Nonlinear Controllers in the Regulation Problem of the Robots ⋆

Aleksandr S. Andreev * **Olga A. Peregudova** **

* *Ulyanovsk State University, Ulyanovsk, Russia (e-mail: asa5208@mail.ru).*
** *Ulyanovsk State University, Ulyanovsk, Russia (e-mail: peregudovaoa@gmail.com)*

Abstract: The report presents the results on the application of nonlinear PI and PID regulators in the position stabilization problem of the robots. As an example the position stabilization problem of a three-wheeled mobile robot with a displaced mass center is considered.

Keywords: Mechanical system, motion control, regulation, PI regulator, robot manipulator, wheeled mobile robot.

1. INTRODUCTION

As has been shown by Astrom and Hagglund (2006) and O'Dwyer (2009), the proportional integral (PI) and proportional integral differential (PID) regulators have been applied in 90–95 % of the control circuits in the technological processes. Mathematical and applied researches on the application of such regulators were conducted during the last 50 years. The complexity of the problem is related to the non-linearity of the controlled system as well as the boundedness of the controller, changes in system parameters over time, incompleteness of controller and measurement of system coordinates, and other conditions.

There have been numerous studies on application of PID controllers in the motion control of robots. It has been shown by Santibanez et al. (2010); Meza et al. (2011) that PID regulators have certain advantages over PD regulators in the accuracy, efficiency and quality of the transient process. The integral component avoids the need to include the gravitational torques in the compensator of the controller, which is an important factor in connection with the possible uncertainty in the system parameters. For the first time this factor was mathematically marked by Arimoto and Miyazaki (1984). Various solutions to the problem of local stabilization of the manipulator position with a linear PID controller were presented by Arimoto and Miyazaki (1984); Arimoto et al. (1990); Qu and Dorsey (1991); Kelly (1995); Arimoto (1996); Rocco (1996); Kelly et al. (2005). The results presented in (Arimoto and Miyazaki, 1984) were developed by Alvarez et al. (2000) to non-local stabilization with an estimate of the region of attraction.

The problem of local stabilization was investigated in (Arimoto, 1994, 1995; Kelly, 1998; Santibanez and Kelly, 1998; Gorez, 1999; Alvarez et al., 2000; Cervantes and Alvarez-Ramirez, 2001; Alvarez et al., 2003; Jafarov et al., 2005; Kelly et al., 2005; Meza et al., 2005, 2007; Alvarez et al., 2008; Santibanez et al., 2008; Sun et al., 2009; Orrante et al., 2010; Santibanez et al., 2010; Meza et al., 2011) on the base of the inclusion of the non-linear function in the proportional with respect to the coordinates and the integral components. A nonlinear control structure based on the classical PID controller and provided the global asymptotic stability was presented in (Arimoto, 1994, 1995). The development of the form of such controller in terms of changing its structure and improving the conditions of applicability has been studied by Kelly (1998); Santibanez and Kelly (1998); Alvarez et al. (2000); Sun et al. (2009). Due to the linearity of the differential component of the regulator, the controller can not be bounded, which is not possible for drives with bounded powers. Nonlinear PID regulators with bounded components (regulators with saturation) were constructed in (Gorez, 1999; Alvarez et al., 2003; Meza et al., 2005, 2007; Alvarez et al., 2008; Santibanez et al., 2008; Orrante et al., 2010; Santibanez et al., 2010). The control parameters are found by Alvarez et al. (2003); Meza et al. (2007); Alvarez et al. (2008) in the problem of semi-global stabilization of the manipulators. The control gains are found by Gorez (1999); Meza et al. (2005); Santibanez et al. (2008); Orrante et al. (2010); Meza et al. (2011) in the problem of global stabilization. Regulators including compensators of gravitational torques were considered by Alvarez et al. (2008). Practical application of the PID regulators in the position stabilization of manipulators is presented in (Santibanez et al., 2010; Meza et al., 2011). Complex and particular solutions were obtained by Arimoto (1994); Cervantes and Alvarez-Ramirez (2001); Jafarov et al. (2005); Santibanez et al. (2010) using PID regulators in the trajectory tracking problem.

