Novel Optimum Magnitude Based Fractional Order Controller Design Method

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Abstract: Due to adding the extra degree of freedom, the fractional order PID controllers can achieve better control performance than the integer order PID controllers. The present paper proposes a novel fractional order controller design method, inspired by the Kessler's optimum magnitude method. The explicit tuning rules are accessible even to less experienced users in fractional calculus, taking only the advantages of the controller and not the disadvantage of complex mathematical background. The advantages of the method are demonstrated by a case study.

Keywords: controller design method, fractional order calculus, fractional order controller, optimum magnitude.

1. INTRODUCTION

Fractional calculus has become very useful over the last 40 years due to its many applications in almost all applied sciences (Chen et al., 2009; Xue, 2017), like acoustic wave propagation in inhomogeneous porous material, diffusive transport, fluid flow, dynamical processes in self-similar structures, dynamics of earthquakes, optics, geology, viscoelastic materials, biomedical engineering, economics, probability and statistics, astrophysics, chemical engineering, physics, fluid mechanics, electromagnetic waves, control engineering, signal processing, chaotic dynamics, polymerscience, electrochemistry, etc. (Copot et al., 2013; Zhou et al., 2015; Ionescu et al., 2017).

The major impact in control engineering represented the work of Podlubny (Podlubny, 1999), who proposed a generalization of the PID controller, namely the $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ. It was demonstrated the better response of this type of controller, in comparison with the classical PID controller, when used for the control of fractional order systems. The fractional order controller design techniques are based in general on extensions of the classical PID control theory, with an emphasis on the increased flexibility in the tuning strategy resulting in an easier way of achieving the control requirements as compared to classical control tuning methods (Caponetto et al., 2002; Monje et al., 2010; Li, 2017). Most of the tuning techniques for fractional-order controllers deal with complex computations and optimization routines (Monje et al., 2010; Li, 2017). In (Valerio and Costa, 2006a; Valerio and Costa, 2006b; Merrikh-Bayat, 2012; Chen et al., 2008; Gude and Kahoraho, 2009), some relatively simple methods are presented, but they are valid only for some particular cases or applications. The authors established a simple yet efficient methodology based on a vector representation of the fractional-order controllers, which simplifies considerably the computations and derivation of the fractional-order controller parameters (Dulf, 2017, 2015, 2012; Muresan, 2016a). As a consequence, the complex equations from the classical fractional-order controller design procedure — introduced by fractional-order derivation and integration — are very much simplified. In fact, the parameters of the fractional-order controllers are easily obtained, and the final results are exactly the same as those that would be obtained using existing techniques that are based on complex computations.

Using these results, the present paper proposes a novel tuning algorithm of the controller's parameter, a generalization of the Kessler's optimum magnitude method (Åström, 1995) using fractional order calculus. Introducing the fractional order one more degree of freedom appears, conducting to better performances. The presented case study highlights the advantages of the proposed algorithm.

The current paper is structured in four parts. After this brief introductory part, the second section presents the new design method, followed by the case study and conclusions.

2. THE CONTROLLER DESIGN METHOD

The form of the fractional order PI controller adopted in this work is:

$$C(s) = K_p + \frac{K_i}{s^{\alpha}}, \ \alpha \in (0,1)$$

or in frequency domain:

$$C(j\omega) = K_p + \frac{K_i}{(j\omega)^{\alpha}}$$

Considering as vectors the proportional and the integrator term, and knowing that $j^{\alpha} = cos\frac{\alpha\pi}{2} + j\,sin\frac{\alpha\pi}{2}$, it is obtained the representation from Fig. 1 (Muresan, 2016b).

