Loop Shaping for PID Controller Design Based on Time and Frequency Specifications ⋆

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Abstract: In this paper it is described a new method to design PID controllers using a linear programming approach for optimizing performance subject to robustness constraints. It is based on the shaping of a reference loop gain transfer function which forms a convex region on the Nyquist diagram which contains and bounds the designed loop gain Nyquist plot. The convex region is approximated by a set of lines in order to formulate a linear optimization problem. It is also presented an optional performance specification related to the crossover frequency of the designed loop gain. The class of stable linear time-invariant single-input simple-output (SISO) systems is considered and the optimization problem is proposed, solved and analyzed.

Keywords: Multiobjective optimization, Optimization problems, Parameter optimization, Linear programming, PID control.

1. INTRODUCTION

Applied in more than 90% of the control loops, the most common solution to practical control problems is the PID controller. This type of controller is not only embedded in special-purpose control systems, like cruise control in cars or DVD players, but also found in large number in all industries Åström and Hägglund (2006), Jelali (2012).

In order to design a controller, it is necessary to obtain a model for the system. As frequency domain models do not have the information condensed into a small set of parameters, unmodeled dynamics errors are avoided Karimi and Galdos (2010) and quantitative information about the plant and the measurement quality are captured. This along with the ease of obtainment are reasons for using this type of models Pintelon and Schoukens (2012).

There are several methods to design controllers, being the formulation as a constrained optimization problem an approach that can capture many factors related to performance and robustness Hast et al. (2013). A linear programming approach based on the shaping of the open loop transfer function in the Nyquist diagram is proposed to tune PID controllers for stable LTI plants in Karimi et al. (2007) and Saeki (2014). Both present optimization problems to maximize load disturbance rejection, being the constraints based on equations of lines and its linearity properties.

More complex optimization problems are presented in Karimi et al. (2008), Karimi and Galdos (2010), Thalmann et al. (2016) and Hast et al. (2013). The robustness constraints are based on the geometric interpretation of the maximum of the sensitivity function and the complementary sensitivity function in the Nyquist plot of the loop transfer function, considering process uncertainties. The result is a optimization problem that can be solved using convex-concave programming Hast et al. (2013) or convex programming Karimi et al. (2008), Karimi and Galdos (2010) and Thalmann et al. (2016).

There are also PID tuning methods by frequency loop shaping that minimize norms of the difference between a desired loop gain transfer function and the designed loop gain transfer function, possibly weighted by another transfer function. Some of this methods are presented in Grassi and Tsakalis (2000) and Grassi et al. (2001). The stability margins such as gain margin, phase margin, maximum of the sensibility function or of the complementary sensitivity function are not guaranteed, unlike the method here proposed.

In this paper, it is proposed a new loop shaping method in which a reference loop gain transfer function is chosen

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to represent performance and robustness specifications. The loop gain transfer function is finely bounded by straight lines which form a convex region. This approach generates constraints that are linear with respect to the PID controller gains, making possible to solve the problem using linear programming.

The major contribution of the proposed method is the formulation of the constraints, based on Karimi et al. (2007), with the addition of constraints that not only specifies the stability margins but also the approximated time behavior of the closed loop system.

The optimization problem could also be formulated as a convex optimization problem and solved using convex programming algorithms, but the authors preferred the form presented in order to be solvable by simple software tools.

This paper is organized as follows: in Section II the plant, controller and design specifications. The constraints and the formulation of the optimization problem is presented in Section III. Simulations results are given in Section IV. Finally, Section V presents some conclusions.

2. PROBLEM FORMULATION

2.1 Plant Model

Consider the class of stable linear time-invariant SISO systems represented by transfer function $G(s)$. Its frequency response at a finite number N of points $G(j\omega_k)$, $k = 1, ..., N$, is assumed to be known and relevant points of the frequency response of the plant are available.

