Analysis of Actuator Rate Limit Effects on First-Order Plus Time-Delay Systems under Fractional-Order Proportional-Integral Control ⋆

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Abstract: Actuator rate limit deteriorates control performance, and may even lead to system instability in precision process control. In this paper, a first-order plus time-delay (FOPTD) system class with actuator rate limit is considered. The describing function (DF) of the rate limiter is derived to obtain the describing functions of the closed-loop and open-loop systems, and the onset frequency in the Nichols chart is used to analyze the rate limit effects in frequency domain. A fractional-order proportional-integral (FOPI) controller is first designed based on the flat phase constraint, crossover frequency, and phase margin specifications. Then a traditional integer-order PID (IOPID) controller is designed based on the same specifications to compare with the FOPI controller in the presence of actuator rate saturation. A careful simulation study is presented to validate all the conclusions.

Keywords: Process control, actuator rate saturation, FOPI controller, describing function, onset frequency

1. INTRODUCTION

Limited actuator capacity is acknowledged as an important control issue in the high performance-driven industry. Aggressive and accurate performance are now increasingly required in recent years due to the fast development in manufacturing, especially in the semiconductor industry. When the system reference input changes, large and fast commands will always be generated to the actuator at the very beginning, which may lead to long duration of actuator rate saturation. Thus, the guarantee of system performance and stability becomes critical (Nguyen and Jabbari, 2000).

In general, actuator saturation includes amplitude saturation and rate saturation. The amplitude saturation has been widely considered in the literatures (Sun et al., 2015; Kapila and Grigoriadis, 2002; Kapila and Haddad, 1998), while the rate saturation has not drawn much attention in process control. Rate limit is an important issue in category II Pilot-in-the-Loop Oscillation (PIO) in fly-by-fire flight control systems. Phase lag occurs when a rate limiter is activated, and will increase dramatically under the fully activated situation, which deteriorates control performance and may possibly lead to system instability. In the fast and precise process control tasks, rate limit becomes a stumbling block to achieve satisfactory system performance if not fully understood and well handled. Thus, it is essential to have a deeper understanding of the rate limit effects on system performance.

The describing function of the rate limiter was developed by Hanke (1994) to build a theoretical basis to analyze the handling qualities of the open-loop and closed-loop systems in frequency domain. The onset frequency of a rate limiter is defined in this paper as a measure of system input amplitude and frequency. The describing function and the onset frequency concept have been used in many academical research efforts, to predict limit cycles and design a phase compensator to reduce the occurrence possibility of category II PIO (Amato et al., 2000; Meng et al., 2010; Alstrom et al., 2012). Some researchers used this method to analyze the closed-loop stability boundary and predicted the potential oscillations introduced by the rate limiter (Gilbreath, 2001; Katayanagi, 2001). Ackermann and Bunte (1997) analyzed the actuator limits and limit cycles on robust car steering control systems.

PID controllers are the most widely used controllers in industry. Fractional-order proportional-integral-derivative (FOPID) controllers have attracted increasing attentions and developments in control field (Xue and Chen, 2002; Monje et al., 2010). Luo et al. (2010) designed two kinds

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of fractional-order controllers (FO-PI and FO-[PI]) based on three given constraints and applied them to real-time experimental systems, which exhibited the benefits of applying fractional calculus in the PID controller design and tuning. Luo and Chen (2012) designed IOPID and FOPI controllers based on three specifications for a class of FOPTD systems, and generated the feasible combination regions of two specifications (gain crossover frequency and phase margin) for the designed controllers.

Motivated by aforementioned issues, we will reveal the nature and analyze the effects of the rate limiter using the describing function method. Then for a typical FOPTD system, an FOPI controller and an IOPID controller will be designed and the rate limit effects on such system will be carefully analyzed. It will be shown that the FOPI controller shows potentially better performance compared with the IOPID controller.