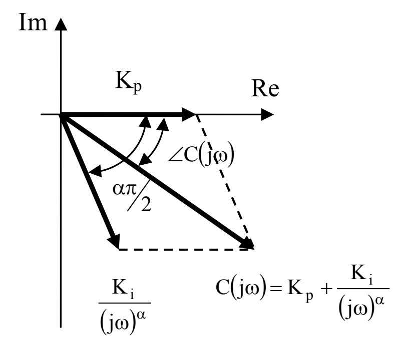

Fig. 1. Vector representation of a fractional order PI controller

From Fig. 1, using the classical vector theory, is obtained:

$$|C(j\omega)| \cdot sin\angle C(j\omega) = \left|\frac{K_i}{(j\omega)^{\alpha}}\right| \cdot sin\frac{\alpha\pi}{2}$$

Replacing the magnitudes it results as follows:

$$sin\angle C(j\omega) = \frac{K_i \cdot \omega^{-\alpha}}{|C(j\omega)|} \cdot sin\frac{\alpha\pi}{2}$$

or

$$K_i = \frac{|C(j\omega)| \cdot sin\angle C(j\omega)}{\omega^{-\alpha} \cdot sin\frac{\alpha\pi}{2}} \tag{1}$$

From the same vector theory it is known that:

$$|C(j\omega)|^2 = |K_p|^2 + \left|\frac{K_i}{(j\omega)^{\alpha}}\right|^2 + 2 \cdot |K_p| \cdot \left|\frac{K_i}{(j\omega)^{\alpha}}\right| \cdot cos\frac{\alpha\pi}{2}$$

Replacing the corresponding magnitudes and rearranging the terms a second order equation results in K_p:

$$K_p^2 - S \cdot K_p + P = 0 \tag{2}$$

with $S = \left(2 \cdot K_i \cdot \omega^{-\alpha} \cdot cos\frac{\alpha\pi}{2}\right)$ and $P = \left(K_i^2\omega^{-2\alpha} - |C(j\omega)|^2\right)$

Starting from equations (1) and (2) any other equation can be derived representing the imposed performance requirements of the system. The controller design became in this manner a mathematical problem: solving an equation system.

In the present research the design goal is to assure the closed loop system magnitude as close as possible to unity for a good reference tracking:

$$|H_o(j\omega)| \cong 1$$

or

$$\left|\frac{C(j\omega) \cdot P(j\omega)}{1 + C(j\omega) \cdot P(j\omega)}\right| = k \cong 1,$$

where P represents the process transfer function.

From this last equation and using the same classical vector theory, it results:

$$\begin{aligned} |C(j\omega) \cdot P(j\omega)|^2 &= k^2\left(1 + |C(j\omega) \cdot P(j\omega)|^2\right) + \\ &+ k^2\left(2 \cdot |C(j\omega) \cdot P(j\omega)| \cdot cos\angle C(j\omega) \cdot P(j\omega)\right) \end{aligned} \tag{3}$$

In order to obtain a simple tuning equation it is imposed:

$$\angle C(j\omega) \cdot P(j\omega) = -\frac{\pi}{2}, \tag{4}$$

If this frequency represents the gain crossover frequency, the above condition means 90° phase margin of the system. In this case $cos\angle C(j\omega) \cdot P(j\omega) = cos\left(-\frac{\pi}{2}\right) = 0$ and equation (2) is simplified to:

$$|C(j\omega) \cdot P(j\omega)|^2 = k^2\left(1 + |C(j\omega) \cdot P(j\omega)|^2\right)$$ or

$$|C(j\omega) \cdot P(j\omega)|^2 = \frac{k^2}{1 - k^2}$$

With this choice the magnitude and phase equations of the controller becomes:

$$|C(j\omega)| = \sqrt{\frac{k^2}{1 - k^2}} \cdot \frac{1}{|P(j\omega)|} = k_1$$ and

$$\angle C(j\omega) = -\frac{\pi}{2} - \angle P(j\omega).$$

Recalling the equations (1) and (2) one can determine the controller's parameters:

$$K_i = \frac{k_1 \cdot \frac{1}{|P(j\omega)|} \sin\left(-\frac{\pi}{2} - \angle P(j\omega)\right)}{\omega^{-\alpha} \cdot \sin\frac{\alpha\pi}{2}} =$$

$$= -\frac{k_1 \cdot \frac{1}{|P(j\omega)|} \cos\angle P(j\omega)}{\omega^{-\alpha} \cdot \sin\frac{\alpha\pi}{2}}$$

$$K_p = \frac{S + \sqrt{S^2 - 4P}}{2}$$

and the fractional order is chosen to fulfill the equation (4).