2.2 PID Controller Form

The class of PID controllers considered is

$$K(s) = K_p + K_i \frac{1}{s} + K_d \frac{s}{1 + T_f s}, \quad (1)$$

with T_f (supposed to be known) as the time constant of the noise filter. The following linearly parameterized form of the controller will be considered

$$K(s) = \rho^T \phi(s), \quad (2)$$

with

$$\rho^T = [K_p, K_i, K_d], \quad (3)$$

$$\phi(s) = [1, \frac{1}{s}, \frac{s}{1 + T_f s}]. \quad (4)$$

Define the designed loop transfer function $L(s) = K(s)G(s)$. Its frequency response can be represented as

$$L(j\omega) = K(j\omega)G(j\omega) = \rho^T \phi(j\omega)G(j\omega) \quad (5)$$

$$= \rho^T \Re(\omega) + j\rho^T \Im(\omega), \quad (6)$$

where $\Re(\omega)$ and $\Im(\omega)$ are, respectively, the real and the imaginary parts of $\phi(j\omega)G(j\omega)$. Define also the associated sensitivity function $S = 1/(1+L)$ and the complementary sensitivity function $T = L/(1+L)$ here also named closed loop transfer function.

2.3 Performance Criteria

The integrated error and the integrated absolute error, defined respectively by

$$IE = \int_0^\infty e(t)dt, \quad IAE = \int_0^\infty \mid e(t) \mid dt, \quad (7)$$

are good measures of load disturbance rejection for controllers with integral action and, consequently, are commonly used performance indices Hast et al. (2013). For well-damped systems:

$$IAE = \int_0^\infty \mid e(t) \mid dt \approx \mid IE \mid . \quad (8)$$

It can be shown Åström and Hägglund (2006) that a unit step disturbance applied at the plant input of a system with an integral acting controller results in

$$IE = \frac{1}{K_i}. \quad (9)$$

Thus, maximizing K_i corresponds to minimizing IE and, therefore, optimizing the closed-loop performance in terms of the load disturbance rejection.

2.4 Design and Robustness Specification

The design and robustness specification used for the method proposed here is given in the format of a curve in the complex plane that form a convex region which will contain the Nyquist diagram of the designed loop transfer function. Observe that the curve can be used to include classical stability margins. For this article, let us choose a reference loop gain transfer functions $\bar{L}(j\omega)$ as follows.

For a **desired** closed loop transfer function specification

$$\bar{T}(s) = \frac{\omega_n^2 e^{-\theta s}}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (10)$$

use the **reference** loop gain

$$\bar{L}(s) = \frac{\omega_n^2 e^{-\theta s}}{s^2 + 2\xi\omega_n s + \omega_n^2(1 - e^{-\theta s})}, \quad (11)$$

with ω_n and ξ being, respectively, the design specification for the desired natural frequency and damping ratio and with θ denoting the time delay.

The idea is to formulate the specifications of the loop gain transfer function based on a reference loop gain with desired robustness margins and derived from a closed loop transfer function with a desired time behavior.

According to Åström and Wittenmark (2013), the poles of the closed-loop characteristic polynomial are normally chosen so that the dominating poles are of the same order of magnitude as the open loop poles. Thus, the value suggested for ω_n is a multiple of the frequency bandwidth of the plant $G(j\omega)$, ω_{Gb}. It follows from the analysis of sensitivity to modeling errors that the closed loop system will be very sensitive to parameter variations if the closed loop bandwidth is chosen much higher than the bandwidth of the open-loop system Åström and Wittenmark (2013).

There are different ways to choose the value of ξ, being the one used in this article based on the maximum value desired for the maximum of the sensitivity transfer function M_S, which is the robustness specification. The relationship between both for a transfer function in the format of 10 is Åström and Wittenmark (2013)

$$M_S = \|S\|_\infty = \sqrt{\frac{1 + 8\xi^2 + (1 + 4\xi^2)\sqrt{1 + 8\xi^2}}{1 + 8\xi^2 + (-1 + 4\xi^2)\sqrt{1 + 8\xi^2}}}. \quad (12)$$

The value $\xi = 0.5$ gives $M_S = 1.5$. Suggested values for M_S are between $1,0$ and $2,0$ Panagopoulos et al. (2002).

As the closed loop system will have at least the same time delay of the plant, the value suggested for θ is the delay of the plant.

2.5 Additional Performance Specification

In order to induce performance, it can be specified the minimum gain crossover frequency ω_{gc}, which is the lowest frequency where the loop transfer function $L(s)$ has unit magnitude. This specification is optional. For this article, the proposed minimum gain crossover frequency is the gain crossover frequency of the reference loop gain transfer functions $\bar{L}(j\omega)$, $\omega_{L_{gc}}$, used for the robustness specification.

