Uncoupled PID Control of Multi-Agent Nonlinear Uncertain Stochastic Systems ⋆

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Abstract: This paper proposes a PID (proportional-integral-derivative) control design method to solve the regulation problem for a class of coupled multi-agent nonlinear uncertain stochastic systems, where each agent only has access to its own regulation error without communicating with others. A three-dimensional manifold will be constructed based on the information about the Lipschitz constants of both the unknown nonlinear drift and diffusion terms, such that the three parameters of each agent's PID controller can be chosen arbitrarily from this common manifold. It will be shown that such an uncoupled PID design method can globally stabilize the whole nonlinear uncertain stochastic multi-agent system with the regulation error of each agent approaching to zero asymptotically.

Keywords: PID controller, coupled multi-agent stochastic system, nonlinear uncertain dynamics, Lipschitz condition, global stability.

1. INTRODUCTION

In the last decades, multi-agent control system has attracted intensive research attention, mostly modelled by uncoupled dynamics. A fundamental issue of multi-agent systems is the regulation problem, which is a common control objective in various real world systems such as multiple aircraft, mobile robots, electric power grids, sensor networks, etc. However, both coupled uncertain dynamics and random disturbances (see Koralov and Sinai (2007)) always exist in practical multi-agent systems, which considerably increases the difficulty in achieving the control objective. Dealing with uncertainties is a challenging problem in control theory, especially for the systems with strongly-coupled nonlinear uncertain dynamics. Various methods including adaptive control and robust control have been proposed and investigated extensively in the literature. Most adaptive control methods deal with parametric uncertain nonlinear systems with unknown parameter and known structure, where the unknown parameters are estimated online by using the measured signals of the systems (see Åström and Wittenmark (1995); Chen and Guo (1991); Krstić et al. (1995)). Robust control usually requires a nominal model for the uncertain systems which are assumed to lie in a certain "ball" centered by the nominal model (see Qu (1998)). There are also many other "model free" methods which are less dependent on the precise mathematical model, such as fuzzy and neuron networks based methods, and the celebrated ADRC (active disturbance rejection control) which uses an extended state observer to estimate the total uncertainties consisting of unmodeled dynamics and external disturbances (see Han (2008); Jiang et al. (2015)). Nevertheless, it is well known that the PID (proportional-integral-derivative) control is still the most widely used approach in engineering systems (see Samad (2017)). The PID control is a feedback control with linear feedback mechanism that can reduce the influence of uncertainties including internal structure uncertainties as well as external disturbances and does not depend on precise mathematical models, while their effectiveness is often limited due to poor tuning (see Åström and Hägglund (1995, 2006); Killingsworth and Krstić (2006)).

The class of the second-order multi-agent systems is perhaps the most extensively studied multi-agent system since the celebrated Newton's second law describing the motion equation is second-order. The first rigorous mathematical theory on global stability and asymptotic regulation for second-order nonlinear uncertain systems controlled by the classical linear PID controller was recently provided in Zhao and Guo (2017a,b); Cong and Guo (2017). Subsequently, in Yuan et al. (2017, 2018), a class of second-order nonlinear deterministic uncertain multi-agent systems was considered, and a decentralized PID controller was designed to stabilized the overall system globally with each agent only using its own regulation error. How to design the PID controller and guarantee the performance of multi-agent control systems with strongly-coupled nonlinear uncertain dynamics as well as random process is not considered previously, which motivates the investigation of this paper.

In this paper, we will study the PID control problem of a class of second-order high-dimensional coupled multi-agent nonlinear uncertain stochastic dynamical systems, where each agent can arbitrarily choose its PID controller parameters from a three-dimensional manifold constructed by using the information on the upper bounds of the

⋆ The paper was supported by the National Natural Science Foundation of China under Grant No. 11688101.

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Lipschitz constants of both the unknown nonlinear drift and diffusion terms. We will show that the overall system can be stabilized globally while each agent can achieve its own regulation objective by using its own regulation error.

The remainder of the paper is organized as follows. The problem formulation will be described in Section 2. Section 3 will present the main results together with mathematical proofs. Section 4 will give a numerical example. Finally, Section 5 will conclude the paper with some remarks.

2. PROBLEM FORMULATION

Let us consider a multi-agent system consists of r agents and each has n degrees of freedom, i.e., the configuration space of these agents is $\mathbb{R}^n$. Denote $p_j(t)$, $v_j(t)$, $a_j(t)$ be the position, velocity, acceleration of agent j at the time instant t, respectively, where $j = 1, 2, \cdots, r$. For simplicity of notations, we denote $p = (p_1^{\mathrm{T}}, p_2^{\mathrm{T}}, \cdots, p_r^{\mathrm{T}})^{\mathrm{T}}$ and $v = (v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \cdots, v_r^{\mathrm{T}})^{\mathrm{T}}$, where α^{T} represents the transposition of a matrix or a vector α throughout this paper. Assume that the external forces acting on agent j consist of u_j, f_j, and the "noise effects" expressed by the multiplication of a drift term σ_j and "white noise", where u_j is the control force acting by agent j, and $f_j = f_j(p, v, t)$ as well as $\sigma_j = \sigma_j(p, v, t)$ are $\mathbb{R}^n$-valued nonlinear functions of all agents' positions p, velocities v and time t. f_j means the unknown dynamic of agent j. The "white noise" in the continuous-time case is not defined mathematically, but can be roughly regarded as the "derivative" of a Brownian motion, and its effects on dynamical systems are usually characterized by stochastic differential equations.

By Newton's second law, we know the following motion equation of agent j at time t,

$$m_j a_j = f_j(p, v, t) + u_j + \sigma_j(p, v, t)\text{"white noise"}, \quad (1)$$

where m_j is agent j's mass. The control objective of each agent j is to design an output feedback controller u_j by using the online information of its own regulation error to guarantee that for any initial position and initial velocity, the position trajectory of agent j reaches a given reference value $y_j^* \in \mathbb{R}^n$ asymptotically.