Numerous studies are devoted to the task of controlling robots without measuring the velocities. The problem is that the velocity sensors are very noisy, and the differentiation of the measured signal is approximate. The methods were obtained that consist of adding a filter by coordinates with obtaining the velocity estimates (Berghuis and Nijmeijer, 1993; Loria et al., 2000; Burkov, 2009; Andreev

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et al., 2016). The use of PI regulators is another approach to solving the control problem of robots without velocity measurement (Andreev and Peregudova, 2017, 2018).

The purpose of this report is to present the results of solving the position stabilization problem of a robot by using the nonlinear regulators with integral components.

2. PRELIMINARIES

Let R^p be p-dimensional linear real space with the norm $|x|$ $(x = (x_1, x_2, \ldots, x_p)'$. The symbol $(\cdot)'$ denotes the transpose operation. Let $h = const > 0$ be some real, C be a Banach space of continuous functions $\varphi : [-h, 0] \to R^p$ with the norm $\|\varphi\| = \sup(|\varphi(s)|, \quad -h \le s \le 0)$. Denote by C_H the space $C_H = \{\varphi \in C : \|\varphi\| < H\}$, $H = const > 0$. Let $x : R \to R^p$ be some continuous function. For all $t \in R$ denote by x_t the function $x_t : [-h, 0] \to R^p$ which is defined by the equality $x_t(s) = x(t+s)$, $-h \le s \le 0$. Denote by $\dot{x}(t)$ the right-hand derivative of the function $x(t)$. The symbol R^+ denote the positive real semi-axis $R^+ = [0, +\infty)$.

Consider the functional-differential equation with finite delay (Hale, 1977)

$$\dot{x}(t) = f(t, x_t), \qquad f(t, 0) \equiv 0 \tag{1}$$

where $f : R^+ \times C_H \to R^p$ is a continuous function which satisfies the following conditions

a) $f(t, \varphi)$ is bounded on each set $R^+ \times \bar{C}_L$ where $\bar{C}_L = \{\varphi \in C_H : \|\varphi\| \le L < H\}$, i.e. for all $(t, \varphi) \in R^+ \times \bar{C}_L$ the following inequality holds

$$|f(t, \varphi)| \le m(L) = const > 0 \tag{2}$$

b) $f(t, \varphi)$ satisfies the Lipschitz condition with respect to φ on each compact set $K \subset C_H$, i.e. for all $t \in R^+$ and $\varphi_1, \varphi_2 \in K$ the following inequality holds

$$|f(t, \varphi_2) - f(t, \varphi_1)| \le l(K)\|\varphi_2 - \varphi_1\| \tag{3}$$

c) $f(t, \varphi)$ is uniformly continuous on each set $R^+ \times K$, where $K \subset C_H$ is an arbitrary compact set, i.e. for all $\varepsilon > 0$ there exists $\delta(\varepsilon, K) > 0$ such that for all $(t_1, \varphi_1), (t_2, \varphi_2) \in R^+ \times K$ which satisfy the conditions $|t_2 - t_1| \le \delta$, $\|\varphi_2 - \varphi_1\| \le \delta$ the following inequality holds

$$|f(t_2, \varphi_2) - f(t_1, \varphi_1)| \le \varepsilon \tag{4}$$

Fact 1. Using the condition a, one can easily obtain (Hale, 1977) the uniform boundedness of solutions of (1), i.e. if $x = x(t, \alpha, \varphi)$ is a solution of (1) satisfying the initial condition $x_\alpha = \varphi$ then for all $t \ge \alpha + h$ the function x_t is such that $x_t \in \Gamma$, where Γ is the set of the family of inserted compact sets, i.e. $\Gamma = \bigcup_{n=1}^{\infty} K_n$, $K_1 \subset K_2 \subset \ldots \subset K_n \subset \ldots$.