3. CONTROLLER DESIGN

It is proposed a new method to design PID controllers using linear programming. Initially, it is presented a base restriction (constraints related to a line) that is used to formulate the set of lines for the proposed constraints. Constraints then are formulated to guarantee robustness and performance. Finally, the optimization problem used to design the controller is formulated.

3.1 The Constraints Formulation

Constraints related to a line Consider the loop gain transfer function as represented in Eq. 6; in order to guarantee that the Nyquist curve of this function lies bellow and to the right of a line with slope α and that intercepts the real axis in $-b$ (Fig. 1), the following constraint is considered

$$\rho\left(cot\alpha\ \Im(\omega_k) - \Re(\omega_k)\right) \leq b, \tag{13}$$

for all ω_k (see Karimi et al. (2007)).

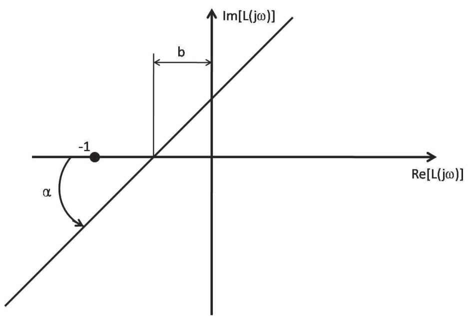

Fig. 1. Line in the Nyquist diagram with slope α and that intercepts the real axis in $-b$

Constraints related to the design and robustness specification To guarantee robustness, it is wanted to have the loop transfer function at any frequency ω inside a convex region which is determined by the robustness specification. The convex region proposed is delimited by the set of lines that is obtained by the interconnection of frequency points in the Nyquist plot of the reference loop gain transfer function (Eq. 11), for a range of frequencies ω_q logarithmically spaced. It is considered a set of lines instead of the curve itself in order to formulate the optimization problem as a linear programming one (Fig. 2).

For the formulation of the constraints for each line q, the distance of the line to the origin at the real axis, b_q, and the inclination between the line and the real axis, α_q, are obtained using two adjacent frequency points of the range of frequencies logarithmically spaced considered on the Nyquist plot.

Thus, varying ω_q from ω_0 to ω_{N_q}, being the range divided in (N_q+1) points logarithmically spaced, one obtains

$$a_q = \frac{Im(\bar{L}(j\omega_q)) - Im(\bar{L}(j\omega_{q-1}))}{Re(\bar{L}(j\omega_q)) - Re(\bar{L}(j\omega_{q-1}))} \tag{14}$$

$$\alpha_q = \arctan(a_q) \tag{15}$$

$$b_q = Im(\bar{L}(j\omega_q)) - a_q Re(\bar{L}(j\omega_q)) \tag{16}$$

The constraints are then given by

$$\rho\left(cot\alpha_q\ \Im(\omega_k) - \Re(\omega_k)\right) \leq b_q, \tag{17}$$

for q varying from 1 to N_q and for all ω_k considered.

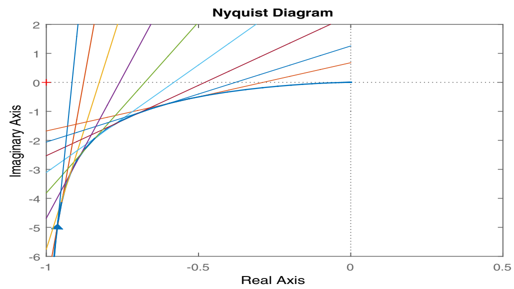

Fig. 2. Graphical illustration of the constraints related to the robustness specification

Constraints related to the additional performance specification To guarantee the performance related to the minimum gain crossover frequency (optional), a set of lines which form an approximation of a circle with radius 1 and center at the origin of the Nyquist diagram is used as constraint for frequencies grater than this minimum specified frequency. The procedure is analogous to the explained for the robustness constraint formulation.

It is considered a set of lines instead of the curve itself in order to formulate the optimization problem as linear.

Thus, varying x_l from $x_0 = -1$ to $x_{N_l} = 0$, being the range divided in (N_l+1) points uniformly spaced (specification only for the third quadrant), one obtains

$$a_l = -\frac{\sqrt{1-x_l^2} - \sqrt{1-x_{l-1}^2}}{x_l - x_{l-1}} \tag{18}$$

$$\alpha_l = \arctan(a_l) \tag{19}$$

$$b_l = -(\sqrt{1-x_l^2} + a_l(-1+x_l))/a_l \tag{20}$$

The constraints are then given by

$$\rho\left(cot\alpha_l\ \Im(\omega_k) - \Re(\omega_k)\right) \leq b_l, \tag{21}$$

for l varying from 1 to N_l and for $\omega_k \geq \omega_{L_{gc}}$.