In this paper, our control force is the classical PID controller,

$$u_j(t) = k_{pj} e_j(t) + k_{ij} \int_0^t e_j(s) ds + k_{dj} \frac{de_j(t)}{dt}, \quad (2)$$

where $e_j(t) = p_j(t) - y_j^*$ is the regulation error of j, $j = 1, \cdots, r$. The control variable is a sum of three terms with three parameters (k_{pj}, k_{ij}, k_{dj}) to be designed. Without loss of generality, we assume that each agent has the unit mass $m_j = 1$. Notice that $v_j = \dot{p}_j$, $a_j = \ddot{p}_j$, then (1) can be rewritten as

$$\ddot{p}_j = f_j(p, \dot{p}, t) + u_j + \sigma_j(p, \dot{p}, t)\text{"white noise"}. \quad (3)$$

Now, let $\{B_j(t)\}_{t\geq 0}$ be an n-dimension standard Brownian motion defined on a complete probability space $(\Omega, \mathscr{F}, P)$ with a natural filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (see Liptser and Shiryaev (2001)), where $j = 1, 2, \cdots, r$ and each $\{B_j(t)\}_{t\geq 0}$ is independent with others. Let the "white noise" be replaced formally by the "derivative" of $B_j(t)$. Then, denote $x_{1j} = p_j, x_{2j} = v_j$, the state space equation of system (3) can be described by the following stochastic differential equation (SDE),

$$\begin{cases} dx_{1j} = x_{2j} dt, \\ dx_{2j} = f_j(x_1, x_2, t) dt + u_j dt + \sigma_j(x_1, x_2, t) dB_j(t), \end{cases} \quad (4)$$

where $x_1 = (x_{11}^{\mathrm{T}}, x_{12}^{\mathrm{T}}, \cdots, x_{1r}^{\mathrm{T}})^{\mathrm{T}}$, $x_2 = (x_{21}^{\mathrm{T}}, x_{22}^{\mathrm{T}}, \cdots, x_{2r}^{\mathrm{T}})^{\mathrm{T}}$. Assume that for all $t \in \mathbb{R}^+$ and $x_1 \in \mathbb{R}^{rn}$, $f_j(x_1, 0, t) = f_j(x_1, 0, 0)$, $\sigma_j(x_1, 0, t) = \sigma_j(x_1, 0, 0)$, and $y^* = (y_1^*, y_2^*, \cdots, y_r^*)$ is the setpoint satisfying $\sigma_j(y^*, 0, t) = 0$.

In this paper, we will show that the three controller parameters (k_{pj}, k_{ij}, k_{dj}) of each j can be designed offline based on some simple global information on the nonlinearities of the multi-agent system such that each agent can achieve its control aim, that is the position of agent j converges to any given constant setpoint y_j^* under the control law (2) for any initial state, as long as all $f_j = f_j(x_1, x_2, t)$ and $\sigma_j = \sigma_j(x_1, x_2, t)$ are Lipschitz continuous functions with known Lipschitz constants.

3. MAIN RESULTS

Firstly, we define a functional space,

$$\mathscr{F}_L = \{f : \mathbb{R}^{2rn} \times \mathbb{R}^+ \to \mathbb{R}^n \,\big|\, \|f(x,t) - f(y,t)\| \leq L\|x - y\|, \\ \forall x, y \in \mathbb{R}^{2rn}, \forall t \in \mathbb{R}^+\},$$

where L is the Lipschitz constant and $\|.\|$ is the standard Euclidean norm.

Theorem 1. Consider the PID control system (2) and (4) with unknown functions $f_j \in \mathscr{F}_{L_j^f}, \sigma_j \in \mathscr{F}_{L_j^\sigma}, j = 1, 2, \cdots, r$, where $\{L_j^f > 0, L_j^\sigma > 0, j = 1, 2, \cdots, r\}$ is a given set of Lipschitz constants. Then, there exists an unbounded open set $\Omega_{pid} \subset \mathbb{R}^3$, such that as long as each agent j takes its PID parameters (k_{pj}, k_{ij}, k_{dj}) from Ω_{pid}, the closed-loop system (2) and (4) will be globally stable, i.e., for any initial state $(x_1(0), x_2(0)) \in \mathbb{R}^{2rn}$,

$$\sup_{t\geq 0} \mathrm{E}[x_1^2(t) + x_2^2(t) + u^2(t)] < \infty,$$

and for each agent j, the regulation error asymptotically approaches to zero in the sense that

$$\lim_{t\to\infty} \mathrm{E}|x_{1j}(t) - y_j^*|^2 = 0,$$

where y_j^* is any given setpoint in $\mathbb{R}^n$.

Remark 2. Theorem 1 designs an uncoupled PID controller for a class of coupled multi-agent nonlinear uncertain stochastic systems, where the PID feedback signal of each control channel depends on its own regulation error only.

Remark 3. Notice that the openness of Ω_{pid} is of significant in practical applications, making the selection of the three controller parameters quite flexible and small perturbations of these parameters can not change the qualitative performance of the system. From the proof of Theorem 1, the concrete definition of Ω_{pid} in $\mathbb{R}^3$ can be given as

$$\Omega_{pid} = \left\{ \begin{bmatrix} k_p \\ k_i \\ k_d \end{bmatrix} \middle| \begin{bmatrix} k_p \\ k_i \\ k_d \end{bmatrix} = \begin{bmatrix} -(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) \\ \lambda_1\lambda_2\lambda_3 \\ \lambda_1 + \lambda_2 + \lambda_3 \end{bmatrix}, (\lambda_1, \lambda_2, \lambda_3) \in \Omega_\theta \right\}, \quad (5)$$

with the set Ω_θ defined by

$$\Omega_\theta = \left\{ \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \middle| \alpha < \lambda_1 < 0, \beta_1 < \lambda_2 < \beta_2, \lambda_3 < \gamma, \eta_1 < \lambda_1\lambda_2\lambda_3 < \eta_2 \right\},$$

