Modified PI controller for the stabilization of high-order unstable delayed systems with complex conjugate poles and a minimum phase zero

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Abstract: This work deals with the stabilization problem of a particular class of high-order unstable linear systems with time delay. In particular, systems with one unstable pole, $q-1$ complex conjugate stable poles and one minimum phase zero. To solve this problem a modified version for the traditional PI controller, called in this work the Proporcional Integral Filtered PI_f, is proposed. This new scheme includes a low-pass first order filter and allows improving the existing results on controlling high-order systems with time delay. Necessary and sufficient conditions for the existence of the PI_f controller are expressed in terms of the parameters of the system and the maximum allowable time-delay magnitude. The proposed control scheme is illustrated by a numerical example applied to the temperature control of an unstable Continuously Stirred Tank Reactor (CSTR) linear model focused on the production of propylene glycol.

Keywords: Time-delay, unstable system, complex conjugate poles, modified PI controller.

1. INTRODUCTION

Time delay systems are common in industrial processes and are often used to model different kinds of engineering systems, where propagation and transmission of information or material are involved in computing control signals, data acquisition, etc Niculescu (2001). The presence of large delays makes system analysis and control design much more complex. In general, control system performance is very sensitive to these delays, which reflects on poor performance in transient response, oscillations, among others is obtained. In fact, a closed-loop control system may become unstable as a consequence of delays, Zhong (2006).

The problem becomes more complicated when the system not only has a time delay but also it is unstable. Therefore, the interest of tackling unstable processes containing a delay term has been growing in the control community due to its complexity in achieving stabilization and an adequate performance Sipahi et al. (2011). In fact, this kind of unstable systems with time delay are commonly found in chemical processes such as in liquid storage tanks Pierdomenico and Ciccio (2011), continuously stirred tank reactors (CSTR) Bequette (2003), among others.

A solution to deal with time-delay systems is to use a Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers. For instance, Silva et al. (2004) have solved the stabilization of first-order systems with time-delay using a version of the Hermite-Biehler Theorem derived by Pontryagin (1995) applicable to a quasi-polynomial. Moreover, this method was generalized to the second-order integrating processes with time-delay by Ou et al. (2006). For a first-order unstable system with time-delay, the D-partition technique was applied to characterize the stability domain in the space of system and controller parameters as shown in Hwang and Hwang (2004). However, in these references, necessary and sufficient conditions for the existence of stabilizer controllers are not provided. With a different perspective, in Lee et al. (2010), a frequency domain approach is exploited to analyze the stability of a particular class of high-order system by means of $P/PI/PD/PID$ controllers. The generalization of this result is provided in Hernandez-Perez et al. (2015) where unstable delayed systems with possible complex conjugate poles are addressed. Recent works focuses on a more particular class of systems, for instance

⋆ This work was partially supported by the National Council for Science and Technology, CONACyT-Mexico, under the grant 254329.
⋆⋆This work was partially supported by the Secretary of Sciences, Technology and Innovation of Mexico City, under the grant SECITI/079/2017

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in Novella et al. (2017), controllers for the stabilization of high order delayed systems with two unstable poles are considered. It is important to note that all previously cited works consider the use of the classical structure of the PID controller.

However, although PID controllers are the most used in the industry, they have some disadvantages even without the presence of delays. For example, the derivative term in the PID controller causes the controller to have implementation problems because, from a transfer function point of view, the controller is an improper function. Moreover, the derivative term response is highly sensitive to measurement noise in the process variable signal. If the sensor feedback signal is noisy or if the control loop rate is too slow, the derivative response can make the control system unstable. Likewise, in the case of processes with large time-delays, the anticipatory action of the derivative term is no longer working since the linear approximation is only valid for small values of the derivative term. For this reason, it is common to avoid the use of the derivative term in the control strategy and just keep the proportional-integral action. However, the design of a feedback control law by using a PI controller becomes more complicated when besides the time-delay and unstable systems, the processes have a minimum phase zero. Notice that the performance is further limited when the process contains a zero in the right half plane (RHP).