The remainder of this paper is organized as follows: section 2 introduces the nature of the rate limiter, including the time responses of the rate limiter under different saturation situations, and the detailed derivation of the describing function of the rate limiter. Section 3 presents the concept of rate limiter onset frequency and closed-loop onset frequency, and the procedure to deduce the actuator saturated closed-loop describing function and open-loop describing function. The design specifications for the IOPID and FOPI controllers are given in section 4, and the tuning rules of these two controllers are also presented in this section. A typical example of an FOPTD system is provided in section 5. The whole paper is concluded in section 6.

2. THE NATURE OF RATE LIMITER

2.1 Time response for the sinusoidal input signal in the presence of rate limit

An actuator with rate limit has four different behaviors, which are, respectively, not active, partly active with amplitude reduction, partly active with amplitude reduction and phase delay, and fully active with amplitude reduction and phase delay. These four behaviors in the steady state oscillation excited by a sinusoidal input are illustrated in Fig. 1.

For the sinusoidal input

$$x_o(t) = A\sin(wt), \tag{1}$$

the rate of the input is

$$\dot{y}(t) = Aw\cos(wt). \tag{2}$$

The output rate cannot exceed the rate limit value R, which satisfies

$$|\dot{y}| \leq R. \tag{3}$$

For $Aw \leq R$, the rate limiter is deactivated and the output is identical to the input, which is illustrated by Fig. 1 (a). When the rate of input signal gets slight greater than the rate limit value, the rate limiter is activated and the output will meet the input signals before the peak of input is reached. After the meeting point, the output follows the input signals, where there is no amplitude reduction and

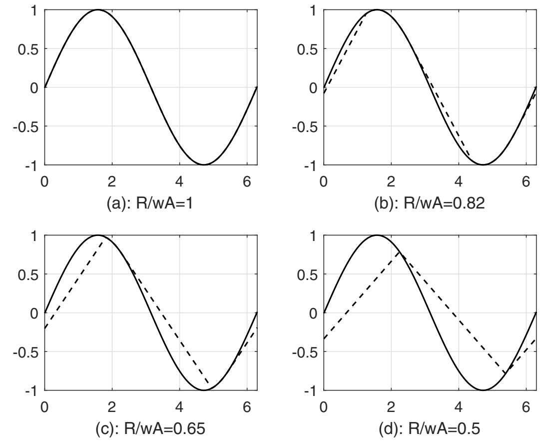

Fig. 1. Steady-state responses (dashed) of the rate limiter to sinusoidal inputs

phase delay. An example is shown in Fig. 1 (b). If the rate of input signal continues to increase, as seen in Fig. 1 (c), the output signal will meet the input after the peak value. At the meeting point, the input rate is smaller than the rate limit value, and then the output will follow the input until the rate of input is bigger than R. Amplitude attenuation and phase delay both exit in such case. In Fig. 1 (d), the rate of input signal is always greater than the rate limit R, thus the rate limiter keeps activated all the time. The output becomes a pure triangular wave under the situation $R/(wA) \leq 1/\sqrt{(\pi/2)^2+1}$ (Hanke, 1994).

2.2 Describing function of the rate limiter

In control system theory, the describing function method was developed in 1930s and extended by Kochenburger (1950), as an approximate method to analyze the nonlinear control problems. The describing function is defined as the magnitude ratio of the fundamental component of the system output to the input, which is based on the quasi-linearization. The describing function of the fully activated rate limiter, whose input-output relationship is illustrated in Fig. 1 (d), can be expressed as

$$N(jw, w_{onset}) = \frac{4}{\pi}\frac{w_{onset}}{w}\mathrm{e}^{-j\arccos\frac{\pi}{2}\frac{w_{onset}}{w}}, \tag{4}$$

where $w_{onset} = R/A$, which is defined as the frequency where the actuator saturation first occurs.

Equation (4) is only valid when the frequency w of input signal is no less than $1.862w_{onset}$ (Duda, 1994). The Bode plot of (4) is shown in Fig. 2. For $w \leq w_{onset}$, the rate limiter is not activated, and the output will follow the input signal exactly. Thus there is no amplitude reduction nor phase delay. A cubic spline interpolation is used to draw the magnitude and phase in "partly saturated" region. The dramatic magnitude attenuation and phase delay when the rate limiter is fully activated will deteriorate the system response and may possibly lead to system instability.

