Tuning of Fractional Order $PI^{\lambda}D^{\mu}$ Controllers using Evolutionary Optimization for PID Tuned Synchronous Generator Excitation System

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Abstract: This paper has propounded the notion of the design of cascaded integer order (IO) PID - fractional order (FO) $PI^{\lambda}D^{\mu}$ controller by evolutionary multi-objective based optimization approach for a synchronous generator excitation system. The three contradicting performance indices have been framed in time domain as well as in frequency domain to minimize error, escalate the robust stability and to minimize the energy consumption. This paper propounded the issue of contradiction in minimizing error, escalation of robust stability and minimization of energy consumption by framing cascaded IO PID - FO PID controllers as multi-objective optimization problem. The optimization problem is solved to generate the design parameter that meets the competitive multi-objective specifications relating to performance, robust stability and to optimal control by making trade-off between them and respective weightage given to each objective function. The solution generates the non-dominated set of Pareto-optimal solutions and allows the designer to select a particular controller configuration with respective weightages. With the application of this proposed design to the excitation system of synchronous generator to a power plant's, the dynamic robust stability enhanced explicitly with minimum energy consumption.

Keywords: Fractional Order Calculus, Multiobjective Optimization, PID Controller, Pareto Front, Sinchronous Generator Excitation System.

1. INTRODUCTION

The work is propounded the notion of design of cascaded IO PID-FO PID controller for excitation system of synchronous generator used for power generation. To maintain the terminal voltage and stability in operation for a synchronous generator used in power generation is the basic need of safe and economic power system operation. Fluctuation voltage can be reduced by the excitation control system of the synchronous generator also balancing of inactive power distribution, anti-interference expansion and sturdiness operation is improved by excitation control system of the synchronous generator. PID controller is the most common controller for the control purpose of the excitation system of the synchronous generator. However, contemporarily the design of FOPID is new-fangled and pronounced. The strong adaptability, robustness achievements, feasible operation and convenient debugging of fractional order FOPID controller makes it applicable substantially for precise control. Numerous design techniques and algorithms for FOPID controllers have been reported in the history. An improved differential evolution algorithm for utter parameter optimization has been proposed for FO*PID* controller design in [1]. I. Podlubny has proposed the notion of FOPID controllers and demonstrated the efficaciousness of those controllers for triggering the responses of FO systems in [2]. Tuning of the proportionality constant, integral parameter, derivative parameter, the order of integral controller and order of differential controller is a complicated process as compared to IO conventional PID controller. Tuning of parameters of $PI^{\lambda}D^{\mu}$ controllers has been done for Magnetic Bearing System of solid core with the application of numerical search method and the commendatory dynamic performance has been achieved as compared to IO PID controllers [3]. Loop Shaping Trade-offs proposed for FOPID control with the help of multi-objective optimization for AVR system in [4], in which the objective function formulation is done in the frequency domain and fuzzy logic technique has been used to find the best solution from the set of non-dominated solution. Control system for trajectory tracking has been propounded for 3- DOF parallel robotic manipulator in [5] the $PI^{\lambda}D^{\mu}$ controller has been designed to achieve the desired performance. PI controller design is proposed as multi-objective optimization problem for parameter adjustment in [6]. In [7] an IO PID controller design has been proposed for controlling purpose of excitation system of the synchronous generator using a genetic algorithm. The present research work is motivated by the ideas proposed in [7], [13] and [14], where in later case cascaded FO PI controller is designed using internal model control approach in frequency domain and the design is extended for cascaded IOPID-FOPID controller with the multi-objective contradicting problem formulation for precise control than other existing methods. Fractional order calculus gains impetus for research in these days and this concept is used for the dealing of the fractional order controller design. In [15] fractional control theory is bestowed for MATLAB simulation. The remaining parts of the paper are organized as follows: the next section focuses on the system modelling and IO PID stability constraints, design of cascaded IOPID-FOPID, multiobjective optimization and fractional order calculus has been presented in Section 3, Section 4 deals with the simulation results of designed controllers and section 5 is concluded the paper.