A modified version of the traditional PI control scheme is proposed which is called the Proportional Integral Filtered (PI_f) controller. This control scheme is composed of a traditional PI adding as a third term a simple first order filter instead of the derivative term used in the traditional PID. This modification allows having some advantages such as reducing the measurement noise that may be present in the system since it does not resort on a derivative term. Moreover, it also allows the stabilization and control of the same family of unstable systems with time delay considered in Lee et al. (2010) and Hernandez-Perez et al. (2015), but with the advantage that the delays supported by this new scheme are larger than those supported by the traditional PI. Additionally, one of the main advantages of the PI_f is that it can deal, with unstable systems which include a transmission zero, which is an interesting feature from the control point of view. This new scheme keeps the basic properties of a conventional PI controller such as disturbance rejection and reference tracking of step type signals. Hence, conditions to stabilize high-order unstable delayed systems with one unstable pole, a pair of possible complex conjugate poles and a minimum phase zero are presented in this work. The stabilization conditions are expressed in terms of the maximum allowable time-delay magnitude. Further, a procedure is provided for determining the parameter ranges of the stabilizing controller. The proposed control scheme is illustrated by a numerical example consisting of the temperature in an unstable linear model of a Chemical Process-Propylene Glycol taken from Bequette (2003).

The rest of the work is organized as follows: Section 2 presents the problem statement. Section 3 addresses the proposed control strategy. In Section 4, the results are applied to a numerical example. Finally, Section 5 ends the papaer with some concluiding remarks.

2. PROBLEM STATEMENT

Consider the class of high-order Single-Input Single-Output (SISO) linear unstable delayed systems with possible complex conjugate poles and a single zero given by,

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{\alpha(s+\beta)e^{-\tau s}}{(s-\gamma)\prod_{m=1}^{q}(s^2+2\zeta_m\omega_{n_m}s+{\omega_{n_m}}^2)}, \quad (1)$$

with γ, β and $\tau > 0$, $q = (n-1)/2$, ζ the damping relation and ω_n the undamped natural frequency. In order to simplify the notation, it is assumed that the order of the system is odd, i.e., $n = 2q+1$. The stability analysis of (1) will be carried out by means a modified PI controller (PI_f), defined as

$$H(s) = k_p\left(1+\frac{k_i}{s}+\frac{k_f}{s+\phi}\right) \quad (2)$$

with k_p, k_i and $\phi > 0$. Notice that the obtained closed–loop system will have the general form,

$$\frac{Y(s)}{R(s)} = \frac{H(s)G(s)e^{-\tau s}}{1+H(s)G(s)e^{-\tau s}}. \quad (3)$$

It is clear that the term $e^{-\tau s}$ located at the denominator of the transfer function (3), leads to a system with an infinite number of poles and where the closed–loop stability properties must be carefully stated. Therefore, the objective is to provide conditions to stabilize the class of systems (1) by means of a PI_f controller (2). Also, it is intended to characterize all possible values for the parameters k_p, k_i, k_f and ϕ that render the closed–loop system asymptotically stable. Notice that from (3), the corresponding open-loop transfer function can be expresed as,

$$Q(s) = H(s)G(s)e^{-\tau s}. \quad (4)$$

Notice that instead of adding a derivative term to the classical PI, in order to get a PID controller, in this work it is add a first order filter where its cutoff frequency is not determined by the approximation of the derivative term. Despite this, notice that even if the derivative term is not explicitly included in (2), after elementary algebra, this expression can be rewritten as a filtered controller or if it is necessary, it can be seen as a PID with the derivative term numerically approximated. However it is interesting to note that, the main advantage provided by the PI_f against most of filtered-PID schemes is that the PI_f encloses in its core the cutoff frequency of the implemented first order filter. This cutoff frequency is directly related to the necessary and sufficient stability ranges required to deal with time-delayed unstable systems which contain a zero with non-minimum phase characteristic.