The controller design algorithm can be summarized as follows:

It is supposed that the transfer function of the process P(s) is known and the controller transfer function is $C(s) = K_p + \frac{K_i}{s^\alpha}$, $\alpha \in (0,1)$. Then:

1. Choose the proper gain crossover frequency (ensuring the desired settling time) ω_{gc}
2. Establish the fractional order α from the equation $\angle C(j\omega_{gc}) = -\frac{\pi}{2} - \angle P(j\omega_{gc})$
3. Establish the integral gain using the equation $K_i = \frac{k_1 \cdot \frac{1}{|P(j\omega_{gc})|} \cos \angle P(j\omega_{gc})}{\omega_{gc}^{-\alpha} \cdot \sin \frac{\alpha\pi}{2}}$

   $k_1 = \sqrt{\frac{k^2}{1-k^2}} \cdot \frac{1}{|P(j\omega_{gc})|}$ and k a parameter very close to unity.
4. Determine the proportional gain using the equation $K_p = \frac{S + \sqrt{S^2 - 4P}}{2}$, $S = \left(2 \cdot K_i \cdot \omega_{gc}^{-\alpha} \cdot \cos \frac{\alpha\pi}{2}\right)$, with, $P = \left(K_i^2 \omega_{gc}^{-2\alpha} - k_1^{\,2}\right)$.

3. CASE STUDY

The experimental unit consists in the modular servo system designed by Inteco (Inteco, 2008). The system is used in a particular configuration (Fig. 2), consisting in a tachogenerator, inertia load, damping module, backlash, incremental encoder, and gearbox with output disk, each module being added or removed from the experimental plant.

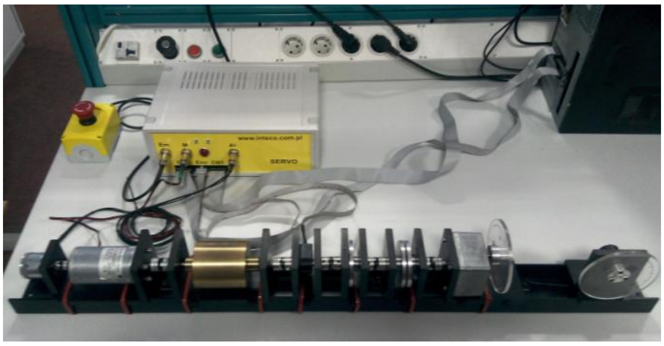

Fig. 2. The experimental unit: the modular servo system

The mathematical model of the modular servo system without backlash and loads has been determined experimentally for the operating point of 100 rad/s as:

$$H(s) = \frac{\omega(s)}{u(s)} = \frac{k}{Ts+1} \qquad (5)$$

where ω is the angular velocity of the rotor and u is the voltage, k=194 and T=60.25 ms are the motor nominal gain and time constant, respectively.

Using this model, a classical, integer order PI controller was designed, based on the optimal magnitude criterion of Kessler (Åström, 1995):

$$H_{PI}(s) = \frac{1}{Ti \cdot s} = \frac{1}{23.377s}.$$

Imposing as performance criteria the unity magnitude of the closed loop in frequency domain (as in the case of the classical Kessler method), a fractional order controller was designed using the tuning procedure presented in section 2. The resulted controller is:

$$H_{FO\text{-}PI}(s) = 0.0016 + \frac{0.0447}{s^{0.95}}$$

Using the Oustaloup Recursive Approximation method (Oustaloup, 2000), the equivalent transfer function becomes:

$$H_{FO\text{-}PI}(s) = \frac{0.0034s^2 + 0.0627s + 0.1488}{s^2 + 3.4955s + 1.0098}.$$

The closed loop Bode diagram is presented in Fig. 3, highlighting the unity magnitude for low frequencies.

