_S = 0.079,\ w_S = 3.5,\ \epsilon_T = 0.54,\ w_T = 16. \tag{8}$$

The design specifications create a robustness / performance trade-off (Kurokawa et al., 2017) and cannot be

[5] $\mathcal{P}$ could contain e.g. four vertex processes resulting from the Kharitonov's theorem.

[6] Point out, that such open loop provides infinite gain margin and a phase margin is determined by m.

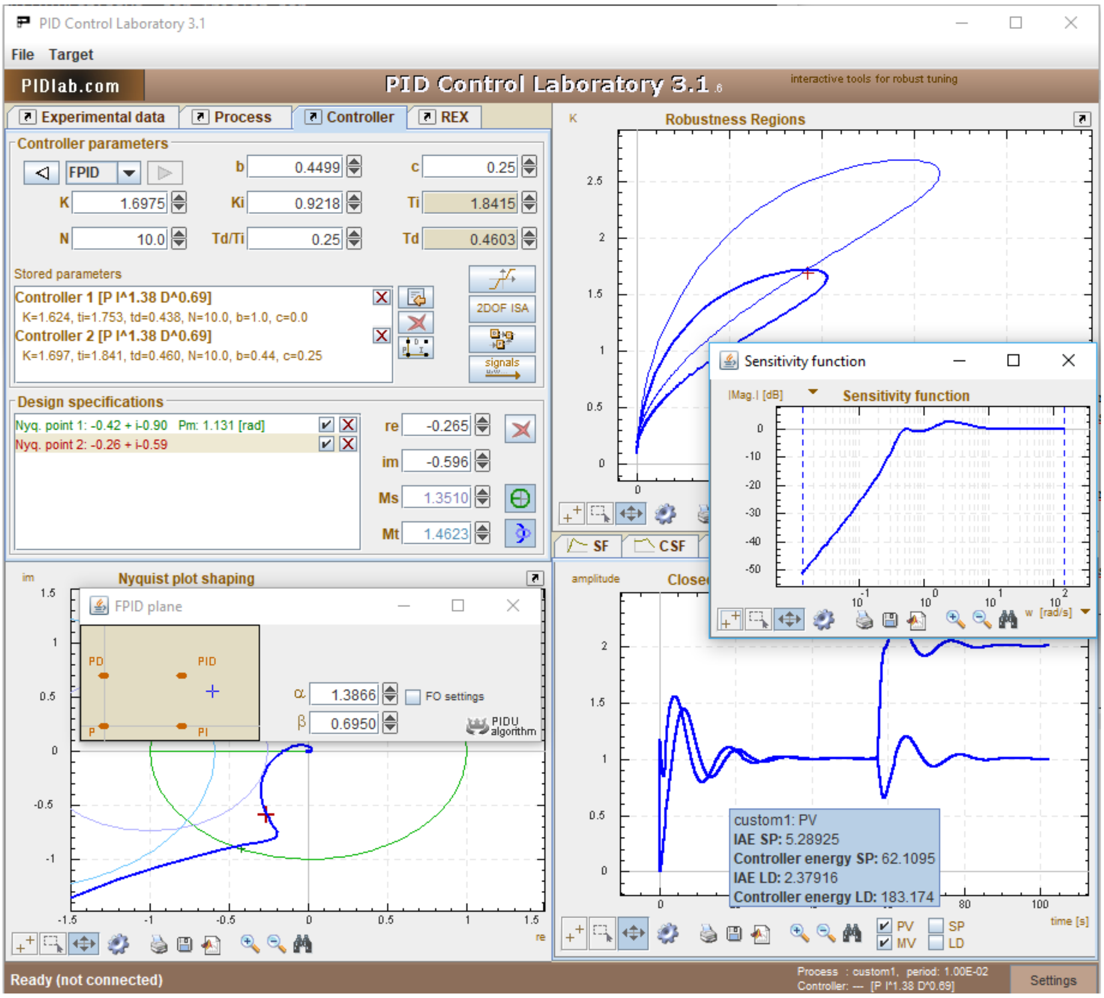

Fig. 4. General overview of virtual laboratory GUI – Initial 4-window layout with mutually connected plots

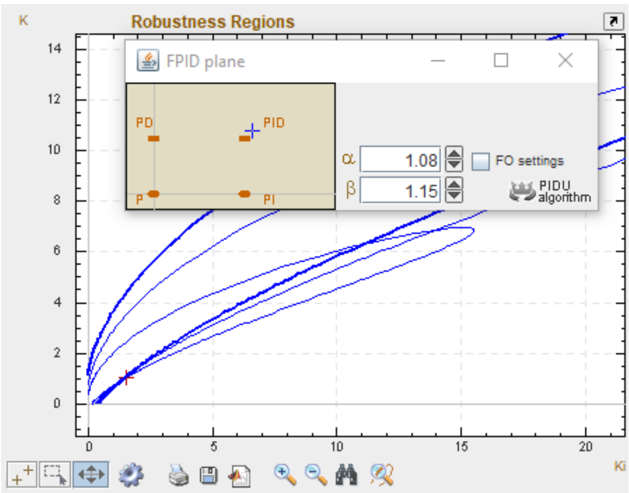

Fig. 5. Example 1: Robustness regions in the interactive GUI, FPID plane allowing smooth change of FPID orders α and β