where the six parameters denoted by $\theta \triangleq (\alpha, \beta_1, \beta_2, \gamma, \eta_1, \eta_2)$ can be taken arbitrarily from the following set,

$$\Theta = \Big\{\theta=(\alpha,\beta_1,\beta_2,\gamma,\eta_1,\eta_2)|\gamma<\beta_1<\beta_2<\alpha<0, \eta_1<\eta_2<0,$$
$$\eta_2-\sqrt{\sum_{j=1}^{r}(L_j^f)^2\eta_1\,\Phi(\theta)\Xi(\theta)-\sum_{j=1}^{r}(L_j^\sigma)^2\eta_1\,\Psi(\theta)\Xi^2(\theta)}<0\Big\},$$

and

$$\Xi(\theta) = \sqrt{3+\alpha^2+\beta_1^2+\frac{1}{\gamma^2}},$$
$$\Phi(\theta) = \sqrt{\frac{(\gamma-\beta_1)^2+(\gamma-\alpha)^2+\gamma^2(\beta_2-\alpha)^2}{(\gamma-\alpha)^2(\gamma-\beta_1)^2(\beta_2-\alpha)^2}},$$
$$\Psi(\theta) = -\frac{\alpha(\gamma-\beta_1)^2+\beta_2(\gamma-\alpha)^2+\gamma^3(\beta_2-\alpha)^2}{2(\gamma-\alpha)^2(\gamma-\beta_1)^2(\beta_2-\alpha)^2}.$$

It can be verified that Θ is nonempty similar to the case in Yuan et al. (2017), and therefore, Ω_{pid} is nonempty. If the system (4) does not have the diffusion term, i.e., $L_j^\sigma=0, j=1,2,\cdots,r$, then the result reduces to Theorem 1 in Yuan et al. (2017), i.e., the deterministic case.

We give the following example to simply Ω_{pid}.

Example. Let γ be a negative number which satisfies $\gamma \le -16\sqrt{\sum_{j=1}^r(L_j^f)^2-176\sum_{j=1}^r(L_j^\sigma)^2-4}$. Then, in Theorem 1, we can chose the following parameter set,

$$\Omega_{pid} = \Big\{\begin{bmatrix}k_p\\k_i\\k_d\end{bmatrix}\Big|\begin{bmatrix}k_p\\k_i\\k_d\end{bmatrix}=\begin{bmatrix}-(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)\\ \lambda_1\lambda_2\lambda_3\\ \lambda_1+\lambda_2+\lambda_3\end{bmatrix},$$
$$-1<\lambda_1<0, -3<\lambda_2<-2, \lambda_3<\gamma, -2<\lambda_1\lambda_2\lambda_3<-1\Big\}.$$

Proof of Theorem 1.

First, denote $F=(f_1^{\mathrm{T}}, f_2^{\mathrm{T}},\cdots,f_r^{\mathrm{T}})^{\mathrm{T}}$, $\Sigma=(\sigma_1^{\mathrm{T}},\sigma_2^{\mathrm{T}},\cdots,\sigma_r^{\mathrm{T}})^{\mathrm{T}}$, $u=(u_1^{\mathrm{T}},u_2^{\mathrm{T}},\cdots,u_r^{\mathrm{T}})^{\mathrm{T}}$, $B=(B_1^{\mathrm{T}},B_2^{\mathrm{T}},\cdots,B_r^{\mathrm{T}})^{\mathrm{T}}$. Then (4) can be rewritten as

$$\begin{cases} dx_1 = x_2dt,\\ dx_2 = F(x_1,x_2,t)dt+u(t)dt+\Sigma(x_1,x_2,t)dB(t),\\ u(t)=K_pe(t)+K_i\displaystyle\int_0^t e(s)ds+K_d\frac{de(t)}{dt},\end{cases} \tag{6}$$

where $e=(e_1^{\mathrm{T}},e_2^{\mathrm{T}},\cdots,e_r^{\mathrm{T}})^{\mathrm{T}}=x_1-y^*$, $K_p=\mathrm{diag}(k_{p1}I,\cdots,k_{pr}I)$, $K_i=\mathrm{diag}(k_{i1}I,\cdots,k_{ir}I)$, $K_d=\mathrm{diag}(k_{d1}I,\cdots,k_{dr}I)$. Here I is an $n\times n$ unit matrix. We will not say the dimension of I in the following, and I denotes the unit matrix of appropriate dimension. Next, denote $y_1(t)=e(t)$, $y_2(t)=\frac{de(t)}{dt}$,

$$y_0=\int_0^t e(s)ds+\Big[\big(\frac{f_1(y^*,0,0)}{k_{i1}}\big)^{\mathrm{T}},\cdots,\big(\frac{f_r(y^*,0,0)}{k_{ir}}\big)^{\mathrm{T}}\Big]^{\mathrm{T}},$$

$$G_1(y_1,y_2,t)=\begin{bmatrix}g_{11}(y_1,y_2,t)\\ \vdots\\ g_{1r}(y_1,y_2,t)\end{bmatrix}=\begin{bmatrix}f_1(y_1+y^*,y_2,t)-f_1(y^*,0,t)\\ \vdots\\ f_r(y_1+y^*,y_2,t)-f_r(y^*,0,t)\end{bmatrix},$$

$$G_2(y_1,y_2,t)=\begin{bmatrix}g_{21}(y_1,y_2,t)\\ \vdots\\ g_{2r}(y_1,y_2,t)\end{bmatrix}=\begin{bmatrix}\sigma_1(y_1+y^*,y_2,t)\\ \vdots\\ \sigma_r(y_1+y^*,y_2,t)\end{bmatrix},$$

then (6) is equivalent to

$$\begin{cases} dy_0=y_1dt,\\ dy_1=y_2dt,\\ dy_2=(G_1(y_1,y_2,t)+K_iy_0+K_py_1+K_dy_2)dt+G_2(y_1,y_2,t)dB(t).\end{cases} \tag{7}$$

Denote $\bar{r}=\{1,2,\cdots,r\}$. Because $f_j\in\mathscr{F}_{L_j^f}$, $\sigma_j\in\mathscr{F}_{L_j^\sigma}$, it is not difficult to see that $g_{1j}\in\mathscr{F}_{L_j^f}, g_{2j}\in\mathscr{F}_{L_j^\sigma}, j\in\bar{r}$, and $G_i(0,0,t)=0, i=1,2$. Hence 0 is an equilibrium of (7).