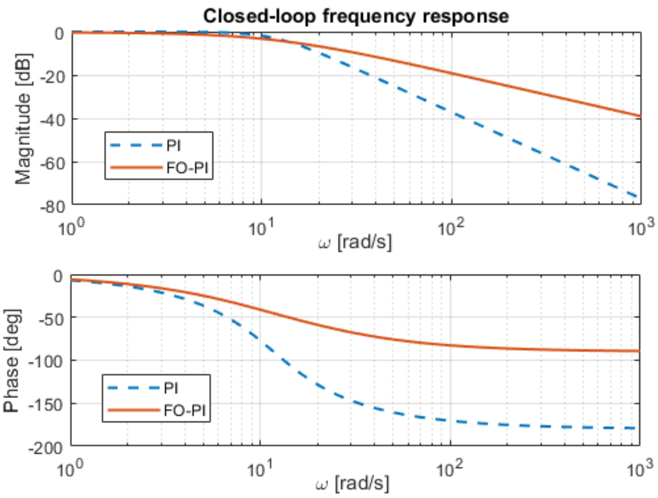

Fig. 3. Closed loop frequency response of the integer order and the fractional order controller

The open loop Bode diagram is depicted in Fig.4. One can observe that the system with integer order controller exhibits a phase margin of 65°, as it is demonstrated in (Åström, 1995), while with the fractional order controller the phase margin is increased to 95°.

The step response of the closed loop system has the behaviour of a first-order system for the proposed method, giving a phase margin of 95° (Fig. 5). Given that the algorithm imposes a coefficient k which indicates how much the closed-loop system approaches the unit magnitude, one can observe a steady-state error in the step response (for the current case study, k = 0.95).

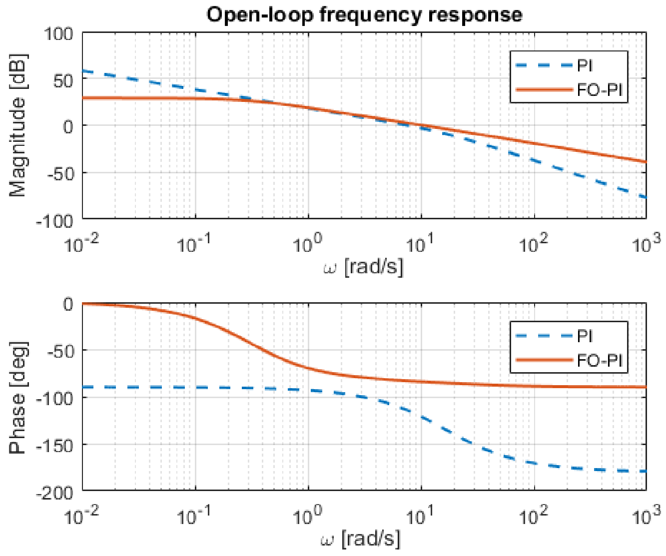

Fig. 4. Open loop frequency response of the integer order and the fractional order controller

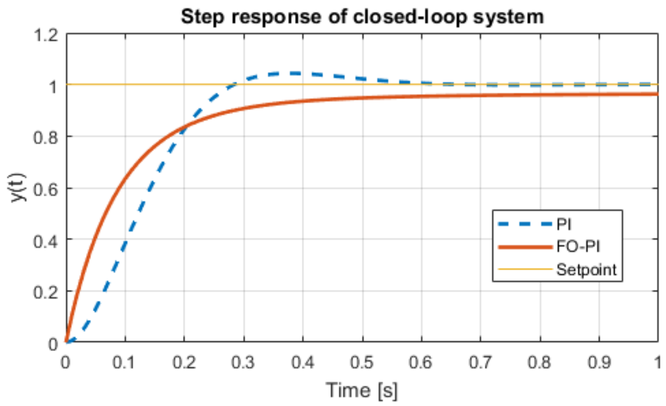

Fig. 5. Simulated step responses of the two designed controllers

The robustness of the system is tested for different setpoints and system configurations including the backlash element in the experimental unit. Both the integer order and the fractional order controller were designed for the plant model with the DC Motor only. For the experimental setpoint of 100 rad/s a reference offset of 4.1% leads to zero steady-state error. The same percentage was used for the other setpoints. The experimental results using only the DC motor are presented in Fig.6, highlighting a 0% overshoot instead of approximately 11% overshoot with the original Kessler's method and a decreasing of settling time from 502 ms to 450 ms.

The same experiment was carried out including the DC motor damping load. The results (output and control signal) are presented in Fig.7 for the same setpoints of 50, 100 and 160 rad/s. One can observe the same overshoot and settling time decrease using the proposed method.