2. SYSTEM DESCRIPTION AND STABILITY CONSTRAINTS FOR IO PID CONTROLLER

2.1 Mathematical Model of the System

For the mathematical modelling of the system, the block diagram of feedback system shown in figure 1 is considered. Generator's transfer function is considered for the running condition of the generator at no load and current the stator winding current is assumed to be zero. Saturation effect has been ignored and the maximum voltage is assumed near to the rated voltage. Since only excitation voltage V_f (s) and generator voltage V_G (s) has been considered so transfer function derived from its no-load characteristics is given as follows:

$$G(s) = V_{G0}(s) \big/ V_f(s) = K_G \big/ 1 + T_{d0}^{'} s \tag{1}$$

Where $V_{G0}(s)$ is the no-load voltage of generator, $K_G = V_{G0} \big/ R_f i_f$, $T_{d0}^{'} = L_f \big/ R_f i_f$, i_f , R_f and L_f are the excitation current, resistance and field winding reactance respectively. Voltage measurement unit is defined as first order lag element and the transfer function is taken as

$$G_e(s) = K_e \big/ 1 + T_e s \tag{2}$$

The transfer function of the power amplifier module is taken as follows:

$$G_f(s) = u_f \big/ u_c = K_f \left(1 - T_f s \big/ 2\right) \Big/ \left(1 + T_f s \big/ 2\right) \tag{3}$$

Where, $K_f = 1.35 * \left[K_a \big/ K_{tb} \right]$, $K_a = \left[u_t \big/ V_G \right]$ and $K_{tb} = \left[u_{tb} \big/ V_G \right]$, where u_f is the output voltage and u_p is the control signal, u_t is effective secondary voltage of excitation transformer and u_{tb} is the synchronous voltage peak value procured by the synchronous transformer.

2.2 Stability Constraints with IO PID Controller

The structure of IO PID has been taken as given in equation (11), where Kp, Kd and KI are proportional, derivative and integral constants and a large value of N will be feasible.

Stability constraints have been determined from the Routh-Hurwitz stability according to which necessary and sufficient condition for a system to be stable is that each and every element of the first column of Routh array of its characteristic equation should be positive. Now the characteristic function of the system with IO PID controller is given as below:

$$a_1 s^5 + a_2 s^4 + a_3 s^3 + a_4 s^2 + a_5 s^1 + a_6 = 0 \tag{4}$$

Equation (4) represents the characteristic equation of the system with IO PID controller where a_1, a_2, a_3, a_4, a_5, a_6 are the function of K_G, K_e, K_f, T_e, T_f, $T_{d0}^{'}$.

Now the Routh array can be written as

$$\begin{array}{c|ccc}
s^5 & a_1 & a_3 & a_5 \\
s^4 & a_2 & a_4 & a_6 \\
s^3 & \dfrac{a_2 a_3 - a_1 a_4}{a_2} & \dfrac{a_2 a_5 - a_1 a_6}{a_2} & \\
s^2 & a_4 - \left\{ \dfrac{(a_2 a_5 - a_1 a_6) a_2}{a_2 a_3 - a_1 a_4} \right\} & a_6 & \\
s^1 & \left[\dfrac{a_2 a_5 - a_1 a_6}{a_2} \right] - \left[\dfrac{\left(\dfrac{a_2 a_3 - a_1 a_4}{a_2} \right) a_6}{a_4 - \left\{ \dfrac{(a_2 a_5 - a_1 a_6) a_2}{a_2 a_3 - a_1 a_4} \right\}} \right] & & \\
s^0 & a_6 & &
\end{array}$$

Now according to Routh the systems will be stable if all the elements of the first column must have positive sign:

Hence, $a_1 > 0$ (5)

$$a_2 > 0 \tag{6}$$

$$\frac{a_2 a_3 - a_1 a_4}{a_2} > 0 \tag{7}$$

$$a_4 - \left\{ \frac{(a_2 a_5 - a_1 a_6) a_2}{a_2 a_3 - a_1 a_4} \right\} > 0 \tag{8}$$

$$\left[\frac{a_2 a_5 - a_1 a_6}{a_2} \right] - \left[\frac{\left(\dfrac{a_2 a_3 - a_1 a_4}{a_2} \right) a_6}{a_4 - \left\{ \dfrac{(a_2 a_5 - a_1 a_6) a_2}{a_2 a_3 - a_1 a_4} \right\}} \right] > 0 \tag{9}$$