Denote

$$Y=\begin{bmatrix}y_0\\y_1\\y_2\end{bmatrix},\ A=\begin{bmatrix}0&I&0\\0&0&I\\K_i&K_p&K_d\end{bmatrix},$$

A is a $3rn\times 3rn$ matrix. Then (7) can be rewritten as

$$dY=AYdt+\begin{bmatrix}0\\0\\G_1(y_1,y_2,t)\end{bmatrix}dt+\begin{bmatrix}0\\0\\G_2(y_1,y_2,t)\end{bmatrix}dB(t). \tag{8}$$

Using the properties of determinant, we have

$$\det(\lambda I_{3rn\times 3rn}-A)=\prod_{j=1}^{r}(\lambda^3-k_{dj}\lambda^2-k_{pj}\lambda-k_{ij})^n.$$

For each $j\in\bar{r}$, we take (k_{pj},k_{ij},k_{dj}) such that $\lambda^3-k_{dj}\lambda^2-k_{pj}\lambda-k_{ij}=0$ has three distinct negative real roots $\lambda_{1j},\lambda_{2j},\lambda_{3j}$, which is feasible because we can adjust all the coefficients of this cubic equation.

Now, we introduce some notations that will be used throughout the sequel.

$$a_j=\frac{1}{(\lambda_{3j}-\lambda_{1j})(\lambda_{2j}-\lambda_{1j})},\ b_j=\frac{1}{(\lambda_{3j}-\lambda_{2j})(\lambda_{1j}-\lambda_{2j})},$$
$$c_j=\frac{\lambda_{3j}}{(\lambda_{3j}-\lambda_{1j})(\lambda_{3j}-\lambda_{2j})},\ d_j=\lambda_{1j}\lambda_{2j}\lambda_{3j},\ \Lambda_j=(\lambda_{1j},\lambda_{2j},\lambda_{3j}),$$

$$\xi(\Lambda_j)=\sqrt{3+\lambda_{1j}^2+\lambda_{2j}^2+\frac{1}{\lambda_{3j}^2}}, \tag{9}$$

$$\phi(\Lambda_j)=\sqrt{\frac{(\lambda_{3j}-\lambda_{2j})^2+(\lambda_{3j}-\lambda_{1j})^2+\lambda_{3j}^2(\lambda_{2j}-\lambda_{1j})^2}{(\lambda_{3j}-\lambda_{1j})^2(\lambda_{2j}-\lambda_{1j})^2(\lambda_{3j}-\lambda_{2j})^2}}=\sqrt{a_j^2+b_j^2+c_j^2}, \tag{10}$$

$$\psi(\Lambda_j)=-\frac{\lambda_{1j}(\lambda_{3j}-\lambda_{2j})^2+\lambda_{2j}(\lambda_{3j}-\lambda_{1j})^2+\lambda_{3j}^3(\lambda_{2j}-\lambda_{1j})^2}{2(\lambda_{3j}-\lambda_{1j})^2(\lambda_{2j}-\lambda_{1j})^2(\lambda_{3j}-\lambda_{2j})^2}=-\frac{1}{2}(\lambda_{1j}a_j^2+\lambda_{2j}b_j^2+\lambda_{3j}c_j^2),\quad j\in\bar{r}. \tag{11}$$

Let $C_i=\mathrm{diag}(\lambda_{i1},\cdots,\lambda_{ir})\otimes I_{n\times n}, i=1,2,3$, where $\otimes$ is the Kronecker product. Define three matrices $P'_{2rn\times 3rn}$, $P_{3rn\times 3rn}$ and $J_{3rn\times 3rn}$,

$$P'=\begin{bmatrix}I&I&C_3^{-1}\\C_1&C_2&I\end{bmatrix},\quad P=\begin{bmatrix}C_1^{-1}&C_2^{-1}&C_3^{-2}\\I&I&C_3^{-1}\\C_1&C_2&I\end{bmatrix},$$
$$J=\mathrm{diag}(C_1,C_2,C_3).$$

Then, it is easy to see that P is invertible and

$$P^{-1}=\begin{bmatrix}*&*&\mathrm{diag}(\lambda_{11}a_1I,\cdots,\lambda_{1r}a_rI)\\ *&*&\mathrm{diag}(\lambda_{21}b_1I,\cdots,\lambda_{2r}b_rI)\\ *&*&\mathrm{diag}(\lambda_{31}c_1I,\cdots,\lambda_{3r}c_rI)\end{bmatrix},$$

where the $*$ in P^{-1} represents the element we don't care about in our proof of the theorem. We can get $A=PJP^{-1}$ by simple calculations.