3. MAIN RESULTS

In order to state the PI_f control strategy, the following theorem is stated.

Theorem 1. Consider the class of high-order unstable delayed systems with possible complex conjugate poles and a zero in the left half complex plane give by (1). There exists a PI_f controller given by (2) such that the corresponding closed-loop system is stable if and only if,

$$\tau < \frac{1}{\gamma} + \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q}\frac{2\zeta_m^2-1}{\omega_{n_m}^2}} - 2\sum_{m=1}^{q}\frac{\zeta_m}{\omega_{n_m}}. \quad (5)$$

The proof of Theorem 1 is based on the well-known Nyquist stability criterion which is stated below.

Theorem 2. (Nyquist stability criterion). A linear system is stable if and only if $N + P = 0$, where P the number of poles in the right half complex plane and N the number of clockwise rotations to the point $(-1, 0)$ (if N is negative, the rotations are in the counterclockwise direction) in the Nyquist diagram.

Proof. Theorem 1. Let us consider a frequency domain analysis. From (4), the open-loop frequency response is given by,

$$Q(j\omega) = k_p \frac{\alpha\left((j\omega)^2 + (k_f + k_i + \phi)j\omega + k_i\phi\right)(j\omega + \beta)\, e^{-\tau j\omega}}{j\omega\,(j\omega - \gamma)\,(j\omega + \phi)\prod_{m=1}^{q}((j\omega)^2 + 2\zeta_m\omega_{n_m} j\omega + \omega_{n_m}^2)}. \tag{6}$$

In order to simplify the analysis, consider the following definitions,

$$\begin{aligned} \bar{k}_p &= k_p(k_f + k_i + \phi) \\ \bar{k}_f &= \frac{1}{k_f + k_i + \phi} \\ \bar{k}_i &= \frac{k_i\phi}{k_f + k_i + \phi}. \end{aligned} \tag{7}$$

That allow rewriting the open-loop transfer function (6) as,

$$Q(j\omega) = \frac{\bar{k}_p\bar{k}_f\alpha\left((j\omega)^2 + (\frac{1}{\bar{k}_f})j\omega + \frac{\bar{k}_i}{\bar{k}_f}\right)(j\omega + \beta)\, e^{-j\omega\tau}}{j\omega\,(j\omega - \gamma)\,(j\omega + \phi)\prod_{m=1}^{q}((j\omega)^2 + 2\zeta_m\omega_{n_m} j\omega + \omega_{n_m}^2)}. \tag{8}$$

Due to the freedom provided when selecting the parameter ϕ of the proposed controller, let us choose the cutoff frequency of the PI_f as $\phi = \beta$. Therefore, using (7), from (6), the open-loop response is obtained as,

$$Q(j\omega) = \bar{k}_p\bar{k}_f\alpha \frac{\left((j\omega)^2 + (\frac{1}{\bar{k}_f})j\omega + \frac{\bar{k}_i}{\bar{k}_f}\right) e^{-j\omega\tau}}{j\omega\,(j\omega - \gamma)\prod_{m=1}^{q}((j\omega)^2 + 2\zeta_m\omega_{n_m} j\omega + \omega_{n_m}^2)}. \tag{9}$$

For the simplicity, let us consider $\bar{k}_i = 0$. The phase and magnitude expression of (9) are given by,

$$\begin{aligned} \angle Q_{\bar{k}_i=0}(j\omega) = &-\left(\pi - \arctan\left(\tfrac{\omega}{\gamma}\right)\right) - \omega\tau + \\ &\arctan\left(\bar{k}_f\omega\right) - \sum_{m=1}^{q}\arctan\left(\frac{2\zeta_m\left(\frac{\omega}{\omega_{n_m}}\right)}{1-\left(\frac{\omega}{\omega_{n_m}}\right)^2}\right) \end{aligned} \tag{10}$$

$$\begin{aligned} M_{Q_{\bar{k}_i=0}}(j\omega) = \\ \bar{k}_p\alpha\sqrt{\frac{1+\bar{k}_f^2\omega^2}{(\omega^2+\gamma^2)\prod_{m=0}^{q}\left(\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m^2-1)+\omega_{n_m}^4\right)}}. \end{aligned} \tag{11}$$

