Cascade Fractional-Order PI Control of a Linear Positioning System

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Abstract: This paper proposes a method for designing robust fractional-order proportional-integral (FOPI) controllers to be employed in a cascade control system. The FOPI controllers are employed for controlling two nested loops. The design is based on performance and robustness specifications in the frequency domain. Taking inspiration from well-known tuning rules, the open-loop frequency response in the two nested loops is shaped around the gain crossover frequency to obtain a nearly flat phase diagram, then a nearly constant phase margin. The method is tested to control the speed and position of a linearly sliding motor.

Keywords: Fractional systems, infinite-dimensional systems, linear systems, fractional-order PI control, cascade control, tuning rules, regulation, robust control, frequency response.

1. INTRODUCTION

It is well-known that PI/PID are the most applied controllers in industry. Namely, they are in more than 90% of the industrial control loops (Åström and Hägglund, 1995). Moreover, controllers are tuned by trial-and-error or by well-established and widely accepted rules: for example, the Symmetrical Optimum method which is very common for electro-mechanical and thermal plants.

However, the paradigm of fractional calculus allows to replace integer-order differentiation and integration by a noninteger-order one (Podlubny, 1999a). Noninteger-order operators in the Laplace domain are the basic tool to realize this replacement and to achieve what is often named as fractional-order controller (Chen et al., 2009). Operators and controllers of noninteger order have the ability of improving the robustness to gain and load variations because the ideal open-loop gain is of noninteger order (Bode, 1945). Many contributions in the literature have shown that it is possible to achieve a better trade-off between robust stability and dynamic performance (Oustaloup, 1991; Podlubny, 1999b; Monje et al., 2008; Luo and Chen, 2009; Caponetto et al., 2010; Monje et al., 2010; Padula and Visioli, 2011, 2015; De Keyser et al., 2016), as well as some recent ones with reference to peculiar industrial applications (Caponetto et al., 2016; Lino et al., 2017). The challenge is to maintain implementation simplicity of the realization schemes, while obtaining improvements with respect to usually employed controllers. Realization can be easy compared to more complex, integer-order schemes that have higher computational demand and sensitivity to hardware limitations (Maione, 2011a).

Then the idea is to prove the advantages of fractional-order proportional-integral-derivative (FOPID) or fractional-order proportional-integral (FOPI) controllers with respect to PI/PID controllers by considering important applications, benchmark problems, and by developing and testing tuning rules that are similar to or extend the classical ones. Moreover, the proposed settings should be easy-to-use or require a low computational cost and implementation effort. Namely, the purpose is to make them acceptable by practitioners and control engineers. Some researches are then directed towards this simplification issue (Lino and Maione, 2013; Lino et al., 2017).

In this paper, a cascade control scheme is proposed in which PI controllers are replaced by FOPI controllers. The standard PID control is improved by using FOPI control. Specific design rules are developed for the parameters of the FOPI controllers used in two nested loops. The formulas provide the parameters by relating them to the robustness and performance specifications. The implementation of the controller transfer function is based on an approximation technique that computes the coefficients of the approximant (of reduced order) in an easy way. The proposed design method can be applied to a variety of control systems, as represented as in Fig. 1.

As shown by the results, the FOPI controller provides a better frequency response and a better trend of the controlled variables in the time domain. Benefits could derive for many applications relying on PID controllers in the described cascade control scheme. Tests are performed on a linear movement and positioning system, which is similar to systems that are used in many motion control applications. Section 2 describes the control problem and illustrates the tuning rules for PID controllers and the new tuning rules for FOPI controllers. Section 3 shows the experimental results. Section 4 gives some final remarks.

2. THE CONTROL SYSTEM

The control system scheme is shown in Fig. 1. It is a well-established scheme for cascade control of speed and position of electrical drives. Two nested loops can be distinguished. In the first inner one, the plant is a linear