Next, define an invertible linear transformation $Y=PZ$, and denote $Z=(z_1^T,z_2^T,z_3^T)^T$, where $z_i=(z_{i1}^T,z_{i2}^T,\cdots,z_{ir}^T)^T$, and z_{ij} is n-dimensional column vector, $i=1,2,3, j\in\bar{r}$. Then we can rewrite the equation (8) in a diagonal form,

$$dZ=JZdt+P^{-1}\begin{bmatrix}0\\0\\G_1(P'Z,t)\end{bmatrix}dt+P^{-1}\begin{bmatrix}0\\0\\G_2(P'Z,t)\end{bmatrix}dB(t). \tag{12}$$

Therefore, we have

$$\begin{cases} dz_1 = \begin{bmatrix} \lambda_{11}z_{11}+\lambda_{11}a_1g_{11}(P'Z,t) \\ \vdots \\ \lambda_{1r}z_{1r}+\lambda_{1r}a_rg_{1r}(P'Z,t) \end{bmatrix} dt + \begin{bmatrix} \lambda_{11}a_1g_{21}(P'Z,t) \\ \vdots \\ \lambda_{1r}a_rg_{2r}(P'Z,t) \end{bmatrix} dB(t) \\ \quad \triangleq S_1(P'Z,t)dt + R_1(P'Z,t)dB(t), \\ dz_2 = \begin{bmatrix} \lambda_{21}z_{21}+\lambda_{21}b_1g_{11}(P'Z,t) \\ \vdots \\ \lambda_{2r}z_{2r}+\lambda_{2r}b_rg_{1r}(P'Z,t) \end{bmatrix} dt + \begin{bmatrix} \lambda_{21}b_1g_{21}(P'Z,t) \\ \vdots \\ \lambda_{2r}b_rg_{2r}(P'Z,t) \end{bmatrix} dB(t) \\ \quad \triangleq S_2(P'Z,t)dt + R_2(P'Z,t)dB(t), \\ dz_3 = \begin{bmatrix} \lambda_{31}z_{31}+\lambda_{31}c_1g_{11}(P'Z,t) \\ \vdots \\ \lambda_{3r}z_{3r}+\lambda_{3r}a_rg_{1r}(P'Z,t) \end{bmatrix} dt + \begin{bmatrix} \lambda_{31}c_1g_{21}(P'Z,t) \\ \vdots \\ \lambda_{3r}c_rg_{2r}(P'Z,t) \end{bmatrix} dB(t) \\ \quad \triangleq S_3(P'Z,t)dt + R_3(P'Z,t)dB(t). \end{cases} \tag{13}$$

Now, we construct the following Lyapunov function,

$$V(Z) = \frac{1}{2}\sum_{j=1}^{r}(\lambda_{2j}\lambda_{3j}\|z_{1j}\|^2 + \lambda_{1j}\lambda_{3j}\|z_{2j}\|^2 + \lambda_{1j}\lambda_{2j}\|z_{3j}\|^2). \tag{14}$$

$\nabla V = (\frac{\partial V}{\partial z_1}, \frac{\partial V}{\partial z_2}, \frac{\partial V}{\partial z_3})^{\mathrm{T}}$ is the gradient of V, where $\frac{\partial V}{\partial z_1} = (\lambda_{21}\lambda_{31}z_{11}, \cdots, \lambda_{2r}\lambda_{3r}z_{1r})^{\mathrm{T}}$, $\frac{\partial V}{\partial z_2} = (\lambda_{11}\lambda_{31}z_{21}, \cdots, \lambda_{1r}\lambda_{3r}z_{2r})^{\mathrm{T}}$, $\frac{\partial V}{\partial z_3} = (\lambda_{11}\lambda_{21}z_{31}, \cdots, \lambda_{1r}\lambda_{2r}z_{3r})^{\mathrm{T}}$. $H(V)$ is the $3rn \times 3rn$ symmetric Hessian matrix $H(V) = \mathrm{diag}(\lambda_{21}\lambda_{31}I, \cdots, \lambda_{2r}\lambda_{3r}I, \lambda_{11}\lambda_{31}I, \cdots, \lambda_{1r}\lambda_{3r}I, \lambda_{11}\lambda_{21}I, \cdots, \lambda_{1r}\lambda_{2r}I)$.

Next, let us calculate the differential operator L associated with (13),

$$\begin{aligned} \mathrm{L}V(Z) &= \frac{\partial V}{\partial t} + (S_1^{\mathrm{T}}\ S_2^{\mathrm{T}}\ S_3^{\mathrm{T}})\nabla V + \frac{1}{2}\mathrm{Tr}\left[\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}[R_1^{\mathrm{T}}\ R_2^{\mathrm{T}}\ R_3^{\mathrm{T}}]\mathrm{H}(V)\right] \\ &= \sum_{j=1}^{r} d_j(\|z_{1j}\|^2 + \|z_{2j}\|^2 + \|z_{3j}\|^2) \\ &\quad + \left(d_1g_{11}^{\mathrm{T}}, \cdots, d_rg_{1r}^{\mathrm{T}}\right)\begin{bmatrix} a_1z_{11}+b_1z_{21}+c_1z_{31} \\ \vdots \\ a_rz_{1r}+b_rz_{2r}+c_rz_{3r} \end{bmatrix} \\ &\quad + \frac{1}{2}\sum_{j=1}^{r} d_j(\lambda_{1j}a_j^2 + \lambda_{2j}b_j^2 + \lambda_{3j}c_j^2)\|g_{2j}\|^2 \\ &\triangleq \mathrm{I} + \mathrm{II} + \mathrm{III}, \end{aligned} \tag{15}$$

where g_{ij}^{T} is the abbreviation for $(g_{ij}(P'Z,t))^{\mathrm{T}}, i = 1, 2, j \in \bar{r}$, and Tr denotes the trace of a matrix.