Additionally, varying x_l from $x_{N_l} = 0$ to $x_{2N_l} = 1$, being the range divided in $(N_l + 1)$ points uniformly spaced (specification only for the fourth quadrant), one obtains

$$a_l = -\frac{\sqrt{1-x_l^2} - \sqrt{1-x_{l-1}^2}}{x_l - x_{l-1}} \tag{22}$$

$$\alpha_l = \arctan(a_l) \tag{23}$$

$$b_l = -(\sqrt{1-x_l^2} + a_l(-1+x_l))/a_l \tag{24}$$

The constraints are then given by

$$\rho\,(-cot\alpha_l\,\Im(\omega_k) + \Re(\omega_k)) \leq b_l, \tag{25}$$

for l varying from $N_l + 1$ to $2N_l$ and for $\omega_k \geq \omega_{L_{gc}}$.

Fig. 3 shows the set of lines that forms the constraints for the third and fourth quadrants, as specified above, considering $N_l = 25$. In most cases, it is not necessary to formulate the constraints for the first and second quadrants as usually for higher frequencies the magnitude of the loop gain transfer function decays and the Nyquist diagram remains inside the unit circle.

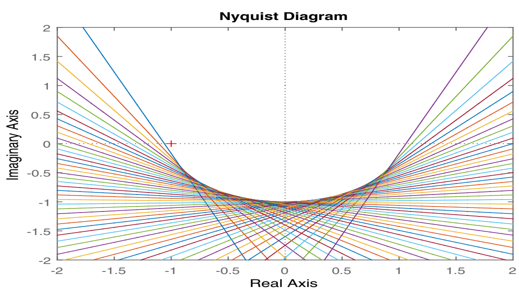

Fig. 3. Graphical illustration of the constraints related to the performance specification

Observations Robustness is obtained as at all frequencies the loop transfer function stays away from the point $(-1, 0)$ with a minimum distance defined by the convex region formed by $\bar{L}$. The maximum possible values for the classical robustness criteria gain margin and phase margin (when it exists), as well as for the sensitivity function, are also defined and are equal to the ones of $\bar{L}$.

Performance related to the gain crossover frequency is obtained in the sense that at frequencies greater than $\omega_{L_{gc}}$ the Nyquist diagram remains inside the unit circle centered at the origin.

3.2 The Optimization Problem

The control objective is to minimize the performance index IE under the constraints in Eq. 17 and possibly in Eq. 25. The proposed optimization problem to obtain the controller's parameters is

$$\begin{aligned} \underset{\rho}{\text{maximize}} \quad & K_i \\ \text{subject to} \quad & \rho\,(cot\alpha_q\,\Im(\omega_k) - \Re(\omega_k)) \leq b_q \text{ for all } \omega_k, \\ & \rho\,(cot\alpha_l\,\Im(\omega_k) - \Re(\omega_k)) \leq b_l \text{ for } \omega_k \geq \omega_{L_{gc}}, \\ & \text{for all } q \text{ and } l \text{ considered.} \end{aligned}$$

The suggested frequencies used to formulate the constraints coefficients α_q and b_q, related to the robustness specification, are around two decades below the gain crossover frequency of $\bar{L}(j\omega)$ and up to a couple decades above it.

The suggested frequencies used to formulate the constraints should be around one decade below the gain crossover frequency of $\bar{G}(j\omega)$ and up to a couple decades above it. If the crossover frequency does not exists or equals zero, it is suggested to use around two decades bellow and up to a decade above the frequency bandwidth.

The values of N_q and N_l depends on the accuracy desired for the permissible region for the Nyquist plot and for the value of the minimum gain crossover frequency $\omega_{L_{gc}}$, respectively.

4. SIMULATION RESULTS

The suggested design method will be illustrated by some examples. The number of evaluation points considered of the open loop transfer function is $N = 200$, being frequencies ω_k logarithmically spaced between a range specified for each case. After defining a reference loop gain transfer function, which contains the wanted behavior, the constraints coefficients for robustness and optionally for performance specifications are obtained and the optimization problem to maximize K_i subject to the constraints is formulated. It is then solved using linear programming and the solution, ρ, contains the PID controller gains.