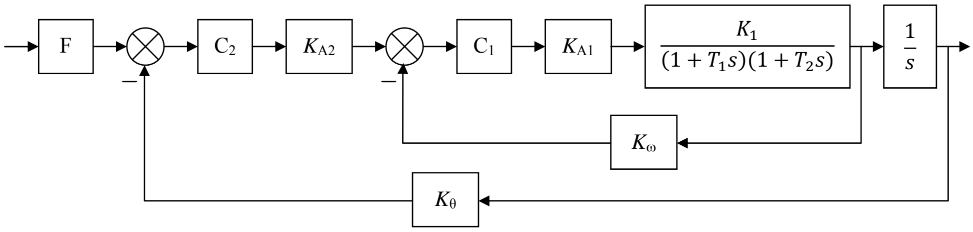

Fig. 1. Scheme of the control system

position system driven by a permanent magnets dc motor described by the usual mechanical and electrical equations that can be associated to a second-order system

$$G_{p1}(s) = \frac{K_1}{(1+T_1\,s)\,(1+T_2\,s)} \tag{1}$$

where K_1 is the dc-gain, $T_1 > T_2$, with T_1 representing a mechanical time constant and T_2 an electrical time constant. Obviously, this model takes into account all the electrical and mechanical variables (current, torque, speed) to derive the relation between the input to the inner loop and the output, i.e. the rotational speed. The gain K_{A1} in the block scheme takes into account the power electronics and the command equipment. The dynamics of the feedback element is neglected so that only a constant gain K_ω is considered to represent speed measurement by an encoder. The controller C_1 is devoted to track a reference speed.

The second outer loop is for position control. The controlled system includes the feedback system from the inner loop plus an integrator. A gain K_{A2} represents an actuator that commands the inner loop. Again, the dynamics of the feedback element is neglected so that only a constant gain K_θ takes into account the measurement and conversion of the angular position to a proper comparable variable. The controller C_2 is used to achieve the desired position. Finally, the set-point filter F is used to damp the closed-loop response and decrease the maximum overshoot.

2.1 Tuning of PID Controllers

The classical scheme employs two PI controllers. In industry, the two integer-order PI controllers are frequently tuned by well-established techniques. The first PI controller is typically tuned by the *Optimum Modulus* criterion to optimize the closed-loop transfer function (Oldenbourg and Sartorius, 1956). The second PI controller is tuned by the *Symmetrical Optimum* method, which must be combined with a smoothing filter F on the set-point (Kessler, 1958; Voda and Landau, 1995).

However, since the proposed control scheme includes two FOPI controllers with three tunable parameters each, two standard PID controllers are considered for a fair comparison of performance, with the same number of tuning parameters. To tune each PID controller, with reference to system stability and robustness, three design specifications are considered concerned with the phase and gain of each open-loop transfer function (Luo and Chen, 2013). Given the generic plant and the controller transfer functions $G_p(s)$ and $G_c(s)$, respectively, the following constraints can be set:

- Specification on phase margin
$$PM = \pi + \angle[G_p(j\omega_c)\cdot G_c(j\omega_c)] \tag{2}$$
- Specification on gain crossover frequency
$$|G_p(j\omega_c)\cdot G_c(j\omega_c)| = 1 \tag{3}$$
- Specification on robustness to loop gain variations, i.e. the phase Bode diagram of the open loop transfer function must be flat around the gain crossover frequency
$$\left.\frac{d\angle[G_p(j\omega)\cdot G_c(j\omega)]}{d\omega}\right|_{\omega=\omega_c} = 0 \tag{4}$$

where PM and ω_c are the required phase margin and gain crossover frequency, respectively.

If the inner PID controller is defined as

$$G_{c1}(s) = K_{P1} + \frac{K_{I1}}{s} + K_{D1}s \tag{5}$$

then the open-loop transfer function of the inner loop is

$$G_{OL1}(s) = \frac{(K_{D1}s^2 + K_{P1}s + K_{I1})K_{A1}\,K_1\,K_\omega}{s\,(1+T_1\,s)\,(1+T_2\,s)} \tag{6}$$

Now, the plant in (1) is approximated by

$$G_{p1}(s) = \frac{K_1}{1+T_e\,s} \tag{7}$$

where $T_e = T_1 + T_2$. By defining $K_e = K_{A1}\,K_1$, and by assuming, without loss of generality, $K_\omega = 1$, then (6) becomes:

$$G_{OL1}(s) = K_e\frac{(K_{D1}s^2 + K_{P1}s + K_{I1})}{s\,(1+T_e\,s)} \tag{8}$$

By setting the inner loop gain crossover frequency ω_{c1} and phase margin PM_{s1}, it holds:

$$\tan(PM_{s1} + \arctan(T_e\omega_{c1})) = \frac{K_{D1}\omega_{c1}^2 - K_{I1}}{K_{P1}\omega_{c1}} \tag{9}$$