Necessity. Suppose that there exists a PI_f controller such that in closed-loop with (1) produces an asymptotically stable system. Then, the Nyquist criterion is satisfied and therefore, there exists a counterclockwise rotation to the point $(-1, 0)$ in the complex plane as it is shown in Fig. 1.

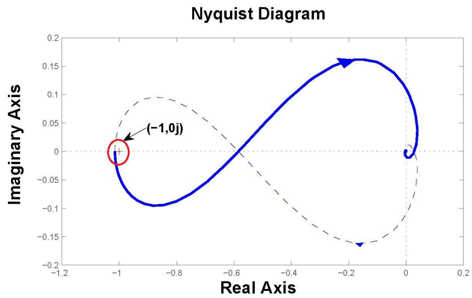

Fig. 1. Nyquist stability criterion.

Therefore, from the Nyquist stability criterion and taking into account $\bar{k}_i = 0$, the phase trajectory begins at $\angle Q_{\bar{k}_i=0}(j0) = -\pi$ for $\omega = 0$ and start its path in a positive direction in the Nyquist diagram, i.e., the phase expression is a increasing function of ω, which has only one change of sign as $\omega \to \infty$. The existence of a counterclockwise rotation to the $(-1, 0)$ point allows concluding the next inequality,

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0} > 0 \tag{12}$$

or equivalently,

$$\begin{aligned} \tfrac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\big|_{\omega=0} = -\tau + \tfrac{\gamma}{\omega^2+\gamma^2} + \tfrac{\bar{k}_f}{\bar{k}_f^2\omega^2+1} - \\ -\sum_{m=1}^{q}\left(\frac{2\zeta_m\omega_{n_m}(\omega^2+\omega_{n_m}^2)}{\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m^2-1)+\omega_{n_m}^4}\right) > 0. \end{aligned}$$

Therefore, evaluating at $\omega = 0$ yields,

$$-\tau + \frac{1}{\gamma} + \bar{k}_f - \sum_{m=1}^{q}\left(\frac{2\zeta_m}{\omega_{n_m}}\right) > 0,$$

from which,

$$\tau < \frac{1}{\gamma} + \bar{k}_f - \sum_{m=1}^{q}\left(\frac{2\zeta_m}{\omega_{n_m}}\right). \tag{13}$$

From the counterclockwise rotation assumption and the fact that (12) is satisfied for $\omega = 0$, it is clear that for this frequency value the magnitude (11) should be a decreasing function. The above is equivalent to ask that,

$$\frac{d}{d\omega}\left(\frac{M^2_{Q_{\bar{k}_i=0}}(j\omega)}{\bar{k}_p^2\alpha^2}\right)\Bigg|_{\omega=0} < 0. \tag{14}$$

After some manipulations inequality (14) produces,

$$\frac{2\omega\bar{k}_f^2 - 2\omega\left(1+\bar{k}_f^2\omega^2\right)\left[\frac{1}{\omega^2+\gamma^2} + \sum_{m=1}^{q}\psi\right]}{(\omega^2+\gamma^2)\prod_{m=1}^{q}\left(\omega^4+2\omega_{n_m}^2\omega^2\left(2\zeta_m{}^2-1\right)+\omega_{n_m}{}^4\right)} < 0, \tag{15}$$

where,

$$\psi = \frac{2\omega\left(2\omega^2+2\omega_{n_m}^2\left(2\zeta_m{}^2-1\right)\right)}{\omega^4+2\omega_{n_m}^2\omega^2\left(2\zeta_m{}^2-1\right)+\omega_{n_m}{}^4}.$$