Fact 2. Using the condition b, one can obtain (Hale, 1977) the uniqueness of solutions of (1) and also each solution $x = x(t, \alpha, \varphi)$ of (1) is defined on maximal interval $[\alpha - h, \beta)$. If $\beta < +\infty$ then $\|x_t(\alpha, \varphi)\| \to H$ for $t \to \beta$.

Fact 3. Using the condition c, one can obtain (Andreev, 2009) the precompactness of the family of translates $\Phi = \{f_\tau : f_\tau(t, \varphi) = f(\tau + t, \varphi), \quad \tau \in R^+\}$ in some space F of continuous functions defined on the set $R \times \Gamma$.

From Fact 3 it follows that for each sequence $t_n \to \infty$ one can find the subsequence $t_{n_k} \to +\infty$ such that the sequence of functions $f_k(t, \varphi) = f(t_{n_k} + t, \varphi)$ converges to some function $f^*(t, \varphi) \in F$ (Andreev, 2009).

Denote by Φ^* the family of limiting functions $f^*(t, \varphi) \in F$. Note that if $f^* \in \Phi^*$ then $f^*(\tau + t, \varphi) = f^*_\tau \in \Phi^*$ for every $\tau \in R$.

The equation

$$\dot{x}(t) = f^*(t, x_t), \qquad f^* \in \Phi^* \tag{5}$$

is called limiting with respect to (1) (Andreev, 2009).

Note that the function $f^* \in \Phi^*$ satisfies the condition b. Therefore, for each initial point $(\alpha, \varphi) \in R \times \Gamma$ the solution $x = x^*(t, \alpha, \varphi)$ of equation (5) is unique. Since $f^*_\alpha \in \Phi^*$ for all $\alpha \in R$, we can define the solutions of (5) for zero initial instant $\alpha = 0$, i.e. $x = x^*(t, \varphi) = x^*(t, 0, \varphi)$.

Let $x = x(t, \alpha, \varphi)$ be a solution of (1) defined for all $t \ge \alpha - h$. Let also $\omega^+(\alpha, \varphi) \in C$ be a positive limit set of solution $x = x(t, \alpha, \varphi)$, i.e. $p \in \omega^+(\alpha, \varphi)$ if there exists the sequence $t_n \to \infty$ such that $x_t^{(n)}(\alpha, \varphi) \to p$ for $n \to \infty$, where $x_t^{(n)}(\alpha, \varphi) = x(t_n + s, \alpha, \varphi)$, $-h \le s \le 0$.

By using the limiting equations (5) we obtain the quasi-invariance property of the set $\omega^+(\alpha, \varphi)$.

Theorem 4. (Andreev, 2009) Let $x = x(t, \alpha, \varphi)$ be a bounded solution of (1) defined for all $t \ge \alpha - h$. Then, for each limit point $p \in \omega^+(\alpha, \varphi)$ there exists the limiting equation (5) such that for its solution $x^*(t, 0, p) = x^*(t, p)$ the following holds $x^*_t(p) \in \omega^+(\alpha, \varphi)$ for all $t \in R$.

Let $V : R^+ \times C_H \to R$ be a continuous functional. Let also $x = x(t, \alpha, \varphi)$ be a solution of (1). Define the upper right hand time derivative of V as follows

$$\dot{V}_{(1)}(\alpha, \varphi) = \limsup_{\tau \to 0+} \frac{1}{2}(V(\alpha + \tau, x_{\alpha+\tau}(\alpha, \varphi)) - V(\alpha, \varphi)) \tag{6}$$

Assume that the time derivative $\dot{V}_{(1)}(t, \varphi)$ of the functional $V(t, \varphi)$ satisfies the following inequality

$$\dot{V}_{(1)}(t, \varphi) \le -W(t, \varphi) \le 0 \tag{7}$$

where $W : R^+ \times C_0 \to R^+$ is a continuous functional which is bounded and uniformly continuous in $R^+ \times K$ for an arbitrary compact set $K \subset C_0$.