Example 1. Third order system

Consider

$$G(s) = \frac{1}{(s+1)^3}. \tag{26}$$

The frequency bandwidth of the plant $G(j\omega)$ is $\omega_{Gb} = 0.5099$ rad/s its crossover frequency is 0.4009 rad/s.

The reference transfer function for this example is derived from the closed loop transfer function with natural frequency ω_n equal to ω_{Gb}, which is 0.5099 rad/s, and damping ratio of 0.5 (in order to guarantee a maximum of the peak of the sensitivity function of 1.5). Thus, the reference function is given by

$$\bar{L}_1(s) = \frac{0.26}{s(s+0.05099)}. \tag{27}$$

Its frequency bandwidth is $\omega_{Lb} = 0.5099$ rad/s and its crossover frequency is $\omega_{Lgc} = 0.4009$ rad/s.

The frequency range used to obtain the coefficients for the robustness constrains is ω_q varying from 0.004 to 0.5099 (from approximately two decades below the crossover frequency of $\bar{L}_1(s)$ and up to its frequency bandwidth ω_{Lb}), with $N_q = 50$ points of frequency logarithmically spaced. The frequency range considered is from 0.01 to 10 rad/s, as the plant's crossover frequency is 0.4009 rad/s.

It can be specified a minimum gain crossover frequency permissible of the designed loop gain transfer function by adding the following performance specification: the number of points used to obtain the coefficients for the performance constrains of $N_l = 50$ and the minimum gain crossover frequency permissible ω_{gc} equal to ω_{Lgc}, which is 0.4009 rad/s.

The proposed linear programming procedure results in the PID controller gains

$$K_p = 1.128, \quad K_i = 0.710, \quad K_d = 3.130. \tag{28}$$

The Nyquist diagrams of the reference function and the obtained loop transfer function are shown in Fig. 4. It can be observed that the imposed margins along with the maximization of K_i approximates the frequency response of the reference and obtained loop gain transfer functions, but as the performance specification imposes an approximated minimum crossover frequency, is it displaced at frequencies nearby this frequency.

The bandwidth frequency of the obtained closed loop transfer function is 0.43 rad/s. The crossover frequency of the loop gain obtained is 0.45 rad/s and the maximum of the sensitivity function is $M_S = 1.32$.

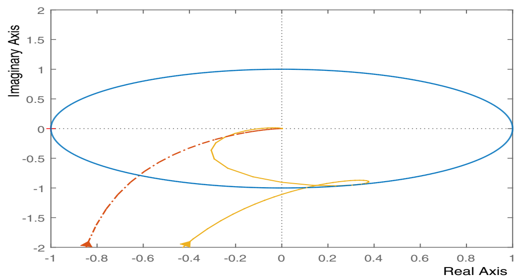

Fig. 4. Nyquist diagram of the reference transfer function $\bar{L}(j\omega)$ (orange dot-slash curve) and the designed loop gain $L(j\omega)$ (yellow solid curve) - example 1b

It can be seen in Fig. 5 that the time response is approximated to the one of the closed loop transfer function used to formulate the specifications, fact justified by the fact that obtained bandwidth frequency of the closed loop system obtained is close to the one of the desired specification.

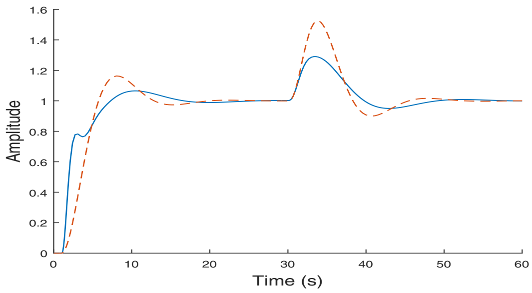

Fig. 5. Set point ($t = 0$s) and load disturbance ($t = 30$s) step responses for the designed control system (blue solid curve) and for the desired closed loop system (orange dot-slash curve) - example 1b

Example 2. Third order system with delay

Consider

$$G(s) = \frac{1}{(s+1)^3} e^{-5s}. \tag{29}$$

The frequency bandwidth of the plant $G(j\omega)$ is $\omega_{Gb} = 0.510$ rad/s.