The third specification gives:

$$\frac{\left(\frac{K_{D1}}{K_{P1}} + \frac{K_{I1}}{K_{P1}\omega_{c1}^2}\right)}{1+\left(\frac{K_{D1}\omega_{c1}^2 - K_{I1}}{K_{P1}\omega_{c1}}\right)^2} - \frac{T_e}{1+T_e^2\omega_{c1}^2} = 0 \tag{10}$$

The specification on gain crossover frequency gives:

$$|G_{OL1}(j\omega_{c1})| = K_e\cdot K_{P1}\sqrt{\frac{1+\left(\frac{K_{D1}\omega_{c1}^2 - K_{I1}}{K_{P1}\omega_{c1}}\right)^2}{1+T_e^2\omega_{c1}^2}} = 1 \tag{11}$$

By putting $A_1 = \tan(PM_{s1}+\arctan(T_e\omega_{c1}))$ and $B_1 = 1+T_e^2\omega_{c1}^2$, then K_{P1} can be calculated by

$$K_{P1} = \frac{1}{K_e}\sqrt{\frac{B_1}{1+A_1^2}} \tag{12}$$

Moreover, (9) and (10) give

$$K_{I1} = \frac{1}{2K_e}\left[\sqrt{\frac{1+A_1^2}{B_1}}T_e\omega_{c1}^2 - A_1\omega_{c1}\sqrt{\frac{B_1}{1+A_1^2}}\right] \tag{13}$$

and

$$K_{D1} = \frac{1}{2K_e}\left[\sqrt{\frac{1+A_1^2}{B_1}}T_e - \frac{A_1}{\omega_{c1}}\sqrt{\frac{B_1}{1+A_1^2}}\right] \tag{14}$$

The outer PID controller is given by

$$G_{c2}(s) = K_{P2} + \frac{K_{I2}}{s} + K_{D2}s \tag{15}$$

then the open-loop transfer function of the outer loop is:

$$G_{OL2}(s) = \frac{G_{c2}(s)(K_{D1}s^2 + K_{P1}s + K_{I1})K_{A2}\,K_e\,K_\theta}{s\,[(T_e + K_{D1}K_e)s^2 + (1 + K_{P1}K_e)s + K_{I1}K_e]} \tag{16}$$

The previously introduced specifications can be also used to design the outer loop PID controller, even if the derivation of the tuning parameters is not as straightforward as before. More in details, by setting the outer loop gain crossover frequency ω_{c2} and the phase margin PM_{s2} and, without loss of generality, after normalizing the gains K_{A2}, K_e, and K_θ, the following symbols can be defined: $A_2 = \tan\left[PM_{s2} + \arctan\frac{(1+K_{P1})\omega_{c2}}{(T_e+K_{D1})\omega_{c2}^2-K_{I1}}\right]$; $B_2 = [(T_e + K_{D1})\omega_{c2}^2 - K_{I1}]^2 + (1+K_{P1})^2\omega_{c2}^2$; $R_1 = A_2\omega_{c2}\sqrt{\frac{B_2}{1+A_2^2}}$; $R_2 = \omega_{c2}^2\sqrt{\frac{1+A_2^2}{B_2}}(1+K_{P1})[(T_e + K_{D1})\omega_{c2}^2 + K_{I1}]$; $R_3 = K_{D1}\omega_{c2}^2 - K_{I1}$; $R_{1+2} = (R_1 + R_2)$; $R_{1-2} = (R_1 - R_2)$.

Then, the PID parameters which meet the design constraints can be computed as follows:

$$K_{P2} = \frac{1}{2}\frac{R_{1-2}R_3 - R_{1+2}(K_{I1} - K_{D1}) + \omega_{c2}R_1K_{P1}A_2^{-1}}{K_{P1}^2 - K_{D1}R_3\omega_{c2}^2 - K_{I1}(K_{I1} - K_{D1})} \tag{17}$$

$$K_{I2} = \frac{R_{1+2} - 2K_{I1}K_{P2}}{2K_{P1}} \tag{18}$$

$$K_{D2} = \frac{-R_{1-2} - 2K_{D1}K_{P2}\omega_{c2}^2}{2K_{P1}\omega_{c2}^2} \tag{19}$$

2.2 Tuning of FOPI Controllers

Consider now the use of FOPI controllers in place of PI controllers. Usually, fractional-order controllers are proposed to improve robustness and also dynamic performance. This is obtained by properly tuning the additional parameters, in this case the noninteger order ν_1 of integration in the controller transfer function

$$G_{c1}(s) = K_{P1} + \frac{K_{I1}}{s^{\nu_1}} = \frac{K_{I1}\,(1 + T_{I1}\,s^{\nu_1})}{s^{\nu_1}} \tag{20}$$

where K_{P1} and K_{I1} are the proportional and integral gain, respectively, $T_{I1} = K_{P1}/K_{I1}$, and $1 < \nu_1 < 2$. Namely, $1/s^{\nu_1} = (1/s)\cdot(1/s^{\mu})$, with $0 < \mu < 1$. Then, $1/s$ is used to reject disturbances on the motor input.