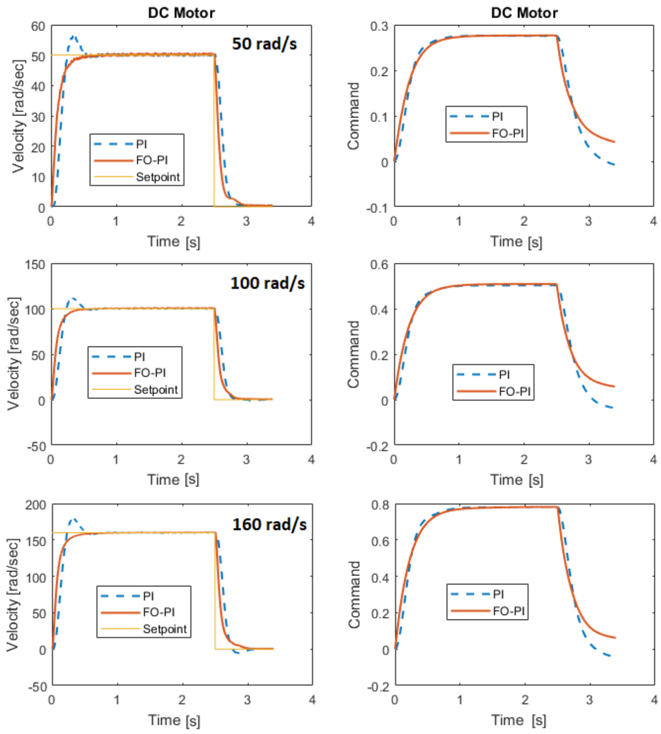

Fig. 6. DC Motor closed loop velocity evolution and the corresponding control signal for three setpoints: a) SP = 50 rad/s; b) SP = 100 rad/s; c) SP = 160 rad/s

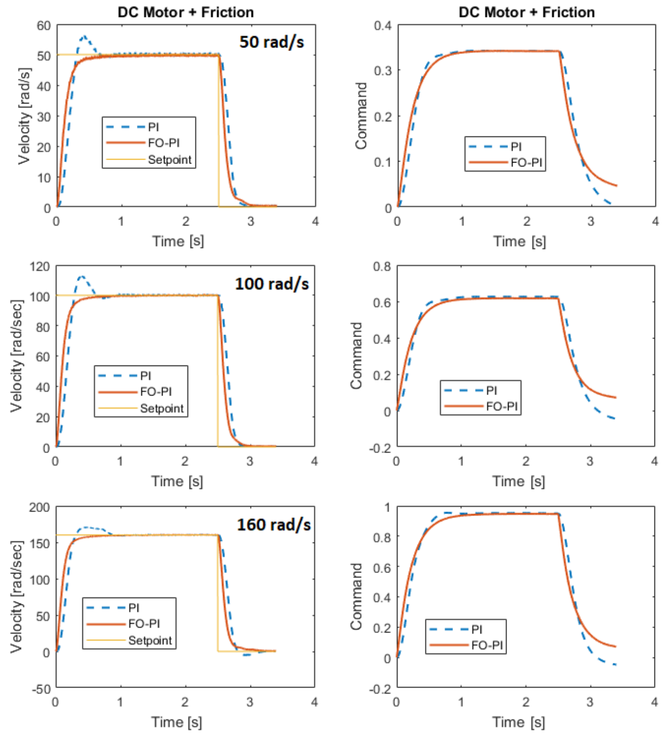

Fig. 7. DC Motor with damping module closed loop velocity evolution and the corresponding control signal for three setpoints: a) SP = 50 rad/s; b) SP = 100 rad/s; c) SP = 160 rad/s

Fig.8 shows the experimental results using the DC motor, damping load and backlash, which includes a considerable difference in the process model. Even in this case both the output and control signal exhibits greater performances than with the classical, integer order controller. For the same 50, 100, 160 rad/s setpoint values, the overshoot with the classical controller is 20.6, 23.25 and 5.1% respectively, while with the fractional order controller is 1.0, 3.1, 4.5%. The obtained settling time values with the integer order controller are: 1.04, 1.23 and 1.927s and with the fractional order controller are reduced to 0.41, 0.475, 0.602s. For the 160 rad/s setpoint the command signal with the classical method is saturated, considerably increasing the settling-time.