The family of translates $\Phi_W = \{W_\tau(t, \varphi) = W(\tau + t, \varphi)\}$ as well as the family Φ is precompact in some space F_W of the functionals $W : R \to \Gamma_0 \to R^+$, $\Gamma_0 = C_0 \bigcap \Gamma$. Therefore, one can find both the family of limiting functionals $\{W^* \in F_W : R \times \Gamma_0 \to R^+\}$ and the family of limiting pairs (f^*, W^*) (Andreev, 2009).

Theorem 5. (Andreev, 2009) Assume that for (1) one can find a continuous functional $V = V(t, \varphi)$ such that for all $(t, \varphi) \in R^+ \times C_0$ the following conditions hold

1) $a_1(|\varphi(0)|) \le V(t,\varphi) \le a_2(\|\varphi\|)$;

2) the inequality (7) is true;

3) for any limiting pair (f^*, W^*) no solution $x^*(t,\varphi)$ of the limiting system (5) can stay forever in the set $\{W^*(t,\varphi) = 0\}$, other than the zero solution $x = 0$;

Then, the zero solution $x = 0$ of (1) is uniformly asymptotically stable.

3. ON REGULATION OF ROBOT MANIPULATORS USING NONLINEAR PID CONTROLLER

The rigid-joints robot kinematic energy is given by $T(q,\dot{q}) = \dot{q}'A(q)\dot{q}/2$, where $q \in R^n$ represents the link positions, $A(q) = A'(q) > 0$ is the robot inertia matrix.

Applying the Lagrange equations we obtain the well-known model

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = Q(t,q,\dot{q}) + U \tag{8}$$

where $Q = Q(t,q,\dot{q})$ is the vector of generalized uncontrollable forces, U is the control torque.

Consider the position stabilization problem of a robot for which without loss of generality we take the following position

$$\dot{q} = 0, \quad q = 0 \tag{9}$$

Decompose the action of uncontrollable forces

$$\begin{array}{l} Q(t,q,\dot{q}) = Q_0(t) + Q_1(t,q) + Q_2(t,q,\dot{q}) \\ Q_0(t) = Q(t,0,0), \quad Q_1(t,q) = Q(t,q,0) - Q_0(t) \\ Q_2(t,q,\dot{q}) = Q(t,q,\dot{q}) - Q_1(t,q) \end{array} \tag{10}$$

Taking into account the actions of uncontrollable forces Q, choose the nonlinear PID controller as follows

$$\begin{array}{l} U = -Q_0(t) + U_1(t,q) + U_2(t,q,\dot{q}) - \\ - \displaystyle\int\limits_{t-h(t)}^{t} R(t,\nu)\dot{q}(\nu)d\nu \end{array} \tag{11}$$

where the components of the instantaneous control $U_1(t,q)$ and $U_2(t,q,\dot{q})$, the hereditarity matrix $R(t,\nu)$ and the function of hereditarity action $h(t)$ are such that the following holds

$$\begin{array}{c} U_1(t,q) + Q_1(t,q) = -\dfrac{\partial \Pi(t,q)}{\partial q}, \quad \dfrac{\partial \Pi(t,0)}{\partial q} = 0 \\ \Pi(t,0) = 0, \quad \dot{q}'(U_2(t,q,\dot{q}) + Q_2(t,q,\dot{q})) \le 0; \\ 0 \le \alpha_1\|\dot{q}\|^2 \le \dot{q}'R(t,\nu)\dot{q} \le \alpha_2\|\dot{q}\|^2 \\ \dfrac{\partial R(t,\nu)}{\partial t} = M(t)R(t,\nu), \quad R(t,t) = R_0(t) \\ \beta_0\|\dot{q}\|^2 \le \dot{q}'R_0(t)\dot{q} \le \beta_1\|\dot{q}\| \\ \dot{q}'\left(R_0(t)M(t) - M(t)R_0(t) - \dfrac{dR_0(t)}{dt}\right)\dot{q} \le -\beta_0\|\dot{q}\|^2 \\ 0 < h_0 \le h(t) \le h_1, \quad \dot{h}(t) \le 1 - \delta \end{array} \tag{12}$$

where β_0, β_1, h_0, h_1 and δ are some positive reals.