The literature shows several tuning methods. Here we extend the one in (Lino and Maione, 2013). Namely, the FOPI controller is very close to the PI controller and the proposed method extends the *Symmetrical Optimum* method, so that both the controller structure and the tuning can be more acceptable by practitioners. The aim is to achieve robustness specifications and a nearly optimal feedback system, which means that a very good input-output tracking is obtained in a specified bandwidth (Kalman, 1964). In particular, a specified phase margin PM_s is obtained around the angular gain crossover frequency, say ω_c, that is expressed in terms of the specified bandwidth ω_B in which optimality is required. The main idea is to take advantage of the noninteger order integrator to shape the overall open-loop gain around the crossover so that a nearly flat phase diagram is obtained. Then the phase margin is more or less constant within a range around the crossover, and robustness is guaranteed even for high gain variations (the seminal Bode's idea). That's why it is said that a loop-shaping is performed.

Now, the plant in (1) is approximated by (7) where $T_e = T_1 + T_2$ and the controller (20) is applied. The approximation is motivated because the time constants are close and small. To set the controller parameters, the tuning procedure starts from the specifications on the bandwidth and on the phase margin, i.e. ω_{B1} and PM_{s1}, as shown in (Lino and Maione, 2013; Lino et al., 2017). Then the required crossover is assumed by $\omega_{c1} \in [\omega_{B1}/1.7, \omega_{B1}/1.3]$, which is a well-known rule of thumb (Maciejowski, 1989).

Since the open-loop frequency response of the inner loop is $G_{OL1}(j\omega) = G_{c1}(j\omega)\,K_{A1}\,G_{p1}(j\omega)\,K_\omega$, establishing that the phase margin $\pi + \angle G_{OL1}(j\omega_{c1})$ is equal to the specified PM_{s1} yields:

$$\begin{aligned} PM_{s1} &= \pi + \arctan\left(\frac{T_{I1}\,\omega_{c1}^{\nu_1}\,\sin(\pi\,\nu_1/2)}{1 + T_{I1}\,\omega_{c1}^{\nu_1}\,\cos(\pi\,\nu_1/2)}\right) \\ &- \arctan(\omega_{c1}\,T_e) - \frac{\pi}{2}\,\nu_1 = \frac{\pi}{2}\,(2-\nu_1) + \varphi_1 - \varphi_2. \end{aligned} \tag{21}$$

where φ_1 and φ_2 are the two phase contributions specified by inverse tangent functions. The idea is then to make the phase margin depend on ν_1 only by putting $\varphi_1 = \varphi_2$ in (21). Given that φ_1 is unknown (function of T_{I1}) and φ_2 known, simple algebra leads to a restriction on the value of T_{I1}, given by (23). Then the following settings are obtained for two controller parameters:

$$\nu_1 = 2 - \frac{PM_{s1}}{\pi/2} \tag{22}$$

$$T_{I1} = \frac{\omega_{c1}\,T_e}{(\omega_{c1})^{\nu_1}\,\left[\sin(\frac{\pi}{2}\nu_1) - \omega_{c1}\,T_e\,\cos(\frac{\pi}{2}\nu_1)\right]} \tag{23}$$

Moreover, the setting on the third controller parameter

$$K_{P1} = \frac{T_{I1}\,\omega_{c1}^{\nu_1}}{K_{A1}\,K_1\,K_\omega}\sqrt{\frac{1 + (\omega_{c1}\,T_e)^2}{1 + 2\,T_{I1}\,\omega_{c1}^{\nu_1}\,\cos(\frac{\pi}{2}\nu_1) + T_{I1}^2\,\omega_{c1}^{2\nu_1}}} \tag{24}$$

is obtained by enforcing the gain crossover at ω_{c1}, i.e. $|G_{OL1}(j\omega_{c1})| = 1$, and taking into account the integral time constant T_{I1} given by (23).