$$a_6 > 0 \tag{10}$$

Where $a_1 = f(K_d, K_i, T_e, T_f, T_{d0}^{'})$,

$a_2 = f(K_d, K_i, T_e, T_f, T_{d0}^{'}, N)$,

$a_3 = f(K_p, K_d, K_i, T_e, T_f, T_{d0}^{'}, N, K_G, K_f, K_e)$,

$a_4 = f(K_p, K_d, K_i, T_e, T_f, T_{d0}^{'}, N, K_G, K_f, K_e)$,

$a_5 = f(K_p, K_d, K_i, T_f, N, K_G, K_f, K_e)$,

$a_6 = f(K_p, N, K_G, K_f, K_e)$

$$PID(s) = K_p \left(1 + \frac{1}{K_I s} + \frac{K_d s}{1 + \dfrac{K_d s}{N}} \right) \tag{11}$$

3. CASCADED IO PID - FO PID DESIGN USING MOO

Fractional order dynamical systems and controllers gained attention for research in recent years. It is based on the fractional-order calculus. In fractional-order PID controllers the order of integral and derivative controllers are usually is in fractions. So in fractional-order controllers, we have two more parameters to tune in addition to normal IO PID controllers: fractional order of integration λ and of derivative μ. The performance of the fractional order $PI^{\lambda}D^{\mu}$ for the excitation systems control of the synchronous the generator along with IO PID controller is anticipated to be palatable and better than IO PID controller acted alone. However, number of parameters in cascaded structure would be more compared to individual tunings of IO PID and FO PID structure. The main drawback of the cascaded design is that the system becomes very complex as compared to individual IO PID controller or FO PID controllers. But due to fractional part of the integral and derivative controllers in $PI^{\lambda}D^{\mu}$ precise control of the system can be possible and if cascaded IO PID-FO PID will be used then both integer and fractional order control will make the system stability, adaptability, debugging capability, robustness and feasibility of operation more precise and less erroneous output can be achieved in this case. So if there is a requirement to provide longer delay or large load change then cascaded control can be used for better control.

Further in this section cascaded design of IO PID – FO PID controller is propounded as the multi-objective problem with the unknown parameters of the system. The problem formulation has been done in the time domain as well as in frequency domain. The design parameters of the multiobjective problem are the K_p, K_I, K_d, Z_p, Z_I, Z_d, λ, μ, parameters of the excitation system, voltage measurement unit and power amplifier module. The transfer function for the FO PID has been taken as

$$PI^{\lambda}D^{\mu}(s) = Z_p\left[1+\frac{1}{Z_I s^{\lambda}}+\frac{Z_d s^{\mu}}{\tau_d s+1}\right] \tag{12}$$

By substituting $s=j\omega$ frequency domain representation of the $PI^{\lambda}D^{\mu}$ can be represented as given below:

$$PI^{\lambda}D^{\mu}(j\omega) = Z_p\left[1+\frac{1}{Z_I(j\omega)^{\lambda}}+\frac{Z_d(j\omega)^{\mu}}{\tau_d(j\omega)+1}\right] \tag{13}$$

Hence convenient form is given as

$(j\omega)^{-\lambda} = \omega^{-\lambda}(\cos\frac{\lambda\pi}{2} - j\sin\frac{\lambda\pi}{2})$ and

$(j\omega)^{\mu} = \omega^{\mu}(\cos\frac{\mu\pi}{2} + j\sin\frac{\mu\pi}{2})$, hence,

$$PI^{\lambda}D^{\mu}(j\omega) = Z_p\left[1+\frac{\omega^{-\lambda}(\cos\frac{\lambda\pi}{2}-j\sin\frac{\lambda\pi}{2})}{Z_I}+\frac{Z_d\omega^{\mu}(\cos\frac{\mu\pi}{2}+j\sin\frac{\mu\pi}{2})}{\tau_d(j\omega)+1}\right] \tag{14}$$