In the interval, $\omega \in (0, \infty)$, the inequality (15) is equivalent to,

$$\bar{k}_f^2 - \left(1+\bar{k}_f^2\omega^2\right)\left[\frac{1}{\omega^2+\gamma^2} + \sum_{m=1}^{q}\psi\right] < 0.$$

Evaluating the right-hand-side of the above inequality for $\omega = 0$ leads to the next condition,

$$\bar{k}_f < \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q}\frac{2\zeta_m^2-1}{\omega_{n_m}^2}}. \tag{16}$$

Consider now the case $\bar{k}_i \neq 0$ that corresponds to the analysis of the transfer function (9). The corresponding magnitude $M_Q(j\omega)$ and phase expressions $\angle Q(j\omega)$ are given as,

$$\begin{aligned} M_{Q(j\omega)} = \\ \bar{k}_p\alpha\sqrt{\frac{1+\left(\bar{k}_f\omega-\frac{\bar{k}_i}{\omega}\right)^2}{(\omega^2+\gamma^2)\prod_{m=1}^{q}\left(\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m{}^2-1)+\omega_{n_m}{}^4\right)}} \end{aligned} \tag{17}$$

$$\begin{aligned} \angle Q(j\omega) = &-\left(\pi - \arctan\left(\tfrac{\omega}{\gamma}\right)\right) - \omega\tau + \\ &\arctan\left(\bar{k}_f\omega - \frac{\bar{k}_i}{\omega}\right) - \sum_{m=1}^{q}\arctan\left(\frac{2\zeta_m\left(\frac{\omega}{\omega_{n_m}}\right)}{1-\left(\frac{\omega}{\omega_{n_m}}\right)^2}\right). \end{aligned} \tag{18}$$

In the above expressions, if $\bar{k}_i \to 0$, then $M_{Q_{\bar{k}_i=0}}(j\omega) \to M_Q(j\omega)$ and $\angle Q(j\omega) \to \angle Q_{\bar{k}_i=0}(j\omega)$. Hence, applying a continuity argument on $\bar{k}_i$ it is always possible to choose a gain $\bar{k}_i$ small enough such that the inequalities (12) and (14) are fulfilled. From this fact, the expression (13) can be rewritten by using (16), such that the following relation,

$$\tau < \frac{1}{\gamma} - 2\sum_{m=1}^{q} \frac{\zeta_m}{\omega_{n_m}} + \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q} \frac{2\zeta_m^2 - 1}{\omega_{n_m}^2}}.$$

is true.

(Sufficiency) Suppose that (5) holds, then there exists a PI_f controller such that the closed-loop (1)-(2) is asymptotically stable. From the Nyquist stability criterion, in order to have a stable system, it is required to have a counterclockwise rotation to the point $(-1, 0)$.

Let us consider $\bar{k}_i = 0$ and a cutting of frequency of the first-order filter of PI_f controller as $\phi = \beta$. Consequently, the fase expression given by (10), has an initial phase angle at $-\pi$ for $\omega = 0$.

Therefore, notice that to get the required counterclockwise rotation around the point $(-1, 0)$ (see Fig. 1), the phase expression should be an increasing function and the magnitude expression should be a decreasing function for $\omega > 0$. Note also that,

$$\frac{d}{d\omega}\left(\frac{M^2_{Q_{\bar{k}_i=0}}(j\omega)}{\bar{k}_p{}^2\alpha^2}\right) = 2\omega\left[\frac{\varphi(\omega)}{(\omega^2+\gamma^2)\prod_{m=1}^{q}\left(\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m{}^2-1)+\omega_{n_m}{}^4\right)}\right] \quad (19)$$