The inner loop generates a closed-loop fractional-order transfer function $G_{FOS}(s)$ which can be written as

$$G_{FOS}(s) = \frac{K_{P1}\,(1 + T_{I1}\,s^{\nu_1})\,K_{A1}\,K_1}{T_{I1}\,s^{\nu_1}\,(1 + T_e\,s) + K_{P1}\,(1 + T_{I1}\,s^{\nu_1})\,K_{A1}\,K_1\,K_\omega} \tag{25}$$

This inner feedback system becomes part of the controlled system (plant) in the outer loop:

$$G_{p2}(s) = \frac{G_{FOS}(s)}{s} \tag{26}$$

for which the second FOPI controller is

$$G_{c2}(s) = K_{P2} + \frac{K_{I2}}{s^{\nu_2}} = \frac{K_{I2}\,(1 + T_{I2}\,s^{\nu_2})}{s^{\nu_2}} \tag{27}$$

Then the open-loop transfer function of the outer loop is $G_{OL2}(s) = G_{c2}(s)\,K_{A2}\,\frac{G_{FOS}(s)}{s}\,K_\theta$.

If the specifications for the outer loop are ω_{B2} and PM_{s2}, then the crossover is specified as $\omega_{c2} \in [\omega_{B2}/1.7, \omega_{B2}/1.3]$ and the phase margin $\pi + \angle G_{OL2}(j\omega_{c2})$ is put equal to PM_{s2}. These specifications yield:

$$PM_{s2} = \frac{\pi}{2}\,(1 - \nu_2) + \phi_1 + \phi_2 - \phi_3 \tag{28}$$

where

$$\phi_1 = \arctan\left(\frac{T_{I2}\,\omega_{c2}^{\nu_2}\,S_2}{1 + T_{I2}\,\omega_{c2}^{\nu_2}\,C_2}\right) \tag{29}$$

$$\phi_2 = \arctan\left(\frac{T_{I1}\,\omega_{c2}^{\nu_1}\,S_1}{1 + T_{I1}\,\omega_{c2}^{\nu_1}\,C_1}\right) \tag{30}$$

$$\phi_3 = \arctan\left(\frac{T_{I1}\,\omega_{c2}^{\nu_1}\,(S_1 + C_1\,\omega_{c2}\,T_e + \overline{K}_1\,S_1)}{T_{I1}\,\omega_{c2}^{\nu_1}\,(C_1 - S_1\,\omega_{c2}\,T_e + \overline{K}_1\,C_1) + \overline{K}_1}\right) \tag{31}$$

with $S_1 = \sin(\pi\,\nu_1/2)$, $C_1 = \cos(\pi\,\nu_1/2)$, $S_2 = \sin(\pi\,\nu_2/2)$, $C_2 = \cos(\pi\,\nu_2/2)$, $\overline{K}_1 = K_{P1}\,K_{A1}\,K_1\,K_\omega$

The idea is again to make the phase margin depend on ν_2 only by putting $\phi_1 + \phi_2 - \phi_3 = 0$ in (28). The condition $\phi_1 = \phi_3 - \phi_2$ implies that T_{I2} is set by using ϕ_2 and ϕ_3 that are known quantities. Simple algebra leads to the following resolving formulas:

$$\nu_2 = 1 - \frac{PM_{s2}}{\pi/2} \tag{32}$$

$$T_{I2} = \frac{\tan(\phi_3 - \phi_2)}{\omega_{c2}^{\nu_2}\,[S_2 - C_2\,\tan(\phi_3 - \phi_2)]} \tag{33}$$

where ϕ_2 and ϕ_3 depend on ω_{c2} and on the parameters in the inner loop ν_1, T_{I1}, and K_{P1} (controller) and K_1, T_e, K_{A1}, and K_ω (plant, actuator and sensor).