Assume that for the closed-loop system (8), (11) the conditions a, b and c hold. The limiting equations defined by the formulas (5) have the form similar to (8), (11) with the following functions

$$\begin{array}{r} Q^*(t,q,\dot{q}) = \lim\limits_{t_k\to+\infty} Q(t_k+t,q,\dot{q}) \\ \Pi^*(t,q) = \lim\limits_{t_k\to+\infty} \Pi(t_k+t,q) \\ R^*(t,\nu) = \lim\limits_{t_k\to+\infty} R(t_k+t,t_k+\nu) \end{array} \tag{13}$$

Claim 6. Assume that the potential function $\Pi = \Pi(t,q)$ is such that the following inequalities hold

$$a_1(\|q\|) \le \Pi(t,q) \le a_2(\|q\|) \tag{14}$$

$$\frac{\partial \Pi(t,q)}{\partial t} \le 0, \quad \left\|\frac{\partial \Pi(t,q)}{\partial q}\right\| \ge \delta(\varepsilon) \tag{15}$$

$$\forall q \in \{0 < \varepsilon \le \|q\| \le \Delta\} \tag{16}$$

and the conditions (12) are valid.

Then, the controller (11) solves the stabilization problem of the zero position (9) of (8) providing its uniform asymptotic stability.

Proof. Consider the Lyapunov functional V

$$\begin{array}{c} V = \dfrac{1}{2}\dot{q}'A(q)\dot{q} + \Pi(t,q) + \\ +\dfrac{1}{2}\left(\displaystyle\int\limits_{t-h}^{t} R(t,\nu)\dot{q}(\nu)d\nu\right)' R_0^{-1}(t)\left(\displaystyle\int\limits_{t-h}^{t} R(t,\nu)\dot{q}(\nu)d\nu\right) \end{array} \tag{17}$$

Using the conditions 1 and 3, one can find the time derivative of the Lyapunov functional V

$$\dot{V} \le -\frac{\beta_0}{2\beta_1^2}\left\|\int\limits_0^t R(t,\nu)\dot{q}(\nu)d\nu\right\|^2 = -\frac{\beta_0}{2\beta_1^2}W(t,\dot{q}_t) \le 0 \tag{18}$$

The limiting to $W(t,\dot{q}_t)$ functional is as follows

$$W^*(t,q_t) = \left\|\int\limits_{t-h}^{t} R^*(t,\nu)\dot{q}(\nu)d\nu\right\|^2 \tag{19}$$

On can easily obtain that $\{W^*(t,q_t) = 0\} = \{\dot{q}(\nu) : \dot{q}(\nu) = 0, \quad t-h \le \nu \le t\}$.

Consider the following nonlinear PID controller

$$U = -Q_0(t) - U_1(t,q,\dot{q}) - \int\limits_{t-h(t)}^{t} P(t,\nu)g(q(\nu))d\nu \tag{20}$$

where

$$U_1(t,q,0) = 0, \quad g \in C^1(R^n \to R^n), \quad g(0) = 0 \tag{21}$$

Transform the integral component of the regulator (20) as follows

$$\int_{t-h(t)}^{t} P(t,\nu)g(q(\nu))d\nu = P_0(t)g(q(t)) - $$
$$- \int_{t-h(t)}^{t} P(t,\nu) \int_{\nu}^{t} \frac{\partial g(q(s))}{\partial q}\dot{q}(s)dsd\nu \quad (22)$$
$$P_0(t) = \int_{t-h(t)}^{t} P(t,\nu)d\nu$$