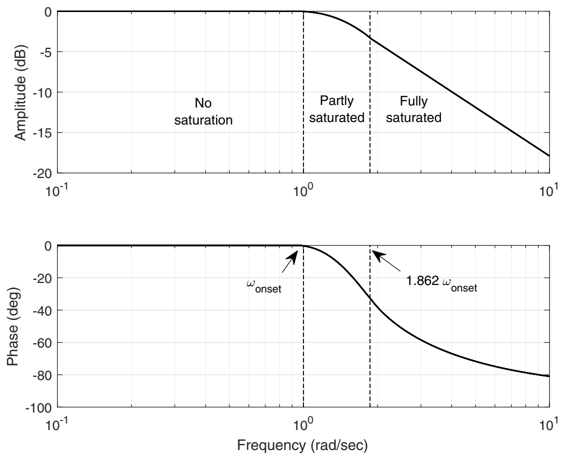

Fig. 2. The Bode plot of the describing function of a rate limiter

3. THE DESCRIBING FUNCTION OF AN OPEN-LOOP RATE LIMITED SYSTEM

This section focuses on the examination of the describing function of an open-loop rate limited system. The system diagram with a rate limiter inside is shown in Fig. 3. The describing function derived in the previous section can be performed to analyze the closed-loop and open-loop describing functions. The closed-loop onset frequency $\widetilde{w}_{onset}$ is defined as the frequency point where the actuator first saturated in the closed-loop system. It can be solved by the following equation (Gilbreath, 2001)

$$r_0 \left| \frac{\delta_c}{r}(j\widetilde{w}_{onset}) \right| = \frac{R}{\widetilde{w}_{onset}}, \quad (5)$$

where, r is the reference signal, r_0 is the amplitude of r, and δ_c is the input signal of the rate limiter in Fig. 3. By looking into the intersection point of both side of Equation (5), the $\widetilde{w}_{onset}$ can be determined.

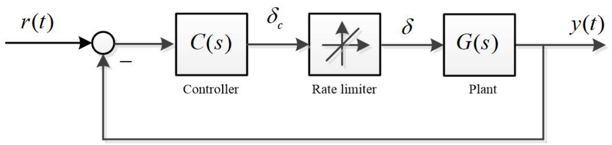

Fig. 3. The system diagram with a rate limiter

For the frequency above $\widetilde{w}_{onset}$, the describing function should be calculated based on the entire closed-loop system which results from that the rate limiter is injected in the feedback loop. Refering to Fig. 3, suppose the input signal in complex time domain is

$$r(t) = r_0 \mathrm{e}^{jwt}. \quad (6)$$

Then the input signal of the rate limiter can be expressed as

$$\delta_c(t) = \delta_{c0} \mathrm{e}^{j(wt+\Phi)}, \quad (7)$$

where, Φ is the additional phase delay generated by the rate limiter and linear model elements in the closed-loop system.

Then the following two nonlinear equations for δ_{c0} and Φ are derived

$$\frac{\delta_{c0}}{|C(w)|}\cos(\Phi_1) + \delta_{c0}\,|G(w)|\,|N(\delta_{c0}, w)|\cos(\Phi_2) = r_0, \quad (8)$$

$$\frac{\delta_{c0}}{|C(w)|}\sin(\Phi_1) + \delta_{c0}\,|G(w)|\,|N(\delta_{c0}, w)|\sin(\Phi_2) = 0, \quad (9)$$

where, $\Phi_1 = \Phi - \angle C(w)$ and $\Phi_2 = \Phi + \angle G(w) + \angle N(\delta_{c0}, w)$, C and G represent the controller and plant in the closed-loop system, Y is the output signal, and N represents the describing function of the rate limiter.