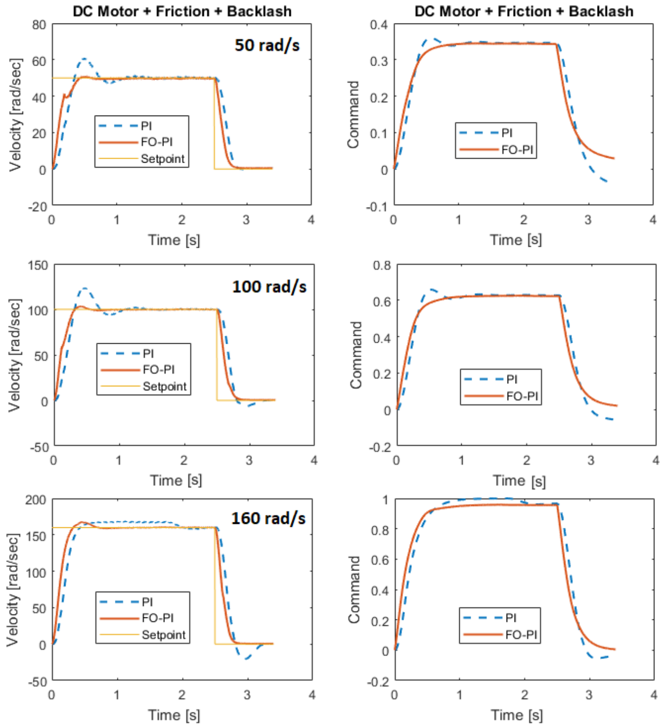

Fig. 8. DC Motor with damping module and backlash closed loop velocity evolution and the corresponding control signal for three setpoints: a) SP = 50 rad/s; b) SP = 100 rad/s; c) SP = 160 rad/s

The experimental results obtained using the DC Motor, the inertia load, damping module and backlash are presented in Fig.9. In this case the performance measures obtained with the classical controller are: overshoot 62, 44.4, 0% and settling time 8, 8, 3.9s, while with the proposed fractional order controller these values are reduced to 35, 33.5, 0% and 4.4, 5.05, 3.7s, respectively.

4. CONCLUSIONS

A new tuning method for fractional order controllers was introduced in the paper, inspired by the Kessler's optimum magnitude method. The major advantages of the method are the simplicity and generality, it is easy to use having no restrictions for the process model. The one more degree of freedom of the fractional PI controller gives improved performances over the traditional Kessler-controller, both in quality coefficients like overshoot, settling-time, and in robustness by setpoint variation or system changes, as it is demonstrated in the case study.

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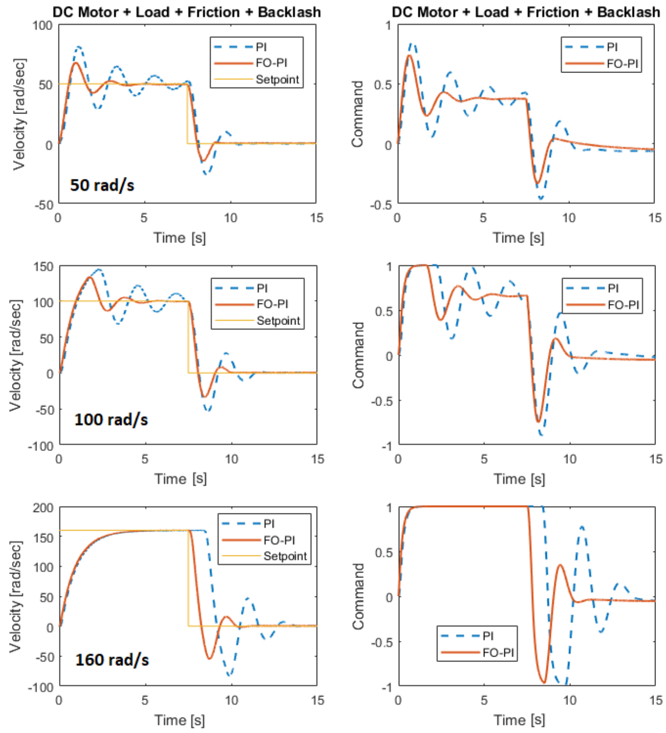

Fig. 9. DC Motor with load, damping module and backlash closed loop velocity evolution and the corresponding control signal for three setpoints: a) SP = 50 rad/s; b) SP = 100 rad/s; c) SP = 160 rad/s

REFERENCES

Åström, K.J. and Hägglund, T. (1995). PID Controllers: Theory, Design, and Tuning. Instrument Society of America, 2nd edition, 1995.