Now, $[\tau_d(j\omega)+1] = \left\lfloor\sqrt{(\tau_d\omega)^2+1}\right\rfloor\angle\tan^{-1}(\tau_d\omega)$ then

$$PI^{\lambda}D^{\mu}(j\omega) = Z_p\left[1+\frac{\omega^{-\lambda}(\cos\frac{\lambda\pi}{2}-j\sin\frac{\lambda\pi}{2})}{Z_I}+\frac{Z_d\omega^{\mu}(\cos\frac{\mu\pi}{2}+j\sin\frac{\mu\pi}{2})}{(\sqrt{(\tau_d\omega)^2+1})\angle\tan^{-1}(\tau_d\omega)}\right] \tag{15}$$

Or,

$$PI^{\lambda}D^{\mu}(j\omega) = Z_p\left[1+\frac{\omega^{-\lambda}(\cos\frac{\lambda\pi}{2}-j\sin\frac{\lambda\pi}{2})}{Z_I}+\frac{Z_d\omega^{\mu}(\cos\frac{\mu\pi}{2}+j\sin\frac{\mu\pi}{2})\angle-\tan^{-1}(\tau_d\omega)}{(\sqrt{(\tau_d\omega)^2+1})}\right] \tag{16}$$

The cascaded PID- $PI^{\lambda}D^{\mu}$ controller structure is shown in figure 2. The closed loop transfer function for inner loop and outer loops are obtained as given below:

$$\frac{Y(s)}{R_2(s)} = \frac{PID(s)G_f(s)G(s)}{1+PID(s)G_f(s)G(s)} \tag{17}$$

$$\frac{Y(s)}{R_2(s)} = \frac{PI^{\lambda}D^{\mu}(s)PID(s)G_f(s)G(s)}{1+PID(s)G_f(s)G(s)\left[1+PI^{\lambda}D^{\mu}(s)G_e(s)\right]} \tag{18}$$

In the above control system structure inner loop control is designed to regulate the disturbance present in the loop and outer loop control is designed to achieve set-point tracking that provides a stable over damped response.

3.1 Evolutionary Optimization of Multi-Objective Function

In a multi-objective optimization problem, the objective functions are optimized in isolation of one another and the palatable solution is based on the individual solutions attained for each objective. In multi-objective optimization, there is no any concept of optimal solution. The solution of these problems is multiple solutions, each of which is palatable based on the need and relative significance of the individual objective functions. The multi-objective optimization problem can be stated as [8]: determination of the vector $p^* = \left[p_1^*, p_2^*, \ldots\ldots\ldots, p_{n1}^*\right]$ that minimizes the vector valued objective function $f(p)$ given as following:

$$f(p) = \left[f_1(p), f_2(p), \ldots\ldots\ldots\ldots, f_j(p)\right]$$

consists of non-identical individual objectives, subjected to a set of constraints represented by equations (5)-(10). The main challenge in evolutionary multi-objective optimization lies in defining the optimal solution because a single vector p^* rarely represents the optimal solution of all the objective functions. In this regard, the Pareto optimal solution concept, which proffers a set of multiple non-dominated solutions, is frequently used. In the context of a minimization problem, a vector p^* can be considered as a Pareto optimal solution, if there exists no p within the feasible region satisfying all the constraints such that:

$f_i(p) \le f_i(p^*); \forall i \in \{1,2,\ldots\ldots\ldots, j\}$ and

$f_q(p) \le f_q(p^*); \exists q \in \{1,2,\ldots\ldots\ldots, j\}$

The equations described above implies that for a Pareto optimal solution there exists no feasible vector which causes a minimization in a certain objective function without simultaneous maximization in any of the remaining objectives. As there is no any single solution p^*, that is better than other vectors p in terms of all the objectives, multi-objective optimization techniques search for a set of dominated solutions in the feasible space, commonly referred to as Pareto set. The projection of the Pareto set in the objective space in known as the Pareto front. Several evolutionary algorithms have been developed in the history for the solution of multi-objective optimization problem based on the iterative generation of the Pareto front. Among them, techniques based on GA (genetic algorithm) have been successfully applied in many engineering applications. The popular technique among them includes MOGA [9], NSGA [10], VEGA [11] and NSGA-II [12]. In this section, NSGA-II technique has been incorporated pertaining to the system identification and design of IO PID controllers. Following random initialization of a population of size M, the iterative steps in NSGA-II for obtaining the Pareto front are detailed below;