After obtaining the values of δ_{c0} and Φ, the closed-loop system output can be derived simply. Thus, the closed-loop describing function, N_c can be determined by the phase delay and magnitude attenuation using the linear system techniques. Once N_c is known, the open-loop describing function N_o, can be derived in order to compare with the linear system frequency response via the Nichols chart. N_o can be determined according to the relationship between open-loop system and closed-loop system. For the linear systems, the relationship is (Gilbreath, 2001)

$$N_c = \frac{N_o}{1+N_o}, \quad (10)$$

thus the open-loop describing function is given as

$$N_o = \frac{N_c}{1-N_c}. \quad (11)$$

The open-loop describing function also contains the non-linearity of the rate limiter due to the loop closure.

4. ROBUST FOPI CONTROLLER DESIGN FOR FOPTD PROCESSES

In practice, many industrial processes can be modeled as an FOPTD system

$$G(s) = \frac{K}{Ts+1}\mathrm{e}^{-Ls}, \quad (12)$$

where K is the steady-state gain of the plant, T is the time constant, and L represents the time delay. Atherton (2007) proposed that the FOPTD model can be normalized as

$$G(s) = \frac{1}{s+1}\mathrm{e}^{-Ls}, \quad (13)$$

which will be mainly considered in this paper.

In order to achieve a fair comparison with the traditional IOPID controller, this paper mainly focuses on the FOPI controller that has the same number of parameters with the IOPID controller

$$C(s) = k_p + \frac{k_i}{s^{\alpha}}, \quad (14)$$

where k_p is the proportional gain, k_i is the integral gain, and $\alpha \in (0, 2)$ is a real number that represents the fractional order (Podlubny, 1998).

In this paper, the FOPI controller is designed based on the linear system without consideration of the rate limiter at first. The design objective is to make the system robust to the loop gain variations. Assume that the desired gain crossover frequency of the linear open-loop system is w_c, the desired phase margin is specified as Φ_m. Three design constraints of the controller are given as follows (Li et al., 2010)

(i) Gain crossover frequency constraint
$$|C(jw_c)G(jw_c)| = 1; \tag{15}$$
(ii) Phase margin constraint
$$\arg(C(jw_c)G(jw_c)) = -\pi + \Phi_m; \tag{16}$$
(iii) Flat phase constraint: in order to make the system robust to the loop gain uncertainty, the Bode phase plot of the loop transfer function should be flat around the gain crossover frequency w_c. With this constraint, the phase margin varies in a very small range when the loop gain changes in a certain interval. The constraint can be written in the form
$$\left|\frac{\mathrm{d}(\arg(C(jw)G(jw)))}{\mathrm{d}w}\right|_{w=w_c} = 0. \tag{17}$$

5. ILLUSTRATIVE EXAMPLE

In order to show the advantages of the FOPI controller, the IOPID controller is also designed based on the same specifications as shown above. In addition, both of the these two controllers have three parameters, thus the comparison between them is fair. A numerical example is given below, processes with other delays also can be studied, here we suppose the process model transfer function is
$$G(s) = \frac{1}{s+1}\mathrm{e}^{-0.1s}. \tag{18}$$

In the reference Luo and Chen (2012), the feasible combination areas of the gain crossover frequency w_c and phase margin Φ_m specification for IOPID and FOPI controllers design are given graphically. Referring to the feasible combination of w_c and Φ_m for system (18), we choose two specifications as $w_c = 4$, $\Phi_m = 50°$. Considering the additional flat phase design constraint, based on the tuning rules introduced in previous sections, the conventional IOPID controller is given as
$$C_i(s) = 3.5298 + \frac{10.3810}{s} + 0.1161s, \tag{19}$$
and the FOPI controller is given as
$$C(s) = 4.223 + \frac{11.830}{s^{1.200}}. \tag{20}$$

The Bode plots of the open-loop frequency responses with these two controllers are shown in Fig. 4. It can be seen the phase Bode plots are flat at the frequency point w_c, which means the closed system is robust to the loop gain variation.