Finally, $|G_{OL2}(j\omega_{c2})| = 1$ provides the setting of K_{P2}:

$$K_{P2} = \frac{T_{I2}\,\omega_{c2}^{\nu_2+1}}{\overline{K}_2}\sqrt{\frac{A(\omega_{c1,2}, PM_{s1,2})}{B(\omega_{c1,2}, PM_{s1,2})\,C(\omega_{c1,2}, PM_{s1,2})}} \tag{34}$$

in which A, B, and C depend on all the specifications ω_{c1}, ω_{c2}, PM_{s1}, and PM_{s2} and where $\overline{K}_2 = K_{A2}\,K_\theta\,K_{P1}\,K_{A1}\,K_1$, $A(\omega_{c1,2}, PM_{s1,2}) = T_{I1}^2\,\omega_{c2}^{2\nu_1}\,(1 + \omega_{c2}^2\,T_e^2) + \overline{K}_1^2\,(1 + 2\,\omega_{c2}^{\nu_1}\,T_{I1}\,C_1 + \omega_{c2}^{2\nu_1}\,T_{I1}^2) + 2\,T_{I1}\,\omega_{c2}^{\nu_1}\,\overline{K}_1\,(C_1 - \omega_{c2}\,T_e\,S_1 + \omega_{c2}^{\nu_1}\,T_{I1})$, $B(\omega_{c1,2}, PM_{s1,2}) = 1 + 2\,T_{I2}\,\omega_{c2}^{\nu_2}\,C_2 + T_{I2}^2\,\omega_{c2}^{2\nu_2}$, $C(\omega_{c1,2}, PM_{s1,2}) = 1 + 2\,T_{I1}\,\omega_{c2}^{\nu_1}\,C_1 + T_{I1}^2\,\omega_{c2}^{2\nu_1}$.

Remark. For practical implementation, the irrational transfer function of the FOPI controllers is approximated by a rational transfer function with N pairs of zeros and poles. To this aim, the method firstly proposed in (Maione, 2008) is employed. Namely, this method guarantees that N minimum-phase zeros and N stable poles, interlaced one with another, are obtained (Maione, 2011b). In this way, the realization is characterized by very important properties for control. More precisely, the approximation of the fractional operator s^ν is given by:

$$s^\nu \approx \frac{\alpha_{N,0}(\nu)\,s^N + \alpha_{N,1}(\nu)\,s^{N-1} + ... + \alpha_{N,N}(\nu)}{\beta_{N,0}(\nu)\,s^N + \beta_{N,1}(\nu)\,s^{N-1} + ... + \beta_{N,N}(\nu)} \tag{35}$$

where the coefficients are $\alpha_{N,j}(\nu) = \beta_{N,N-j}(\nu) = (-1)^j\,B(N,j)\,(\nu + j + 1)_{(N-j)}\,(\nu - N)_{(j)}$, for $j = 0, ..., N$, with $B(N,j) = \frac{N!}{j!\,(N-j)!}$, and Pochhammer functions specified by $(\nu + j + 1)_{(N-j)} = (\nu + j + 1)(\nu + j + 2)...(\nu + N)$, and $(\nu - N)_{(j)} = (\nu - N)(\nu - N + 1)...(\nu - N + j - 1)$, starting with $(\nu + N + 1)_{(0)} = (\nu - N)_{(0)} = 1$. In the experiments, the chosen order of approximation is $N = 5$.

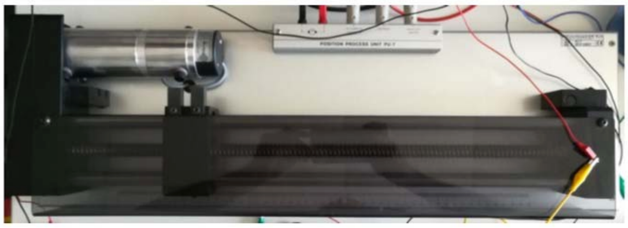

Fig. 2. The controlled system

The benefits of the method can be synthesized as follows. Firstly, expressions in closed form provide the values of the controller parameters, so that tedious trial-and-error procedures or computationally intensive minimization techniques can be avoided. Secondly, the expressions directly connect the specifications to the required values of the controller parameters, and make the method suitable for application. Thirdly, the implementation is not an issue because computation of the coefficients α and β in (35) can be easily coded, also in a recurrent way (see (14) in Maione (2008)). To simplify the controller, N can be reduced to 3 or 4 without significantly affecting the approximation.

In principle, the FOPI control design approach can be applied to all systems that can be represented as shown in Fig. 1. I.e., to all cascade controlled systems in which the inner loop includes a plant modeled as a first or second-order system and the outer loop includes an integrator. This class covers a variety of real problems. However, the approach can be extended to different plant models in the inner loop and different systems in place of the integrator in the outer loop. Namely, the loop-shaping method can be modified by properly manipulating the frequency response and imposing the specifications.