A combined population of the parent and offspring population is sorted to meet the concept of non-domination. Elitism is ensured by contemplating all the individuals of the previous and present population. All the individuals from the dominated set have been considered in the next population. Since the population size *N* is greater than the number of individuals in the dominated set, the remaining elements of the successive population are considered from the non-dominated fronts according to their rank criteria. The population slots from the fronts other than the dominated set are obtained using the crowding distance comparison operator. The basic idea behind the crowding distance scheme is to determine the perimeter of the rectangle in the parameter space, where the nearest neighbours of a particular individual are at diagonally opposite vertices [8]. The larger crowding distance is considered for individuals having the same rank. The new population of size *M* undergoes the operations of selection, crossover, and mutation to create the new offspring population. The tournament selection along with crowding distance operator is utilized to the select the individuals. The crowding comparison operation prevents pre-mature convergence by maintaining diversity among the non-dominated solutions. Simulated binary crossover is used to generate a children string from two randomly selected parent strings from *N* [11]. Finally, during mutation, the children strings are altered using polynomial mutation based on the deviation about the upper and lower limits of the parent components.

3.2 Problem Formulation and Optimization

The multi-objective optimization problem for cascaded IO PID - FO PID design is formulated both in the time domain as well as in frequency domain. In time domain ISE (integral of the squared error) performance index has been considered for minimizing the squared error. Performance index for ISE is

$$J_1 = f_1(x) = \int_0^\infty [e(t)]^2\, dt \tag{19}$$

and $e(t) = |y_d(t) - y_a(t)|$

where, $y_d(t)$ is desired output response with and $y_a(t)$ is actual output response of the systems with controller.

The second and most important goal attainment is to impart robust stability i.e. the closed-loop controlled system will remain stable even if existence of parametric uncertainty is there. However, if the uncertainty exists then infinity-norm of the sensitivity function is formulated to escalate the margin of robust stability. The sensitivity function has been defined as $S(s) = 1/(1 + C(s)B(s))$, where C(s) is controller and B(s) is the forward transfer function of the system. The return difference function is the vectorial length in the Nyquist plot from -1 to the open loop transfer function and is given by the equation $|-1 - C(s)G(s)| = |1 + C(s)G(s)|$. The infinity norm $\|S\|_\infty = \min(\omega)\left[1/|1 + C(j\omega)G(j\omega)|\right]$ transfer function must be minimized to escalate the robust stability. Hence,

$$J_2 = f_2(x) = \min(\omega)\left[1/|1 + C(j\omega)G(j\omega)|\right] \tag{20}$$

The third performance index formulation is ∞-norm of optimal control effort as a function of sensitivity to minimize the energy consumption and is formulated as ∞-norm of singular values of $U(s) = C(s)S(s)R(s)$ as given by following function:

$$J_3 = f_3(x) = \sum_{j=1}^{q} \sum_{i=1}^{p} \left[\sigma_{U(s)}(i, \omega_j)\right]^2 \tag{21}$$

where *p=min(m,n)*; *m* and *n* are the number of inputs and outputs of the MIMO system, and $\sigma_{C(s)S(s)R(s)}(i, \omega_j)$ represents the largest singular values of the $C(s)S(s)R(s)$ matrix that guarantee the minimum energy consumption over the frequency range $\omega_1 \le \omega_j \le \omega_q$.

Now the vector valued function can be written as following

$$\vec{J} = f(.) = [f_1(x)\ \ f_2(x)\ \ f_3(x)] \tag{22}$$

The vector valued function is now solved with the help of multiobjective optimization algorithm NSGA-II as described in the previous section with $\tau_d = 0$. The population size has been taken as 350, number of generations is 1500, the Pareto fraction has been taken as 0.35, cross over fraction is 1.0 and mutation fraction is 0.2.