Suppose that the components of the controller (20) are such that the following holds

$$Q_1(t,q) - P_0(t)f(q) = -\mu(t)\frac{\partial \Pi(t,q)}{\partial q}$$
$$\mu \in C^1, \quad 0 < \mu_0 \le \mu(t) \le \mu_1, \quad \frac{\partial \Pi(t,q)}{\partial t} \le 0$$
$$2\mu(t)\dot{q}'(Q_2(t,q,\dot{q}) - U_1(t,q,\dot{q})) - \dot{\mu}(t)\dot{q}'A(q)\dot{q} \le$$
$$\le -\dot{q}'F(t,q)\dot{q}$$
$$0 \le x'R(t,\nu)x \le \beta_1\|\dot{x}\|^2$$
$$-\beta_1\|x\|^2 \le \frac{\partial R(t,\nu)}{\partial t} \le \beta_2\|x\|^2$$
$$x'\left(F(t,q) - \mu^2(t)\left(\frac{\partial g(q)}{\partial q}\right)' \int_{t-h(t)}^{t} R(t,\nu)d\nu \left(\frac{\partial g(q)}{\partial q}\right) + \right.$$
$$\left. + \int_{t-h(t)}^{t} (t-\nu)P'(t,\nu)\left(\frac{\partial R(t,\nu)}{\partial t}\right)^{-1} P(t,\nu)d\nu\right)x \ge$$
$$\ge \beta_0\|x\|^2$$

where $0 < h_0 \le h(t) \le h_1$, $\dot{h}(t) \le 1-\delta$, $\delta = const > 0$.

The position stabilization problem of (9) of the closed-loop system (8), (20) is solved using the Lyapunov functional

$$V = \frac{1}{\mu(t)}T(q,\dot{q}) + \Pi(t,q) +$$
$$+\frac{1}{2}\int_{t-h}^{t}\int_{\nu}^{t} \dot{g}'(s)\left(\frac{\partial g(q(s))}{\partial q}\right)' R(t,\nu)\frac{\partial g(q(s))}{\partial q}\dot{q}(s)dsd\nu$$

On the basis of Theorem 5 we can get that Claim 6 holds also for the closed-loop system (8), (20).

Consider position stabilization problem of (9) for (8) under the action of nonlinear integral controller (PI-regulator)

$$U = -Q_0(t) + U_1(t,q) -$$
$$-\frac{\partial f(q(t))}{\partial q}\int_{t-h(t)}^{t} P(t,\nu)(f(q(t)) - f(q(\nu)))d\nu$$

the components of which are such that

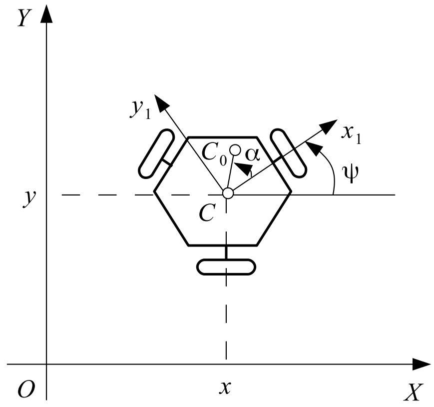

Fig. 1. The omni-wheeled mobile robot

$$U_1(t,q) + Q_1(t,q) = -\frac{\partial \Pi(t,q)}{\partial q}, \quad \frac{\partial \Pi(t,q)}{\partial t} \le 0$$
$$\alpha_1\|x\|^2 \le x'P(t,\nu)x \le \alpha_2\|x\|^2$$
$$x'\frac{\partial P(t,\nu)}{\partial t}x \le -\alpha_3\|x\|^2$$

where α_k are positive reals ($k = 1,2,3$), $0 < h_0 \le h(t) \le h_1$, $\dot{h}(t) \le 1-\delta$, $\delta > 0$.

The function $f \in C^1(R^n \to R^n)$ has a finite number of prototypes in a bounded domain.