3. EXPERIMENTAL RESULTS

The results of some experimental tests are shown to verify effectiveness of the proposed control scheme. Both C_1 and C_2 in Fig. 1 are FOPI controllers tuned as shown in subsection 2.2. The controlled system is a linear position system in which a cart slides along a shaft (see Fig. 2). The system parameters are identified as: $K_1 = 129.97$ and $T_e = 0.306$ s. Moreover, $K_{A1} = 1$ because the gain is included in the plant dc gain and $K_\omega = 1$, for the inner speed loop; in the same way, $K_{A2} = 1$ (the actuator gain is included in the controller C_2), $K_\theta = 1$ for the outer position loop.

The specifications are set as $\omega_{c1} = 4.19$ rad/s, $PM_{s1} = 63°, 54°, 45°, 36°$ ($\nu_1 = 1.3, 1.4, 1.5, 1.6$), for the inner loop.

Instead, $\omega_{c2} = 1.5, 3, 3.5$ rad/s are considered for the outer loop in order to adapt to different performance obtained from the inner loop when varying ν_1. Finally, $PM_{s2} = 45°$ ($\nu_2 = 0.5$) is set. As for the PID controllers, the same specifications are used for the inner loop, whereas ω_{c2} is set equal to 9, 2, 2.9 rad/s to optimize the time step response, while guaranteing a 45° phase margin for the outer loop. Note that if $PM_{s1} = 36°$ is requested, the tuning formulas do not allow to determine an appropriate set of the PID controllers parameters able to stabilize the closed-loop system. By these settings, the Bode diagrams of the open-loop transfer function $G_{OL2}(j\omega)$, including the FOPI controller C_2 and the inner feedback system $G_{FOS}(j\omega)$ resulting from the FOPI controller C_1, is shown in Fig. 3. The PID controller gives the diagrams in Fig. 4.

To perform the tests, an experimental set-up based on a floating point 250 Mhz Motorola PPC dSPACE interface board (DS1104) is used. The linear position system is commanded by an actuation box that receives inputs from a PC equipped with the dSPACE board. The board collects data, provides the reference, and runs the controllers. It includes A/D-D/A converters that can apply and read low voltage signals within the interval ±15 V. A dc-dc converter is used to provide a proper voltage to drive the motor (±30 V). Control algorithms are implemented in MATLAB/Simulink and directly compiled on the board to be executed in real time. The test is to measure the output to a reference position step. The speed and position output are shown in Figures 5 and 6.

As it can be easily verified, the FOPI controllers provide accurate positioning after a relatively short time, with an overshoot below 20% and that can be reduced to less than 10% by using $\nu = 1.4$, or even less with $\nu = 1.6$. High-frequency oscillations in the speed output are due to low encoder resolution and to some nonlinearities (static friction) in the laboratory equipment. Static friction is more evident when the output is close to the set-point.

Conversely, the system performance clearly deteriorates under PID control (Fig. 7). In general, the system controlled by PIDs exhibits larger overshoots and a slowly decaying oscillating behavior. A shorter settling time than with the FOPI controllers is only obtained by setting the phase margin for the inner loop to 63°. This performance is achieved thanks to an overall larger bandwidth, however the overshoot sensibly increases.

4. CONCLUSION

This paper shows how to apply fractional-order PI controllers to cascade control. Both an inner and an outer loop employ a FOPI controller in place of a PI controller. The design of FOPI controllers parameters employs robustness and performance specifications. The same specifications are used for PID controllers for comparison. The new control scheme is tested on a real motor. The results show the improvements achieved by FOPI controllers in the considered class of systems with two nested loops.

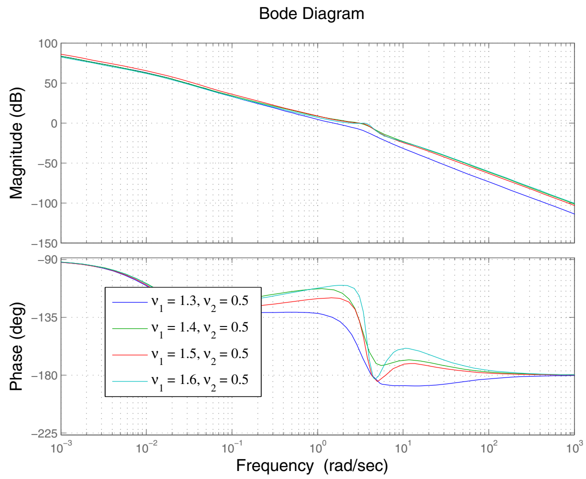

Fig. 3. Bode diagrams of the open-loop transfer function $G_{OL2}(j\omega)$ in the outer loop, including the FOPI controller C_2 and the inner feedback system $G_{FOS}(j\omega)$ resulting from the FOPI controller C_1