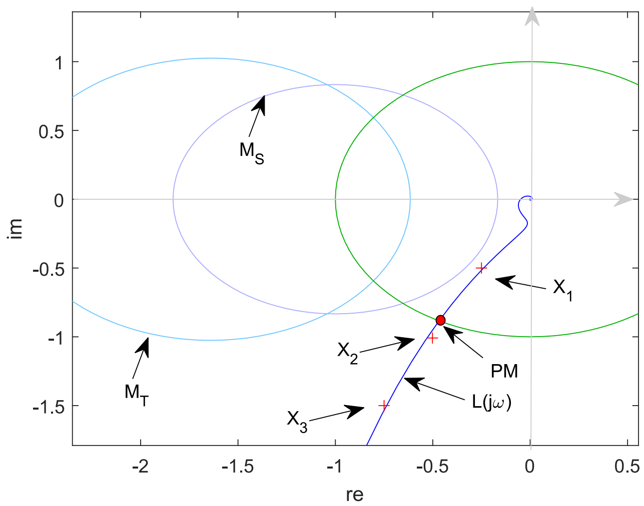

Fig. 6. Example 1: Nyquist plot and general shaping points for loop gain robustness

fulfilled by traditional PID controller even after changing N and T_i/T_d ratio. Using interactive PID plane one can quickly choose $\alpha = 1.05$, $\beta = 1.3$ (see Fig. 8). The remaining controller parameters satisfying design requirements are: $K = 30.0, T_i = 0.1476, T_d = 0.0369, N = 10$.

4.3 Example 3

To show the power of robustness regions, consider a more complex transfer function $P(s)$ defined by (9) which describes a steam turbine model summarized in Reitinger

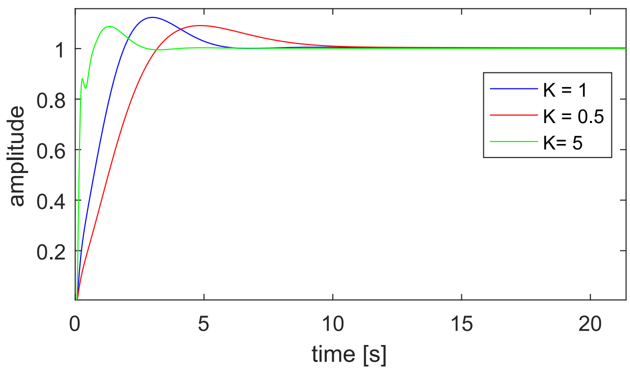

Fig. 7. Example 1: Closed loop step responses showing robustness to large gain variations

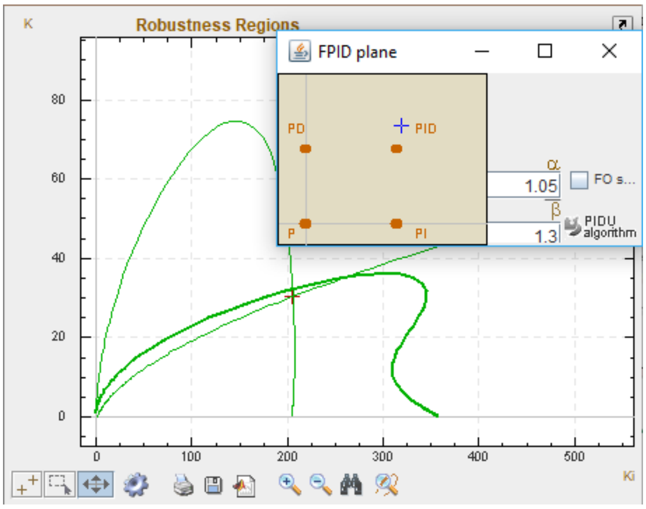

Fig. 8. Example 2: Robustness regions in the GUI

et al. (2017) linearized in a appropriate working point in island mode. Further, the design specifications are as follows:

$$\mathcal{X} : PM = 60^o, \ \ M = 1.3, \epsilon_S = 0.2, \ \ w_S = 0.15. \qquad (10)$$

As in previous cases, the specifications cannot be met by classical PID. Choosing $\alpha = 1.19$, $\beta = 1.31$ leads to satisfactory controller with parameters $K = 1.76e-3, T_i = 9.36, T_d = 2.34, N = 10$, with robust performance documented in Fig. 9 – 12.

5. CONCLUSIONS

The interactive virtual tool for fractional PID controller design was presented. The design procedure implemented is based on general Nyquist plot shaping method and generalized robustness regions. The method effectiveness is demonstrated on three examples: first one showing FPID controller tuning for process with large gain variations; second one showing fast FPID controller tuning for a set of frequency domain requirements creating serious robustness/performance trade-off and fractional order process; third one showing the power robustness regions method to deal with any transfer function of arbitrary order – demonstrated on steam turbine linearized model. It is believed that the presented virtual tool can help to spread FPID controller applicability in wide range of applications.