The reference transfer function for this example is derived from the closed loop transfer function with natural frequency ω_n equal to ω_{Gb}, which is 0.510 rad/s, time delay θ equal to 5 s. The value for the damping ratio considered is 0.9 in order to guarantee a maximum of the peak of the sensitivity function of 1.736.

Thus, the reference function is given by

$$\bar{L}_2(s) = \frac{0.26e^{-5s}}{s^2 + 0.92s + 0.26(1 - e^{-5s})}. \tag{30}$$

Its frequency bandwidth is $\omega_{Lb} = 0.200$ rad/s and its crossover frequency is $\omega_{Lgc} = 0.121$ rad/s.

Two different values of the minimum gain crossover frequency permissible will be considered and then two different controllers will be designed. The number of points used to obtain the coefficients for the performance constrains is $N_l = 50$.

The first value for the minimum gain crossover frequency permissible is ω_{gc} equal to ω_{Lgc}, which is 0.121 rad/s. Thus, the proposed linear programming procedure results in the PID controller gains

$$K_p = 0.504, \quad K_i = 0.122, \quad K_d = 0.780. \tag{31}$$

The bandwidth frequency of the obtained closed loop transfer function is 0.40 rad/s. The crossover frequency of the loop gain obtained is 0.121 rad/s and the maximum of the sensitivity function is $M_S = 1.697$.

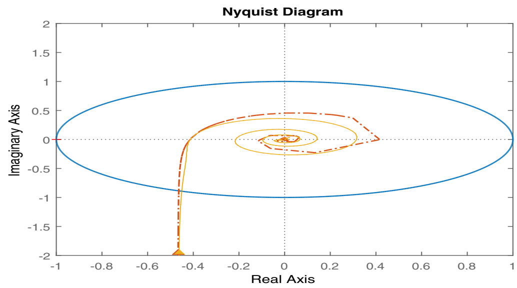

Fig. 6. Nyquist diagram of the reference transfer function $\bar{L}(j\omega)$ (orange dot-slash curve) and the designed loop gain $L(j\omega)$ (yellow solid curve) - example 2 with $\omega_{gc} = 0.121$

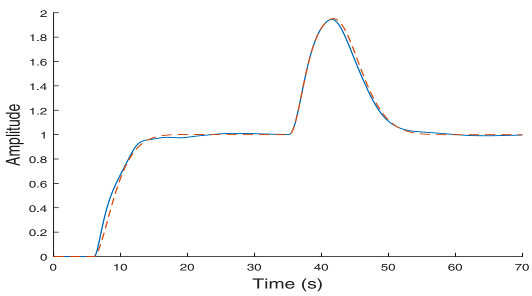

Fig. 7. Set point ($t = 0$s) and load disturbance ($t = 30$s) step responses for the designed control system (blue solid curve) and for the desired closed loop system (orange dot-slash curve) - example 2 with $\omega_{gc} = 0.121$

It can be seen in Fig. 7 that the time response is approximated to the one of the closed loop transfer function used to formulate the specifications, fact justified by the fact that obtained bandwidth frequency of the closed loop system obtained is close to the one of the desired specification.

The second value for the minimum gain crossover frequency permissible is ω_{gc} equal to $0.9\,\omega_{Lgc}$, which is 0.109 rad/s. Thus, the proposed linear programming procedure results in the PID controller gains

$$K_p = 0.437, \quad K_i = 0.113, \quad K_d = 1.014. \tag{32}$$

The bandwidth frequency of the obtained closed loop transfer function is 0.20 rad/s. The crossover frequency of the loop gain obtained is 0.110 rad/s and the maximum of the sensitivity function is $M_S = 1.61$.