In the following part, we will take the rate limiter into consideration. Suppose the amplitude of the sinusoidal input signal is 1 and the rate limit value is 15. Firstly, according to Equation (5), one can plot two curves of both side to get a graphical solution of the closed-loop onset frequency $\widetilde{w}_{onset}$. It can be found that the $\widetilde{w}_{onset}$ for both IDPID and FOPI controllers equals 3.8090 rad/sec. The Nichols charts for the IOPID and FOPI controllers are shown in Fig. 5. It can be noted that dramatic phase jump

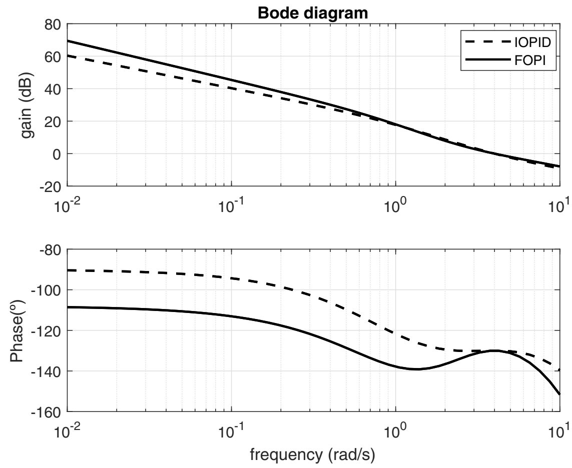

Fig. 4. Bode plots of the open-loop frequency responses with IOPID and FOPI controllers

appears at the closed-loop onset frequency, which means the actuator rate starts getting saturated and generates amplitude reduction and phase lag. Upon reaching the onset frequency, this phase jump pushes the frequency response cross the critical point (180°, 0dB) in the Nichols chart, which indicates the possibility of closed-loop system instability. Before reaching the onset frequency, the magnitude of FOPI controlled linear system (without rate limiter) is partly smaller than the IOPID controlled linear system. After reaching the onset frequency, the magnitude of the FOPI controlled nonlinear system (with rate limiter) is greater than IOPID controlled nonlinear system. All these indicate that the magnitude attenuation introduced by the rate limiter for the FOPI controlled system is less than the IOPID controlled system. Thus, it is observed that the rate limit effect of the FOPI controller is less than the IOPID controller.

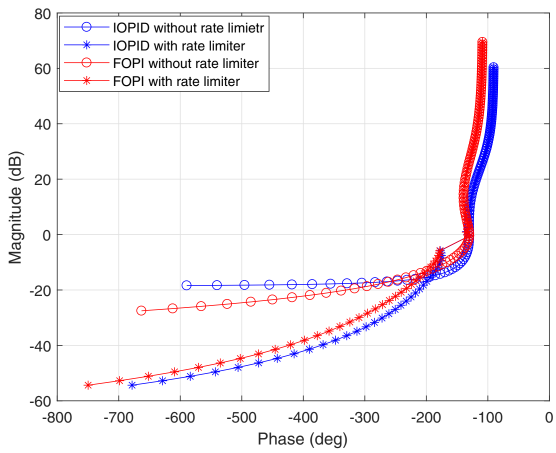

Fig. 5. Nichols chart for open-loop system

Unit step responses of the closed-loop system under different rate limit values are given in Fig. 6 and Fig. 7, and the control signal under the influence of rate limit are shown in Fig. 8 and Fig. 9. From Fig. 6, the step response is very sensitive to the rate limit value. When the rate limit value equals 22, the control signal becomes pure triangular curve and the time response starts to have