Let choose the Lyapunov functional

$$V = T(q(t),\dot{q}(t)) + \Pi(t,q(t)) +$$
$$+\frac{1}{2}\int_{t-h(t)}^{t} (f(q(t)) - f(q(\tau)))'P(t,\nu)(f(q(t)) - f(q(\nu)))d\nu$$

The potential energy $\Pi = \Pi(t,q)$ ($\Pi \in C^1([0,+\infty) \times R^n \to R)$, $\Pi(t,0) \equiv 0$) is non-increasing in time, i.e. $\partial\Pi(t,q)/\partial t \le 0$, the matrix $R(t,\nu)$ ($R \in C^1(R^+ \times R^- \to R^{n\times n})$) and its derivative $\partial R(t,\nu)\partial t$ satisfy the following conditions

$$\alpha_1(\nu - t)\|x\|^2 \le x'R(t,\nu)x \le \alpha_2(\nu - t)\|x\|^2$$
$$\int_{-\infty}^{0} \alpha_k(s)ds < +\infty, \quad k = 1,2$$
$$\frac{\partial R(t,\nu)}{\partial t} = M(t)R(t,\nu), \quad R_0(t) = R(t,t)$$
$$\beta_0\|x\|^2 \le x'R_0(t)x \le \beta_1\|x\|^2$$
$$x'\left(R_0(t)M(t) + M(t)R_0(t) - \frac{dR_0(t)}{dt}\right)x \le -\beta_0\|x\|^2$$

where β_0 and β_1 are positive reals.

4. ON SAMPLED-DATA CONTROL FOR WHEELED MOBILE ROBOTS USING NONLINEAR PID REGULATORS

The dynamic equations of the wheeled mobile robot (see Figure 1) are given by Martynenko (2010)

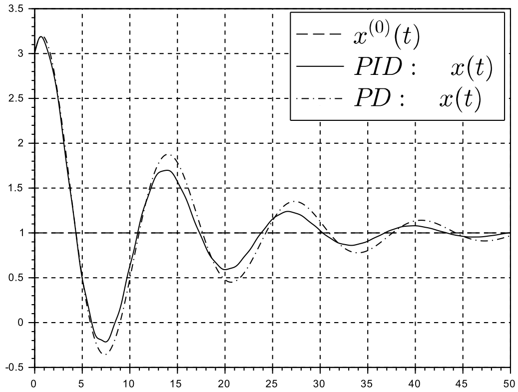

Fig. 2. The coordinate $x(t)$ of the robot center and programme position $x^{(0)}$

$$
\begin{aligned}
& m\ddot{x} - m_0 d\sin(\alpha+\psi)\ddot{\psi} - 3m_1\dot{y}\dot{\psi} - m_0 d\dot{\psi}^2\cos(\alpha+\psi) = \\
&= \frac{1}{2}\left(\sin\psi M_1 + \sin(\psi+\frac{2\pi}{3})M_2 + \sin(\psi+\frac{4\pi}{3})M_3\right) \\
& m\ddot{y} + m_0 d\cos(\alpha+\psi)\ddot{\psi} + 3m_1\dot{x}\dot{\psi} - m_0 d\dot{\psi}^2\sin(\alpha+\psi) = \\
&= \frac{1}{2}\left(-\cos\psi M_1 - \cos(\psi+\frac{2\pi}{3})M_2 - \cos(\psi+\frac{4\pi}{3})M_3\right) \\
& -m_0 d\sin(\alpha+\psi)\ddot{x} + m_0 d\cos(\alpha+\psi)\ddot{y} + I_s\ddot{\psi} = \\
&= -\frac{a}{r}(M_1+M_2+M_3)
\end{aligned}
$$

where x and y are the coordinates of the platform center, ψ is the angle of the platform rotation, m_0, m_1, $m = m_0 + 3m_1$ and I_s are mass-inertia parameters of the robot. The control inputs M_1, M_2 and M_3 are taking to be piecewise constant signals, i.e.