3.3 Fractional Order Calculus

Fractional order calculus gained impetus in the last few decades and becomes ubiquitous for precise control of systems. It is generalization of differentiation and integration operations to the fractional order operator ${}_aD^\alpha{}_t$, where α is fractional order and a and *t* are lower and upper terminals of the operations respectively. Now integro-differential can be expressed as

$${}_aD_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt^\alpha} & \Re(\alpha) > 1 \\ 1 & \Re(\alpha) = 0 \\ \int_a^t (dt)^{-\alpha} & \Re(\alpha) < 0 \end{cases} \tag{23}$$

Where $\alpha \in \Re$ in general but it could be imaginary also. In general there are two definitions of fractional order differetio-integral calculus is in use widely. The Grunwald-Letnikov definition and Riemann-Liouville definition [2], [16] is given below,

$${}_aD_t^\alpha = \lim_{h \to 0} h^{-\alpha} \sum_{i=0}^{\left[\frac{t-a}{h}\right]} (-1)^i \binom{\alpha}{i} f(t - hi) \tag{24}$$

where [.] represents integer value and

$${}_aD_t^\alpha = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\, d\tau \tag{25}$$

For $(n - 1 < \alpha < n)$ *and* $\Gamma(.)$ is the gamma function. The Laplace transformation is used routinely to solve the integral and differential equations of engineering problems. The description of FO PID controller has been done on the basis of the above presented concepts of fractional order calculus.

The complex function concept was used to describe frequency domain concepts.

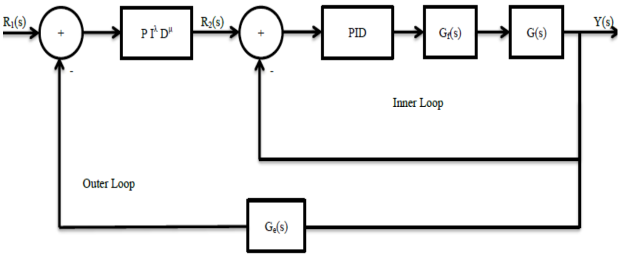

Fig.1. Block diagram of PID- $PI^{\lambda}D^{\mu}$ cascaded system

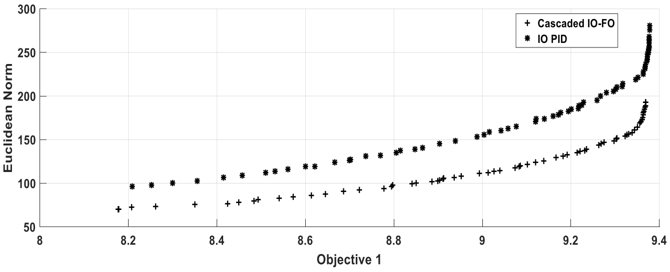

Fig. 2 Pareto Optimal Solution for $f_1(x)$

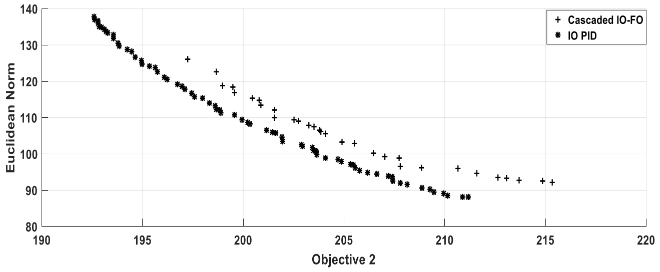

Fig. 3 Pareto Optimal Solution for $f_2(x)$

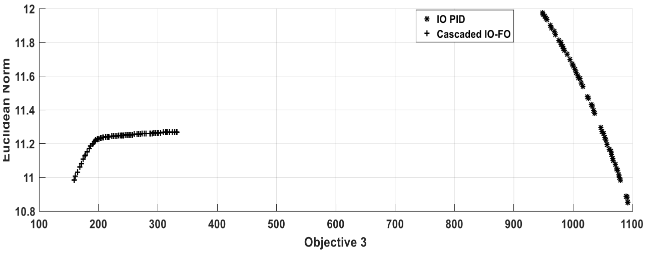

Fig. 4 Pareto Optimal Solution for $f_3(x)$

4. RESULTS AND DISCUSSIONS

The Pareto optimal solutions obtained after the simulation of multiobjective optimization problem as described previously in the present work. The NSGA-II algorithm produced non-dominate solutions. Therefore, the designer has to make some trade-offs in order to choose the best solution which satisfies all the objective functions and constraints simultaneously. The Pareto front has been represented in the figures 2,3 and 4 in which all the normalized objective functions in the form of Euclidean norm is put on the Y axis and each individual objective function is specified on the X axis; therefore, each performance index has its own representation on the graph and Y axis performance index would be the same. Figure 2 represents good ISE while control effort and robustness is worse. Similarly if the systems has minimum value of J_2 then good robustness is achieved whilst worse control effort and ISE is there and is represented by figure 3. If J_3 is minimum then optimal control effort is achieved with worse effect of ISE and robustness which has been represented by figure 4.