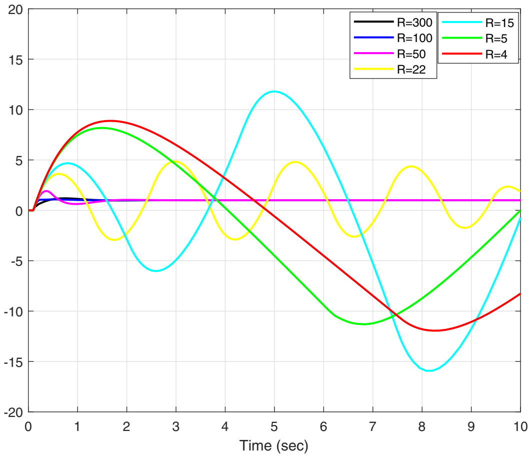

Fig. 6. System step responses for IOPID controller

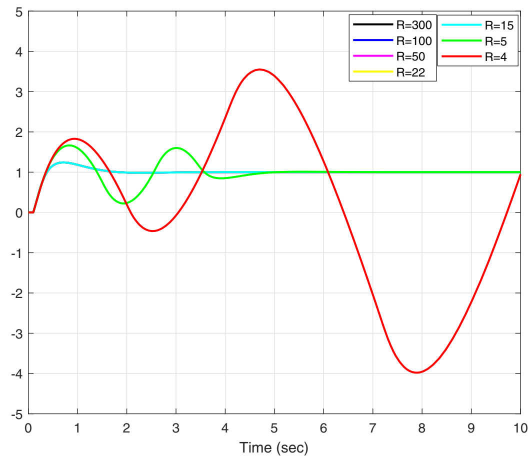

Fig. 7. System step responses for FOPI controller

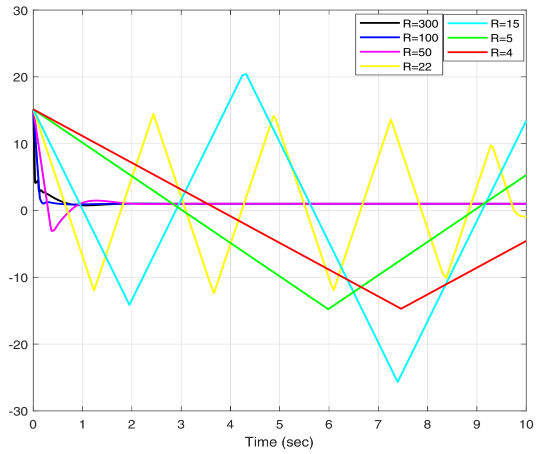

Fig. 8. IOPID control signals after rate limit effect

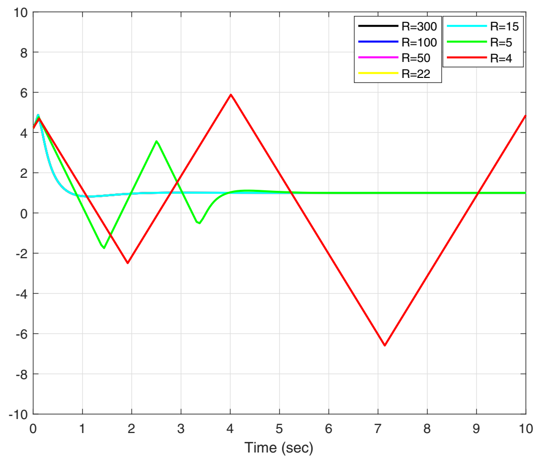

Fig. 9. FOPI control signals after rate limit effect

big oscillations. However, it still holds stable. When R is getting smaller, the actuator saturated system becomes unstable. For the FOPI controlled system, the responses are almost the same when $R \leq 15$ since the control signals are not saturated, as seen in Fig. 9. For $R = 5$, the system still holds stable while the IOPID controlled system becomes unstable. From these four figures, one can get an observation that the FOPI controlled system is more robust to the rate limit variation, and the FOPI controller potentially brings benefits to reliefing the rate limit effects.

6. CONCLUSION

This paper focuses on the analyses on the rate limit effects in precision process control, with the help of describing function method of the rate limiter. An IOPID controller and an FOPI controller are first designed for the linear system without consideration of the rate limiter, according to the gain crossover frequency, phase margin and flat phase specifications. The onset frequency is then used to analyze the effects of a rate limiter. It is found that the rate limiter will generate the amplitude reduction and phase lag when it is saturated, and increases the possibility of system instability. An illustrative example is finally operated from the view of frequency domain analysis as well as the time domain, which shows the effectiveness of the FOPI controller for the rate limit effect in the process control.