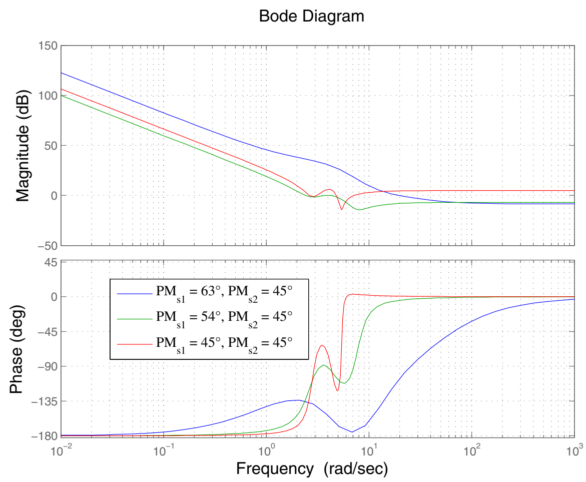

Fig. 4. Bode diagrams of $G_{OL2}(j\omega)$, including the PID controller C_2 and the inner feedback system $G_{IOS}(j\omega)$ resulting from the PID controller C_1

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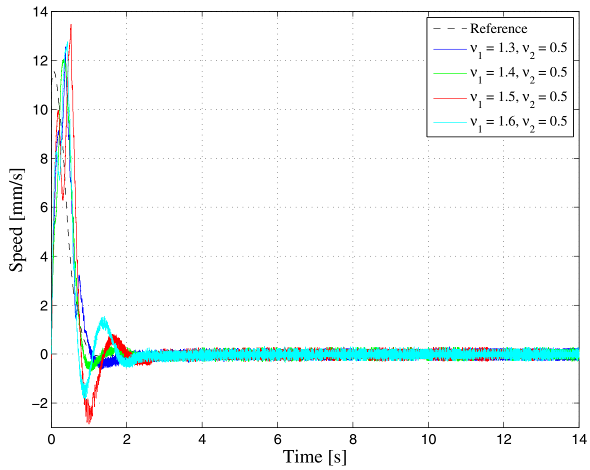

Fig. 5. Speed output from the inner loop

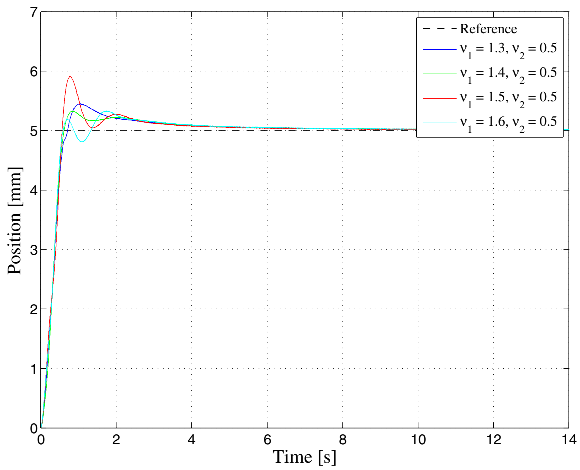

Fig. 6. Position output from the outer loop

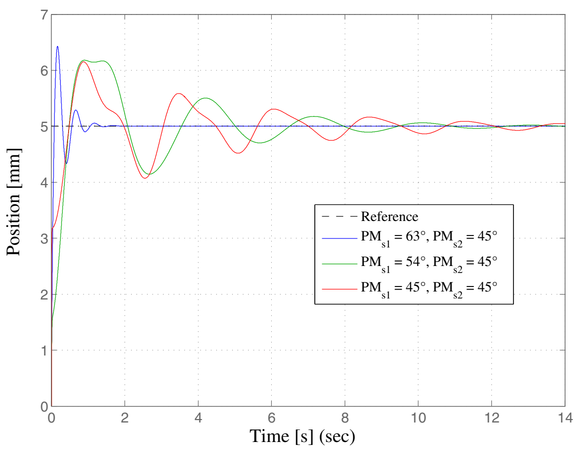

Fig. 7. Position output under PID control

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