Chen, Y., Petras, I. and Xue, D. (2009). Fractional Order Control - A Tutorial, *In Proc. IEEE American Control Confernce (ACC 2009), St. Luis, USA*, 1397–1411.

Chen, Y.Q., Bhaskaran, T., Xue, D. (2008) Practical tuning rule development for fractional order proportional and integral controllers. *ASME J. Comput Nonlin Dyn.*, 3, 0214031–0214037.

Copot, C., Burlacu, A., Ionescu, C.M., Lazar, C., and De Keyser, R. (2013) A fractional order control strategy for visual servoing systems, *Mechatronics*, 23(7), 848-855.

Dulf, E.H., Timis, D. and Muresan, C.-I. (2017). Robust Fractional Order Controllers for Distributed Systems, *Acta Polytech Hung*, 14(1), 163-176.

Dulf E:H., Dulf F.V., Pop C.I. (2015). Fractional Model of the Cryogenic (^{13}C) Isotope Separation Column, *Chemical Engineering Communication*, 202(12), 1600-1606.

Dulf E:H., Pop C.I., Dulf F.V. (2012). Fractional Calculus in ^{13}C Separation Column Control, *Signal, Video and Image Processing*, 6(3), 479-485.

Gude, J.J., Kahoraho, E. (2009). New tuning rules for PI and fractional PI controllers. *IFAC Proceedings Volumes* (European Control Conference), 42(11), 768-773.

Inteco (2008). Modular Servo System-User's Manual. Available at www.inteco.com.pl.

Ionescu, C.M., Lopes, A., Copot, D., (2017) The torle of fractional calculus in modeling biological phenomena: A review, *Communic Nonlin Sci Num Sim*, 51, 141-159.

Li Z., Liu, L., Dehghan, S., Chen, Y. and Xue, D. (2017). A review and evaluation of numerical tools for fractional calculus and fractional order controls, *Int J Control*, 90(6), 1165–1181.

Merrikh-Bayat, F. (2012). General rules for optimal tuning the $PI^{\lambda}D^{\mu}$ controllers with application to first-order plus time delay processes. *Can J Chem Eng*, 90(6), 1400–1410.

Monje, C.A., Chen, Y., Vinagre, B.M., Xue, D. and Feliu-Battle, V. (2010). *Fractional-order Systems and Controls*, Springer Verlag.

Muresan, C.I., Dutta, A., Dulf, E.H., Pinar, Z. and Ionescu, C.M. (2016). A New Tuning Algorithm for Fractional Order IMC Controllers for Multivariable Time Delay Processes, *Int J Control*, 89(3), 579-593.

Muresan, C.I., Dulf, E.H. and Both, R. (2016). Vector Based Tuning and Experimental Validation of Fractional Order PI/PD Controllers, *Nonlin Dynamics*, 84(1), 179–188.

Oustaloup A., Levron, F., Mathieu, B. and Nanot, F.M. (2000). Frequency-band complex noninteger differentiator: characterization and synthesis, *IEEE T Circuits Syst I Fundam Theory Appl*, 47(1), 25–39.

Oustaloup A., Sabatier, J. and Lanusse, P. (1999). From fractional robustness to CRONE control., *Fract Calc Appl Analysis*, 2(1), 1–30.

Valerio, D., Costa, J.S.D. (2006). Tuning-rules for fractional PID controllers. *IFAC Proceedings Volumes* (2nd IFAC Workshop on Fractional Differentiation and its Applications), 39(11). 28-33.

Valerio, D., Costa, J.S.D. (2006). Tuning of fractional PID controllers with Ziegler–Nichols-type rules, *Signal Process*, 86, 2771–2784.

Xue, D. (2017). *Fractional-Order Control Systems*. De Gruyter.

Zhou, Y., Ionescu, C.M. and Tenreiro Machado, J.A. (2015) Fractional dynamics and its applications, *Nonlin Dynamics*, 80(4), 1661-1664.