It is easy to see that

$$\mathrm{I} \le \max_{j\in\bar{r}}\{d_j\}\|Z\|^2.$$

Then, we proceed to estimate the upper bound of $\|P'\|$, where the matrix norm $\|\cdot\|$ is the operator norm induced by the Euclidean norm, i.e., $\|P'\| = \sup_{\|w\|=1}\|P'w\|$. Some calculations using Cauchy inequality give that $\|P'\| \le \max_{j\in\bar{r}}\{\xi(\Lambda_j)\}$, where $\xi(\Lambda_j)$ is defined in (9). Considering the Lipschitz property of g_{1j} and g_{2j}, we have

$$\|g_{1j}(P'Z,t)\| \le L_j^f \max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\|, \|g_{2j}(P'Z,t)\| \le L_j^\sigma \max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\|.$$

Using the above inequality,

$$\begin{aligned} &\|((d_1g_{11}(P'Z,t))^{\mathrm{T}}, \cdots, (d_rg_{1r}(P'Z,t))^{\mathrm{T}})\| \\ &= \sqrt{\sum_{j=1}^{r}\|d_jg_{1j}(P'Z,t)\|^2} \le \sqrt{\sum_{j=1}^{r} d_j^2(L_j^f)^2}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\| \\ &\le -\min_{j\in\bar{r}}\{d_j\}\sqrt{\sum_{j=1}^{r}(L_j^f)^2}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\|. \end{aligned}$$

Next, according to Cauchy inequality,

$$\begin{aligned} &\left\|\begin{bmatrix} a_1z_{11}+b_1z_{21}+c_1z_{31} \\ \vdots \\ a_rz_{1r}+b_rz_{2r}+c_rz_{3r} \end{bmatrix}\right\|^2 = \sum_{j=1}^{r}\|a_jz_{1j}+b_1z_{2j}+c_1z_{3j}\|^2 \\ &\le \sum_{j=1}^{r}(a_j^2+b_j^2+c_j^2)(\|z_{1j}\|^2+\|z_{2j}\|^2+\|z_{3j}\|^2) \\ &\le \max_{j\in\bar{r}}\{\phi^2(\Lambda_j)\}\|Z\|^2, \end{aligned} \tag{16}$$

where $\phi(\Lambda_j)$ is defined in (10). Therefore,

$$|\mathrm{II}| \le -\sqrt{\sum_{j=1}^{r}(L_j^f)^2}\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\phi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\|^2,$$

$$\mathrm{III} \le -\sum_{j=1}^{r}(L_j^\sigma)^2\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\psi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi^2(\Lambda_j)\}\|Z\|^2.$$

Consequently,

$$\begin{aligned} \mathrm{L}V(Z) \le &\Big(\max_{j\in\bar{r}}\{d_j\} - \sqrt{\sum_{j=1}^{r}(L_j^f)^2}\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\phi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\} \\ &- \sum_{j=1}^{r}(L_j^\sigma)^2\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\psi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi^2(\Lambda_j)\}\Big)\|Z\|^2. \end{aligned} \tag{17}$$

Now, we verify that if each j chooses the parameters (k_{pj}, k_{ij}, k_{dj}) from Ω_{pid}, the right hand side of (17) is a negative definite quadratic form of Z. It is easy to see that $\xi(\Lambda_j) < \Xi(\theta)$, $\phi(\Lambda_j) < \Phi(\theta)$, $\psi(\Lambda_j) < \Psi(\theta)$ when $\Lambda_j \in \Omega_\theta$. Therefore, we have

$$\begin{aligned} &\max_{j\in\bar{r}}\{d_j\} - \sqrt{\sum_{j=1}^{r}(L_j^f)^2}\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\phi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\} \\ &- \sum_{j=1}^{r}(L_j^\sigma)^2\min_{j\in\bar{r}}\{d_j\}\max_{j\in\bar{r}}\{\psi(\Lambda_j)\}\max_{j\in\bar{r}}\{\xi^2(\Lambda_j)\} \\ &\le \eta_2 - \sqrt{\sum_{j=1}^{r}(L_j^f)^2\eta_1\Phi(\theta)\Xi(\theta)} - \sum_{j=1}^{r}(L_j^\sigma)^2\eta_1\Psi(\theta)\Xi^2(\theta) < 0, \end{aligned} \tag{18}$$

where the last inequality follows from the definition of the set Θ.

Next, by the Itô formula we have

$$dV = \mathrm{L}V dt + \nabla V^{\mathrm{T}}\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} dB(t). \tag{19}$$

Let us denote the diffusion term as $R(P'Z,t)$,

$$\begin{aligned} R(P'Z,t) &= \sum_{j=1}^{r} d_j(a_jg_{2j}^{\mathrm{T}}z_{1j} + b_jg_{2j}^{\mathrm{T}}z_{2j} + c_jg_{2j}^{\mathrm{T}}z_{3j}) \\ &= \left(d_1g_{21}^{\mathrm{T}}, \cdots, d_rg_{2r}^{\mathrm{T}}\right)\begin{bmatrix} a_1z_{11}+b_1z_{21}+c_1z_{31} \\ \vdots \\ a_rz_{1r}+b_rz_{2r}+c_rz_{3r} \end{bmatrix}. \end{aligned}$$

Because

$$\begin{aligned} &\|((d_1g_{21}(P'Z,t))^{\mathrm{T}}, \cdots, (d_rg_{2r}(P'Z,t))^{\mathrm{T}})\| \\ &= \sqrt{\sum_{j=1}^{r}\|d_jg_{2j}(P'Z,t)\|^2} \le \sqrt{\sum_{j=1}^{r} d_j^2(L_j^\sigma)^2}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\| \\ &\le -\min_{j\in\bar{r}}\{d_j\}\sqrt{\sum_{j=1}^{r}(L_j^\sigma)^2}\max_{j\in\bar{r}}\{\xi(\Lambda_j)\}\|Z\|, \end{aligned}$$

and (16), we have

$$\big|R(P'Z,t)\big| \leq -\sqrt{\sum_{j=1}^{r}(L_j^{\sigma})^2 \min_{j\in\bar r}\{d_j\}\max_{j\in\bar r}\{\phi(\Lambda_j)\}\max_{j\in\bar r}\{\xi(\Lambda_j)\}\|Z\|^2}.$$