The time response of the best controller after making trade-off with cascaded FO PID-IO PID is shown in figures 5, 6 and 7 respectively. It is seen from the responses that the cascaded PID- $PI^{\lambda}D^{\mu}$ controller is performed better as compared to integer-order PID controller alone in almost all the cases.

The settling time for the IO PID controller designed alone with MOO for the synchronous generator excitation system are 9.65sec, 13sec and 4.13sec respectively and for the cascaded IO PID-FO PID controller structure it is 5.73sec, 11sec and 3.86sec respectively. From settling time data we can say that controlling of synchronous generator excitation system with cascaded IO PID and FO PID controller settles down the system expeditiously as compared to integer-order PID controller alone.

The frequency domain responses (bode plots) are also shown in figures 8, 9 and 10 respectively. The gain margin and phase margin for the integer-order controlled system are (6.86dB, 93.8^0), (6.88dB, inf) and (12.076dB, inf) respectively and for cascaded *PID-* $PI^{\lambda}D^{\mu}$ controlled system the gain margin and the phase margin are (10.001dB, 60.984^0), (6.82, 86.9761^0) and (12.762dB, 60^0) respectively. From the frequency domain data it can be said again that the stability extent of the cascaded controlled synchronous generator excitation system is more than that of integer-order controlled synchronous generator excitation system. Therefore, it would be demonstrated that for improving stability margin and bandwidth of closed-loop system significantly for the low-frequency disturbance and noise rejection FO PID controller is advantageous that IO PID controller for synchronous generator excitation system.

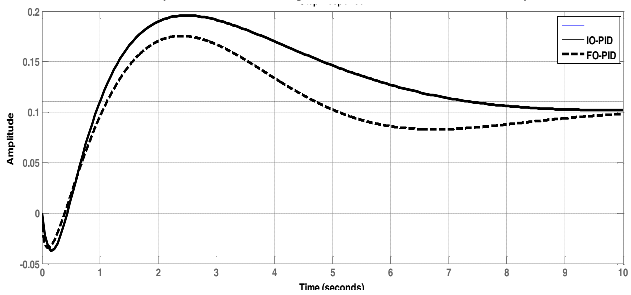

Fig. 5 Output response y(t) of the system after trade-off made for $f_1(x)$

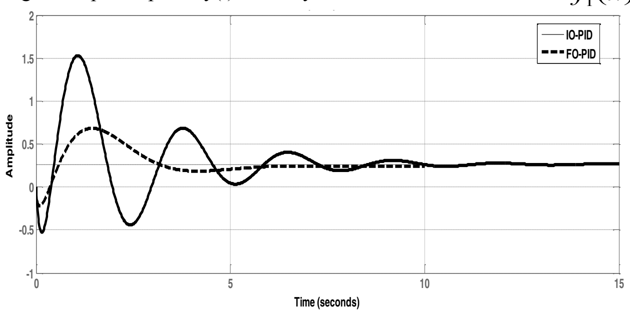

Fig. 6 Output response y(t) of the system after trade-off made for $f_2(x)$

The optimized values of design parameters are found to be

$$K_p = (0.255, 0.189, 0.199), \mathrm{K_I} = (4.395, 3.961, 4.207),$$
$$\mathrm{K_d} = (1.080, 1.103, 1.112), \mathrm{N} = (4.513, 4.457, 4.469),$$
$$\mathrm{K_f} = (1.764, 1.2541.037), \mathrm{T_f} = (0.670, 0.688, 0.759)$$
$$K_G = (2.006, 1.594, 1.644), \mathrm{T'_{d0}} = (0.568, 3.913, 0.557)$$
$$K_e = (3.797, 9.094, 4.349), \mathrm{T_e} = (3.246, 2.716, 3.145)$$
$$Z_p = (0.008, 1.563\mathrm{x}10^{-6}, 0.0035), \mathrm{Z_i} = (0.125, 0.0674, 0.218)$$