$$
\begin{aligned}
& M_i(t) = M_{isd}(k\tilde{T}), \quad i = 1,2,3 \\
& \forall t \in [k\tilde{T}, (k+1)\tilde{T}), \quad k = 0,1,2,...
\end{aligned} \tag{23}
$$

where $\tilde{T} = const > 0$ is a sampling period, $M_{isd}(t)$ $(i = 1,2,3)$ are stabilizing control torques as follows

$$
M_{sd}(t) = diag(M_{1sd}(t), M_{2sd}(t), M_{3sd}(t)) = P(\psi)M_0
$$

$$
P(\psi) = \frac{2}{3}\begin{pmatrix} \sin\psi & -\cos\psi & -\frac{1}{2a} \\ \sin\left(\psi+\frac{2\pi}{3}\right) & -\cos\left(\psi+\frac{2\pi}{3}\right) & -\frac{1}{2a} \\ \sin\left(\psi+\frac{4\pi}{3}\right) & -\cos\left(\psi+\frac{4\pi}{3}\right) & -\frac{1}{2a} \end{pmatrix}
$$

$$
\begin{aligned}
& M_0(t) = -diag(f_1(x-x^{(0)}), f_2(y-y^{(0)}), f_3(\psi-\psi^{(0)}) - \\
& -diag\left(\mu_1\int_{t-h}^{t} e^{\alpha_1(\tau-t)}\dot{x}(\tau)d\tau,\right. \\
& \left.\mu_2\int_{t-h}^{t} e^{\alpha_2(\tau-t)}\dot{y}(\tau)d\tau, \mu_3\int_{t-h}^{t} e^{\alpha_3(\tau-t)}\dot{\psi}(\tau)d\tau\right)
\end{aligned}
$$

where f_i, α_i and $\mu_i > 0$ are some constants satisfying the inequality $f_i\alpha_i > \mu_i$ $(i = 1,2,3)$.

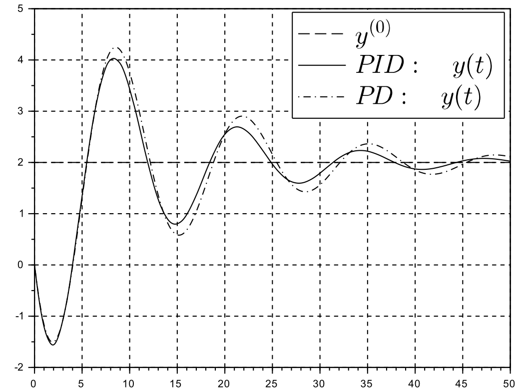

Fig. 3. The coordinate $y(t)$ of the robot center and programme position $y^{(0)}$

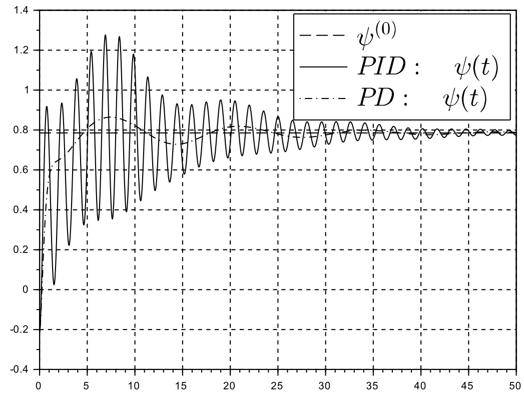

Fig. 4. Angular position $\psi(t)$ and programme position $\psi^{(0)}$

These graphs on Figures 2, 3 and 4 show that the controller (23) solves the regulation problem for the robot. The coordinates $x(t)$, $y(t)$ and $\psi(t)$ converge to the corresponding reference positions. In order to compare the performances the simulation tests were also performed with the standard PD regulator. The control gains of PD regulator were chosen such that the actuator torques evolved inside the same limits as those of (23). Comparison analysis of the graphs on Figures 2, 3 confirmed that the proposed controller provided better performance than PD regulator. Note that the controller (23) can be used for wheeled mobile robots which are not equipped with the tachometers.

5. CONCLUSION

This paper presents the solution to the regulation problem for holonomical mechanical systems. Nonlinear PI and PID controllers have been proposed which solve this problem. An asymptotic stability of the closed-loop system has been studied by constructing a Lyapunov functional with a semidefinite time derivative. The performance of the PID controller was illustrated via simulation on a wheeled mobile robot. The proposed design is shown to have some advantages over the standard PD regulator.

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