REFERENCES

Ackermann, J. and Bunte, T. (1997). Actuator rate limits in robust car steering control. In *Proceedings of the 36th IEEE Conference on Decision and Control*, volume 5, 4726–4731.

Alstrom, R.B., Bollt, E., Marzocca, P., and Ahmadi, G. (2012). The application of nonlinear pre-filters to prevent aeroservoelastic interactions due to actuator rate limiting. In *Proceedings of the 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*.

Amato, F., Iervolino, R., Pandit, M., Scala, S., and Verde, L. (2000). Analysis of pilot-in-the-loop oscillations due to position and rate saturations. In *Proceedings of the 39th IEEE Conference on Decision and Control*, volume 4, 3564–3569.

Atherton, D.P. (2007). Feedback. *IEEE Control Systems*, 27(4), 17–18. `http://ieeexplore.ieee.org/document/4272322/`.

Duda, H. (1994). Frequency domain analysis of rate limiting elements in flight control systems. *Forschungsbericht- Deutsche Forschungsanstalt fuer Luft- und Raumfahrt, DLR-FB*, (94), 40.

Gilbreath, G.P. (2001). Prediction of pilot-induced oscillations (PIO) due to actuator rate limiting using the open-loop onset point (OLOP) criterion. Technical report, Air Force Inst of Tech Wright-Pattersonafb Oh.

Hanke, D. (1994). Handling qualities analysis on rate limiting elements in flight control systems. In *Flight Vehicle Integration Panel Workshop on Pilot Induced Oscillations*. AGARD-AR-335.

Kapila, V. and Grigoriadis, K. (2002). *Actuator saturation control*. CRC Press.

Kapila, V. and Haddad, W.M. (1998). Actuator amplitude saturation control for systems with exogenous disturbances. In *Proceedings of the 1998 American Control Conference*, volume 3, 1468–1472.

Katayanagi, R. (2001). Pilot-induced oscillation analysis with actuator rate limiting and feedback control loop. *Transactions of the Japan Society for Aeronautical and Space Sciences*, 44(143), 48–53.

Kochenburger, R.J. (1950). A frequency response method for analyzing and synthesizing contactor servomechanisms. *Transactions of the American Institute of Electrical Engineers*, 69(1), 270–284. doi:10.1109/T-AIEE.1950.5060149.

Li, H.S., Luo, Y., and Chen, Y.Q. (2010). A fractional order proportional and derivative (FOPD) motion controller: tuning rule and experiments. *IEEE Transactions on control systems technology*, 18(2), 516–520.

Luo, Y. and Chen, Y.Q. (2012). Stabilizing and robust fractional order PI controller synthesis for first order plus time delay systems. *Automatica*, 48(9), 2159–2167.

Luo, Y., Chen, Y.Q., Wang, C.Y., and Pi, Y.G. (2010). Tuning fractional order proportional integral controllers for fractional order systems. *Journal of Process Control*, 20(7), 823–831.

Meng, J., Xu, H., and Zhang, J. (2010). A comparison of rate-limit phase compensator to prevent category ii pilot induced oscillations. In *Proceedings of the 8th World Congress on Intelligent Control and Automation*, 3872–3877.

Monje, C.A., Chen, Y.Q., Vinagre, B.M., Xue, D.Y., and Feliu-Batlle, V. (2010). *Fractional-order systems and controls: fundamentals and applications*. Springer Science & Business Media.

Nguyen, T. and Jabbari, F. (2000). Output feedback controllers for disturbance attenuation with actuator amplitude and rate saturation. *Automatica*, 36(9), 1339–1346.

Podlubny, I. (1998). *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*, volume 198. Academic press.

Sun, W., Gao, H., and Kaynak, O. (2015). Vibration isolation for active suspensions with performance constraints and actuator saturation. *IEEE/ASME Transactions on Mechatronics*, 20(2), 675–683. doi:10.1109/TMECH.2014.2319355.

Xue, D.Y. and Chen, Y.Q. (2002). A comparative introduction of four fractional order controllers. In *Proceedings of the 4th World Congress on Intelligent Control and Automation*, volume 4, 3228–3235.