$\lambda = (0.501, 0.502, 0.501), Z_d = (0.163, 0.284, 0.258)$
$\mu = (0.497, 0.502, 0.499)$

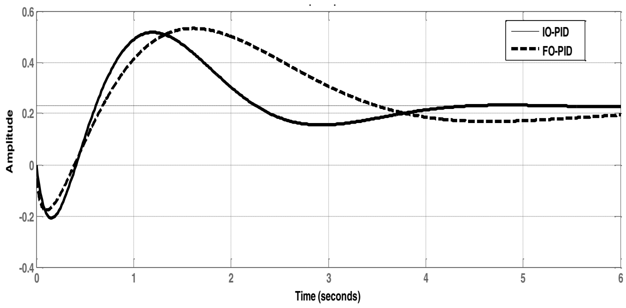

Fig. 7 Output response y(t) of the system after trade-off made for $f_3(x)$

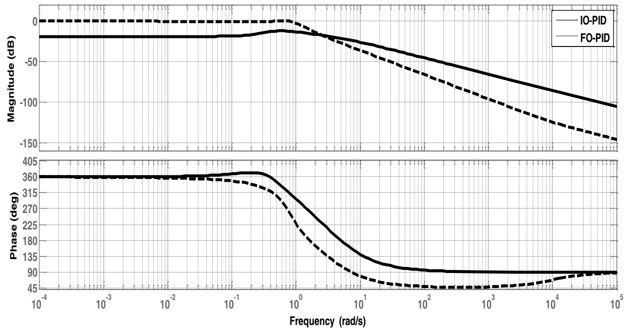

Fig. 8 Frequency plot after trade-off for best solution of $f_1(x)$

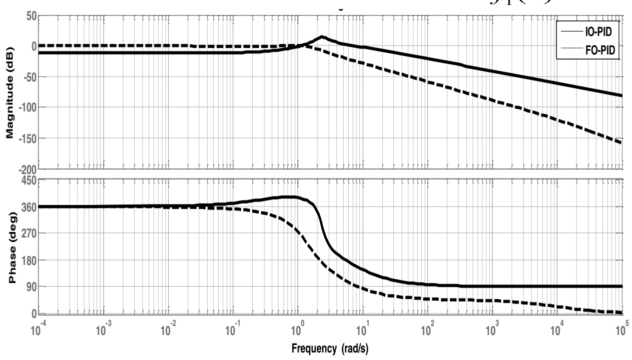

Fig. 9 Frequency response after trade-off for best solution of $f_2(x)$

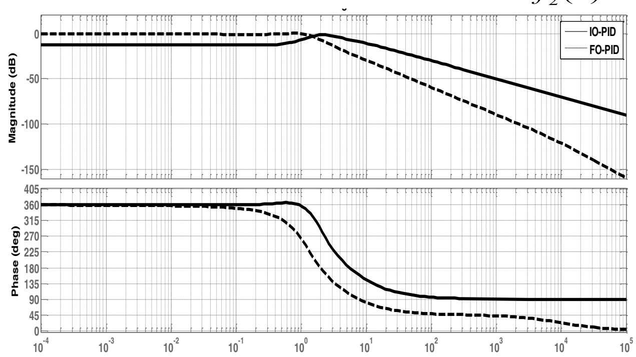

Fig. 10 Frequency response after trade-off for best solution of $f_3(x)$

5. CONCLUSION

This research paper bestows the advantage of cascaded IO PID-FO PID controller structure over IO PID controller when acted alone for a synchronous generator excitation system. The problem has been formulated as multiobjective problem with contradicting objectives for the achievement of ISE, robust stability and optimal control effort. The problem has been solved for each individual objective functions with respect to Euclidean norm of all the three objective functions. The value of fractional order operators are taken as λ<1 and μ<1. From the time response specification and frequency response specification of some best non-dominated Pareto solution the conclusion has drawn that fractional-order controller in cascade with integer-order controller is fast and more stable as compared to integer-order controller when acted alone. However, the cascaded controller system is more complex than integer order controller system for practical implementation due to cascaded structure of $PID - PI^{\lambda}D^{\mu}$ but the improvement in the system performance and stability confirms the usefulness of cascaded structure of IO PID-FO PID controller.

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