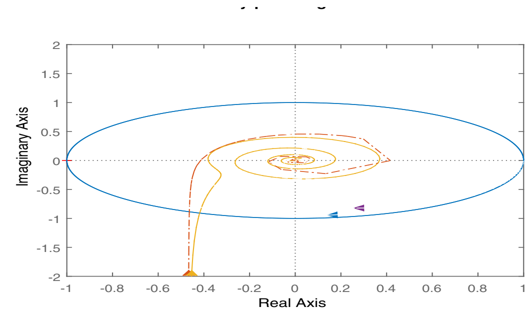

Fig. 8. Nyquist diagram of the reference transfer function $\bar{L}(j\omega)$ (orange dot-slash curve) and the designed loop gain $L(j\omega)$ (yellow solid curve) - example 2 with $\omega_{gc} = 0.109$

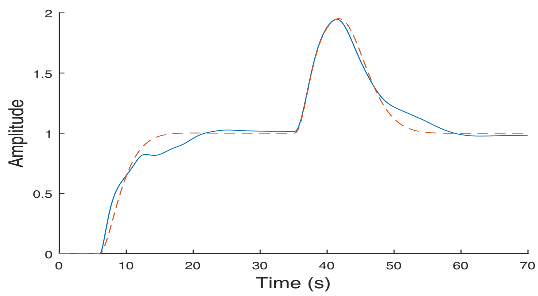

Fig. 9. Set point ($t = 0$s) and load disturbance ($t = 30$s) step responses for the designed control system (blue solid curve) and for the desired closed loop system (orange dot-slash curve) - example 2 with $\omega_{gc} = 0.109$

It can be seen in Fig. 8 that depending on the minimum gain crossover frequency permissible, the Nyquist diagram of the designed loop gain transfer function can be deformed in order to achieve the performance specification. The choice of lower minimum gain crossover frequency permissible results in slower step responses with possibly some deformations, but still with a format similar to the specification (Fig. 9).

5. CONCLUSIONS

Robust PID controller design as formulated as a linear optimization problem. The control objective of the proposed method is to minimize the IE or equivalently to maximize the integral gain. Robustness constraints are formulated based on the shaping of a reference loop gain transfer function at specific frequency points. Simulations demonstrates the robustness and efficiency of the method. For all examples and simulations, the Nyquist plots obtained by the proposed method respect the constraints. The method is simple to implement, since it uses a linear programming approach (compared to convex and nonconvex optimization). The implementation of the proposed design technique can be done using standard linear optimization tools, is simple and requires only the frequency response of the plant.

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REFERENCES

Åström, K.J. and Wittenmark, B. (2013). *Computer-controlled systems: theory and design*. Courier Corporation.

Åström, K.J. and Hägglund, T. (2006). *Advanced PID control*. ISA-The Instrumentation, Systems and Automation Society.

Grassi, E. and Tsakalis, K. (2000). Pid controller tuning by frequency loop-shaping: application to diffusion furnace temperature control. *IEEE Transactions on Control Systems Technology*, 8(5), 842–847.

Grassi, E., Tsakalis, K.S., Dash, S., Gaikwad, S.V., MacArthur, W., and Stein, G. (2001). Integrated system identification and pid controller tuning by frequency loop-shaping. *IEEE Transactions on Control Systems Technology*, 9(2), 285–294.

Hast, M., Astrom, K., Bernhardsson, B., and Boyd, S. (2013). Pid design by convex-concave optimization. In *Control Conference (ECC), 2013 European*, 4460–4465. IEEE.

Jelali, M. (2012). *Control performance management in industrial automation: assessment, diagnosis and improvement of control loop performance*. Springer Science & Business Media.

Karimi, A. and Galdos, G. (2010). Fixed-order h controller design for nonparametric models by convex optimization. *Automatica*, 46(8), 1388–1394.

Karimi, A., Galdos, G., and Longchamp, R. (2008). Robust fixed-order h controller design for spectral models by convex optimization. In *Decision and Control, 2008. CDC 2008. 47th IEEE Conference on*, 921–926. IEEE.

Karimi, A., Kunze, M., and Longchamp, R. (2007). Robust controller design by linear programming with application to a double-axis positioning system. *Control Engineering Practice*, 15(2), 197–208.

Panagopoulos, H., Astrom, K., and Hagglund, T. (2002). Design of pid controllers based on constrained optimisation. *IEE Proceedings-Control Theory and Applications*, 149(1), 32–40.

Pintelon, R. and Schoukens, J. (2012). *System identification: a frequency domain approach*. John Wiley & Sons.

Saeki, M. (2014). Data-driven loop-shaping design of pid controllers for stable plants. *International Journal of Adaptive Control and Signal Processing*, 28(12), 1325–1340.

Thalmann, E., Regamey, Y.J., and Karimi, A. (2016). High performance control of a corner cube reflector by a frequency-domain data-driven robust control method. In *American Control Conference (ACC), 2016*, 455–460. IEEE.