Therefore,

$$|R(P'Z,t)|^2 = O(\|Z\|^4).$$

Hence, according to Theorem 4.1 in Mao (2008), we can get

$$\mathrm{E}\int_0^T |R(P'Z,t)|^2 dt < \infty.$$

So,

$$\mathrm{E}\int_0^T R(P'Z,t)dB(t) = 0.$$

According to (19), we obtained that for any $T \geq 0$,

$$V(Z(T)) = V(Z(0)) + \int_0^T \mathrm{L}V(Z(t))dt + \int_0^T R(P'Z,t)dB(t). \tag{20}$$

Taking the expectation on both sides of (20) and making use of (17) and (18), we obtain

$$\begin{aligned}\mathrm{E}V(Z(T)) &\leq V(Z(0)) + \mathrm{E}\int_0^T \mathrm{L}V(Z(t))dt + \mathrm{E}\int_0^T R(P'Z,t)dB(t)\\ &\leq V(Z(0)) + \int_0^T\Big(\eta_2 - \sqrt{\sum_{j=1}^{r} L_{1j}^2\eta_1\,\Phi(\theta)\Xi(\theta)}\\ &\quad - \sum_{j=1}^{r} L_{2j}^2\eta_1\,\Psi(\theta)\Xi^2(\theta)\Big)\|Z(t)\|^2 dt.\end{aligned}$$

From this and (18), by the definition of $V(Z)$ and its positive define property, we have for all $T \geq 0$, there exist constants $c_1 > 0, c_2 > 0$ such that

$$\mathrm{E}(\|Z(T)\|^2) \leq c_1 V(Z(0)), \quad \int_0^T \mathrm{E}(\|Z(t)\|^2)dt \leq c_2 V(Z(0)).$$

As $\|Y\|^2 = \|PZ\|^2 = \|Z^{\mathrm{T}}P^{\mathrm{T}}PZ\| \leq \|P\|^2\|Z\|^2$, there exist constants $c_3 > 0, c_4 > 0$ such that

$$\mathrm{E}(\|Y(T)\|^2) \leq c_3 V(Z(0)), \tag{21}$$

$$\int_0^T \mathrm{E}(\|Y(t)\|^2)dt \leq c_4 V(Z(0)). \tag{22}$$

Consequently, we have

$$\sup_{t\geq 0}\mathrm{E}[y_0^2(t) + y_1^2(t) + y_2^2(t)] < \infty, \tag{23}$$

which proves the global stability of (4).

Now, let $T \to \infty$ in (22), we have

$$\int_0^\infty \mathrm{E}(\|y_1(t)\|^2)dt \leq \int_0^\infty \mathrm{E}(\|Y(t)\|^2)dt \leq c_4 V(Z(0)).$$

Next, we use the Barbalat Lemma (see Lemma A.6 in Reissig et al. (1974)) to conclude that $\mathrm{E}(\|y_1(t)\|^2) \to 0$ as $t \to \infty$. For that we need to prove the uniform continuity of $\mathrm{E}(\|y_1(t)\|^2)$ on $(0,\infty)$, i.e., for each $j \in \bar r, k \in \{1,\cdots,n\}$, for $\forall\epsilon > 0$ and $\forall t_1, t_2 \in (0,\infty)$, there exists $\delta = \delta(\epsilon) > 0$ such that when $|t_1 - t_2| < \delta$, it must have $|\mathrm{E}y_{1jk}^2(t_1) - \mathrm{E}y_{1jk}^2(t_2)| \leq \epsilon$, where $y_{1jk} \in \mathbb{R}$ is the $[j(n-1)+k]^{\mathrm{th}}$ element of y_1. For simplicity, we use y to denote y_{1jk}.

Firstly, we have the inequality

$$|\mathrm{E}y^2(t_1) - \mathrm{E}y^2(t_2)| \leq \mathrm{E}|y^2(t_1) - y^2(t_2)|.$$

By the mean value theorem, there exists a random variable $\bar y \in [y(t_1), y(t_2)]$ such that

$$\mathrm{E}|y^2(t_1) - y^2(t_2)| \leq \mathrm{E}[2\bar y|y(t_1) - y(t_2)|].$$

According to the Schwarz inequality, we have

$$\mathrm{E}[2\bar y|y(t_1) - y(t_2)|] \leq 2\sqrt{\mathrm{E}\bar y^2}\sqrt{\mathrm{E}|y(t_1) - y(t_2)|^2}. \tag{24}$$

Note that

$$|\bar y| \leq \max\{|y(t_1)|, |y(t_2)|\} \leq |y(t_1)| + |y(t_2)|,$$

by (23), there exists a constant $M_1 > 0$ such that

$$\mathrm{E}|\bar y|^2 \leq 2\mathrm{E}\{|y(t_1)|^2 + |y(t_1)|^2\} \leq 4\sup_{t\geq 0}\mathrm{E}|y(t)|^2 \leq M_1. \tag{25}$$

Furthermore, by the fact that $dy_{1jk}/dt = y_{2jk}$, where $y_{2jk} \in \mathbb{R}$ is the $[j(n-1)+k]^{\mathrm{th}}$ element of y_2, we use the mean value theorem again to get a random point $\bar t \in [t_1, t_2]$ such that

$$\mathrm{E}|y(t_1) - y(t_2)|^2 = \mathrm{E}[y_{2jk}^2(\bar t)(t_1 - t_2)^2] \leq \delta^2\mathrm{E}(\sup_{t\in[t_1,t_2]}\{|y_{2jk}(t)|^2\}).$$

Then, by the boundedness property (23), from Lemma 3.2 in Mao (2008), we know that there exists a constant $M_2 > 0$ such that

$$\mathrm{E}(\sup_{t\in[t_1,t_2]}\{|z_t|^2\}) \leq M_2. \tag{26}$$

Consequently, by (24) $-$ (26), we can get

$$\big|\mathrm{E}y^2(t_1) - \mathrm{E}y^2(t_2)\big| \leq 2\delta\sqrt{M_1 M_2}.$$