with

$$\varphi(\omega) = \bar{k}_f^2 - (1+\bar{k}_f^2\omega^2)\left[\frac{1}{\omega^2+\gamma^2} + \sum_{m=1}^{q}\frac{2\omega\left(2\omega^2+2\omega_{n_m}^2\left(2\zeta_m{}^2-1\right)\right)}{\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m{}^2-1)+\omega_{n_m}{}^4}\right] \quad (20)$$

and

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0} = -\left[\tau - \frac{1}{\gamma} + \sum_{m=1}^{q}\left(\frac{2\zeta_m\omega_{n_m}(\omega^2+\omega_{n_m}^2)}{\omega^4+2\omega_{n_m}^2\omega^2(2\zeta_m^2-1)+\omega_{n_m}^4}\right)\right] + \bar{k}_f. \quad (21)$$

It is clear that (21) could be a positive or negative function depending on the value of the involved parameters, however, if $d(\angle Q_{\bar{k}_i=0})/d\omega < 0$, then equation (10) is a decreasing function for all ω, and only if $d(\angle Q_{\bar{k}_i=0})/d\omega > 0$ the function has only one change of sign as $\omega \to \infty$, this allows as a first instance to conclude that,

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0} > 0.$$

On the other hand, since, $\omega > 0$, the sign of (19) is determined equivalently by the sign of the function given in (20), where, evaluating around $\omega = 0$, it is produced the following expression,

$$\varphi(\omega)|_{\omega=0} = k_f^2 - \left(\frac{1}{\gamma^2} + \sum_{m=1}^{q}\frac{2\left(2\zeta_m^2-1\right)}{\omega_{n_m}^2}\right). \quad (22)$$

In addition, rewriting inequality (5) allows,

$$\tau - \frac{1}{\gamma} + 2\sum_{m=1}^{q}\frac{\zeta_m}{\omega_{n_m}} < \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q}\frac{2\zeta_m^2-1}{\omega_{n_m}^2}}. \quad (23)$$

From (23), it is possible to choose $\bar{k}_f$ such that,

$$\tau - \frac{1}{\gamma} + 2\sum_{m=1}^{q}\frac{\zeta_m}{\omega_{n_m}} < \bar{k}_f < \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q}\frac{2\zeta_m^2-1}{\omega_{n_m}^2}}. \quad (24)$$

Therefore, the above inequality allows obtaining,

$$\frac{d}{d\omega}\left(\angle Q_{\bar{k}_i=0}(j\omega)\right)\Big|_{\omega=0} > 0$$
$$\frac{d}{d\omega}\left(\frac{M^2_{Q_{\bar{k}_i=0}}(j\omega)}{\bar{k}_p^2\alpha^2}\right)\Bigg|_{\omega=0} < 0.$$

Then, only if the magnitude $M_Q(j\omega)$ is monotonically decreasing and that the phase $\angle Q(j\omega)$ has a change of sign, then the existence of a counterclockwise rotation is established in the Nyquist diagram and the closed-loop stability is ensured.

In a similar way as in the necessity part, notice that if $\bar{k}_i \neq 0$, it is always possible to choose a gain $\bar{k}_i$ small enough such that $M_Q(j\omega)$ is monotonically decreasing and $\angle Q(j\omega)$ is an increasing function for $\omega = 0$.

Finally, assuming that condition of Theorem 1 is satisfied. For a small enough $\bar{k}_i$, there exists an adequate $\bar{k}_f$ such that the existence of a counterclockwise rotation in the Nyquist diagram is ensured. To guarantee the anticlockwise rounding be located around the point $(-1, 0)$ in the Nyquist diagram, the parameter $\bar{k}_p$ should be selected such that,

$$\bar{k}_p(\omega_{c_1}) < \bar{k}_p < \bar{k}_p(\omega_{c_2})$$

where $\omega_{c_1}, \omega_{c_2}$ are the first two phase crossover frequencies solutions of