The future work will be focused on connecting the virtual tool with real process data and implement computation of FPID control loop performance assessment indices. In terms of HMI quality the automatic computation of region intersection is assumed. Also, computation of 3D areas in $K - K_I - K_D$ space will be provided.

ACKNOWLEDGEMENTS

This work was supported by the project LO1506 of the Czech Ministry of Education, Youth and Sports under the program NPU I. The support is gratefully acknowledged.

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$$P(s) = \frac{380s^4 + 1963s^3 + 3831s^2 + 4639s + 1583}{s^7 + 9.213s^6 + 31.97s^5 + 53.11s^4 + 45.25s^3 + 20.18s^2 + 4.431s + 0.3692} \tag{9}$$

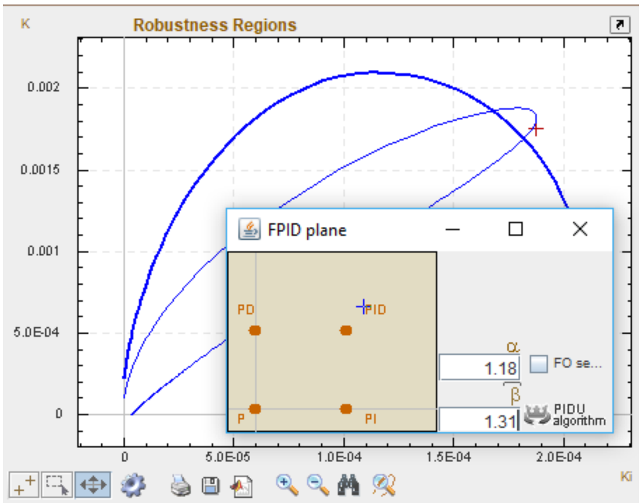

Fig. 9. Example 3: Robustness regions in the GUI

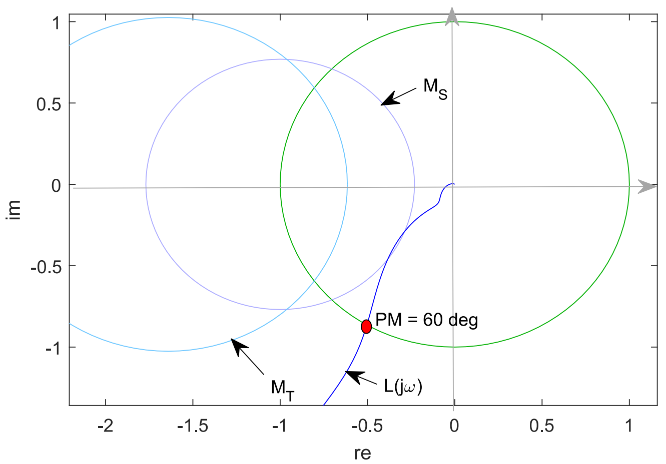

Fig. 10. Example 3: Nyquist plot – open loop with a designed FPID controller

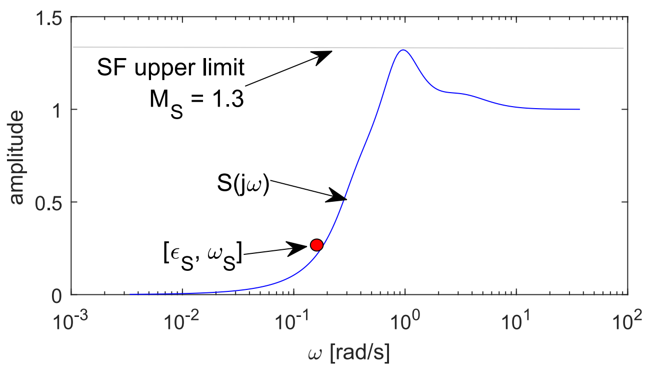

Fig. 11. Example 3: Fulfillment of sensitivity function requirements: upper limits M_S, low frequency disturbance damping (ω_S, ϵ_S)

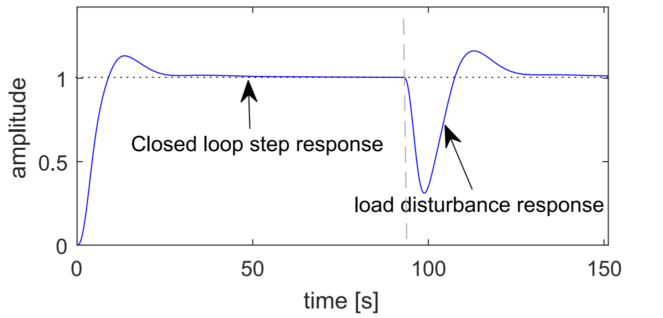

Fig. 12. Example 3: Closed loop simulation with designed FPID controller: step and load disturbance responses

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