Hence, taking $\delta = \frac{\epsilon}{2\sqrt{M_1 M_2}}$, we have

$$\big|\mathrm{E}y^2(t_1) - \mathrm{E}y^2(t_2)\big| \leq \epsilon.$$

By Barbalat Lemma, we conclude that

$$\lim_{t\to\infty}\mathrm{E}|x_1(t) - y^*|^2 = 0.$$

Therefore, for each agent j, the regulation error asymptotically approaches to zero in the sense that

$$\lim_{t\to\infty}\mathrm{E}|x_{1j}(t) - y_j^*|^2 = 0.$$

Finally, to complete the proof of Theorem 1, we need to show that Ω_{pid} is unbounded and open. To begin with, we show that Θ is nonempty. In fact, for any $\theta = (\alpha, \beta_1, \beta_2, \gamma, \eta_1, \eta_2)$ with property $\beta_1 < \beta_2 < \alpha < 0$, $\eta_1 < \eta_2 < 0$, we can chose sufficiently negative γ such that $\theta \in \Theta$, since $\lim_{\gamma\to-\infty}\Phi(\theta) = 0$, $\lim_{\gamma\to-\infty}\Psi(\theta) = 0$ and $\sup_{\gamma<\beta_1}H(\theta)$ is bounded. Obviously, Ω_θ is nonempty, unbounded and open for $\theta \in \Theta$.

Moreover, from the relationship,

$$\begin{cases} k_p = -(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3),\\ k_i = \lambda_1\lambda_2\lambda_3,\\ k_d = \lambda_1 + \lambda_2 + \lambda_3,\end{cases} \tag{27}$$

we claim that Ω_{pid} is also an open set in $\mathbb{R}^3$ since the Jacobian matrix $\bar J$ of the mapping defined by (27) is nonsingular at every point $(\lambda_1, \lambda_2, \lambda_3) \in \Omega_\theta$.

$$\begin{aligned}\det\bar J &= \det\begin{bmatrix} -(\lambda_2+\lambda_3) & -(\lambda_1+\lambda_3) & -(\lambda_1+\lambda_2)\\ \lambda_2\lambda_3 & \lambda_1\lambda_3 & \lambda_1\lambda_2\\ 1 & 1 & 1\end{bmatrix}\\ &= (\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_3-\lambda_2) \neq 0.\end{aligned}$$

The unboundedness of Ω_{pid} is easily to certified by its definition and Ω_θ's properties.

This completes the proof of the theorem.

4. SIMULATION

We illustrate the theoretical results by a numerical example. We consider the following system,

$$\begin{cases} dx_{11} = x_{21}dt, \\ dx_{21} = f_1(x,t)dt + u_1 dt + \sigma_1(x,t)dB_1(t), \\ dx_{12} = x_{22}dt, \\ dx_{22} = f_2(x,t)dt + u_2 dt + \sigma_2(x,t)dB_2(t), \end{cases}$$

where $x = (x_{11}, x_{12}, x_{21}, x_{22})$, u_j is the control input of agent j, $j = 1, 2$. Assume that f_j and σ_j are unknown functions satisfying $f_j \in \mathscr{F}_{L_j^f}, \sigma_j \in \mathscr{F}_{L_j^\sigma}$, where L_j^f and L_j^σ are known positive constants, $j = 1, 2$. Here we take $f_1 = l_1 \sin x_{11} \cos(x_{12} - x_{11}) + l_2 \cos x_{21}, f_2 = l_3 \cos x_{12} + l_4 x_{22}, \sigma_1 = l_5 \cos x_{11} \sin x_{22} + l_6 \cos x_{21}, \sigma_2 = l_7 \sin x_{12} + l_8 x_{22}$, where $l_1, l_2, \cdots, l_8$ are unknown constants, and assume that we only know the Lipschitz constants $L_j^f = L_j^\sigma = 1, j = 1, 2$ of the functions rather than the explicit forms. The objective of agent j is using the following PID controller,

$$u_j(t) = k_{pj}(x_{1j}(t) - y_j^*) + k_{ij}\int_0^t (x_{1j}(s) - y_j^*)ds + k_{dj}\frac{dx_{1j}(t)}{dt},$$

such that x_{1j} converges to the given constant setpoint y_j^*. Assume that the initial state is $x(0) = (8, 6, 1, 4)$ and $y_1^* = 3, y_2^* = 0$. According to the Example in Section 3, we chose the PID parameters $(k_{p1}, k_{i1}, k_{d1}) = (-950.765, -1.9, -382.502) \in \Omega_{pid}$ and $(k_{p2}, k_{i2}, k_{d2}) = (-988.295, -1.975, -397.502) \in \Omega_{pid}$. Then, Fig.1 shows that both agents achieve their regulation objectives, which demonstrates the effectiveness of the theoretical results.

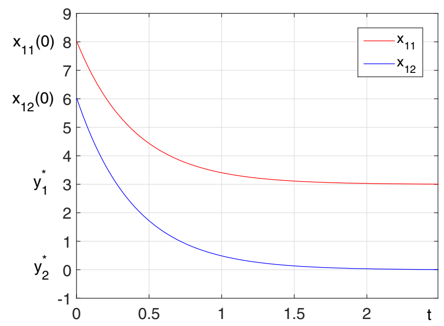

Fig. 1. The integral curves of x_{11} and x_{12}

5. CONCLUSION

In this paper, we have provided a design method for uncoupled PID controllers of a class of second-order coupled multi-agent nonlinear uncertain stochastic systems, and have presented a theoretical result on global stability and asymptotically regulation of the closed-loop control systems. It has been shown that as long as each agent takes its PID parameters arbitrarily from a three-dimensional manifold constructed by using certain global information about the Lipschitz constants of the unknown nonlinear functions, and each agent only uses its own regulation error in its PID feedback loop, the whole closed-loop control system will be globally stabilized and the regulation error of each agent will asymptotically approach to zero. Of course, there are still many problems remain to be solved concerning more general uncertain nonlinear stochastic dynamical systems, which belongs to further investigation.

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