$$\arctan\left(\frac{\omega_{c_i}}{\gamma}\right) - \omega_c\tau + \arctan\left(\bar{k}_f\omega_{c_i} - \frac{\bar{k}_i}{\omega_{c_i}}\right) - \sum_{m=1}^{q}\arctan\left(\frac{2\zeta_m\left(\frac{\omega_{c_i}}{\omega_{n_m}}\right)}{1-\left(\frac{\omega_{c_i}}{\omega_{n_m}}\right)^2}\right) = 0. \quad (25)$$

and $\bar{k}_p(\omega_{c_{i=1,2}})$ are given by,

$$\bar{k}_p(\omega_{c_i}) = \frac{1}{\alpha}\sqrt{\frac{(\omega_{c_i}{}^2+\gamma^2)\prod_{m=1}^{q}\left(\omega_{c_i}{}^4+2\omega_{n_m}^2\omega_{c_i}{}^2(2\zeta_m{}^2-1)+\omega_{n_m}{}^4\right)}{1+\left(\bar{k}_f\omega_{c_i} - \frac{\bar{k}_i}{\omega_{c_i}}\right)^2}}. \quad (26)$$

Remark 1. Let us consider the case where the pole-zero cancellation is not exact, i.e., $\phi \approx \beta$. Under this condition the resulting phase expression $\angle Q^*(j\omega)$ is represented as,

$$\angle Q^*(j\omega) = \angle Q(j\omega) + \arctan\left(\frac{\omega}{\beta}\right) - \arctan\left(\frac{\omega}{\phi}\right). \quad (27)$$

Since $\phi \approx \beta$ then,

$$\arctan\left(\frac{\omega}{\beta}\right) - \arctan\left(\frac{\omega}{\phi}\right) \approx 0$$

and $\angle Q^*_{\bar{k}_i=0}(j\omega) \approx \angle Q_{\bar{k}_i=0}(j\omega)$ and inequality (12) is satisfied. It is also possible to prove that under the condition $\phi \approx \beta$, the decreasing property (14) holds. Therefore, if condition (5) is satisfied, then there exist $\bar{k}_f$, $\bar{k}_i$ and $\bar{k}_p$ gains that stabilize system (1) in closed-loop with a PI_f controller.

4. NUMERICAL EXAMPLE APPLIED TO A CHEMICAL PROCESS-PROPYLENE GLYCOL PRODUCTION

Roughly 1.3 billion pounds of propylene glycol are produced per year. It has a wide variety of uses, including:

anti-freeze applications, aircraft deicing; solvents for a number of drugs; moisturizers; and artificial smoke or fog, for fire-fighting training or theatrical productions. Propylene glycol is produced by a Continuously Stirred Tank Reactor (CSTR) Pierdomenico and Ciccio (2011). This kind of reactor is dynamically modeled as a simplified kinetic mechanism that describes the conversion of reactant A to product B with an irreversible and exothermic reaction. A more detailed model of a CSTR includes the effect of cooling jacket dynamics.

A general description of the mathematical model of a CSTR has been taken from Bequette (2003), and is described by three differencial equations presented below,

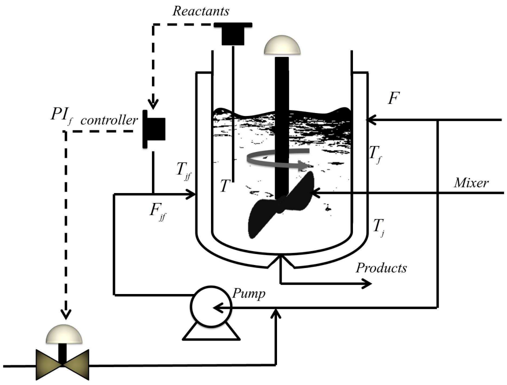

Fig. 2. Unstable CSTR with cooling jacket.

(1) Balance of the mass on component A is

$$\frac{dC_A}{dt} = \frac{F}{V}\left(C_{Af} - C_A\right) - k_0 e^{\frac{-\Delta E}{RT}} C_A.$$

(2) Energy balance in the reactor,

$$\frac{dT}{dt} = \frac{F}{V}\left(T_f - T\right) + \frac{-\Delta H}{\rho c_p} k_0 e^{\left(\frac{-\Delta E}{RT}\right)} C_A - \frac{UA}{V\rho c_p}\left(T - T_j\right),$$

(3) Energy balance in the cooling liquid,

$$\frac{dT_j}{dt} = \frac{F_{jf}}{V_j}\left(T_{jf} - T_j\right) + \frac{UA}{V_j \rho_j c_{pj}}\left(T - T_j\right),$$

System parameters are listed in Table 1.

Table 1. CSTR parameters.

Description	Variables	Values	Unit
Volume of the reactor	V	85	ft^3
Activation energy	$-\Delta E$	32,400	$Btu/lbmol$
Heat transfer coefficient	U	75	$btu/hr^\circ F$
Heat of Reaction	$-\Delta H$	39,000	$Btu/lbmolPO$
Heat transfer area	A	88	ft^2
Frequency factor	k_0	$16.96x10^{12}$	hr^{-1}
Ideal gas constant	R	1.987	$Btu/lbmol^\circ F$
Volume cooling liquid in the chamber (reactor jacket)	Vj	21.25	ft^3
Heat capacity on reactor with density of the reagent	ρc_p	53.25	$Btu/ft^3\ ^\circ F$
Density of the cooling liquid with heat capacity of the cooling liquid	$\rho_j c_{pj}$	55.6	$Btu/ft^3\ ^\circ F$

To get a linear representation of the CSTR model, the cooling jacket feed rate flow F_{jf} is considered as the manipulated (input) variable and the temperature T, as the controlled (output) variable. A tangent approximation will be obtained by considering the operating point described in Table 2 (detailed descriptions of these parameters can be found in Bequette (2003)),

The linear representation around the considered operating point it is obtained as,

$$\frac{F_{jf}(s)}{T(s)} = \frac{-4.747s - 37.940}{(s - 1,177)(s^2 + 10.509s + 29.260)}. \qquad (28)$$

Table 2. Values at the operating point of CSTR.

Description	Variables	Values	Unit
Feed concentration of product A	C^o_{Af}	0.132	$lbmol/ft^3$
Concentration of the product A	C^o_A	0.066	$lbmol/ft^3$
Product Flow A	F^o	340	ft^3/hr
Cooling jacket feed rate flow	F^o_{jf}	28.75	ft^3/hr
Temperature in the reactor	T^o	101.1	$^\circ F$
Feed temperature	T^o_f	55	$^\circ F$
Cooling jacket temperature	T^o_j	60	$^\circ F$
Cooling jacket feed temperature	T^o_{jf}	50	$^\circ F$

Notice that (28) is of the same class of systems described by equation (1), in Section 2, with $\tau = 0$ since it has an unstable pole, a pair of complex conjugate poles and a zero. Using high quality sensors to determine the temperature (T) in CSRT reactors usually means high costs. This disadvantage in the sensor generates a dead-time in the measurement of the temperature, which creates a problem for the control strategy design, that increases its difficulty when the system is unstable. For that reason, it is possible to consider this effect by adding an adequate time-lag to equation (28). For this kind of reactor, the time–delay in the temperature measurement is approximately 0.25 hr. taking into accoutn the residence time over the operation point, this produces,

$$\frac{F_{jf}}{T(s)} = \frac{-4.73(s + 7.992)}{(s - 1,177)(s^2 + 10.509s + 29.260)} e^{-0.25s}. \qquad (29)$$

The parameters of the system are $\beta = -7.992$, $\gamma = 1.1772$, $\zeta = 0.971$, $\omega_n = 5.409$ and $\tau = 0.25$. From the stability condition stated in Theorem 1 and assuming that $m = 1$,

$$\tau = 0.25 < \frac{1}{\gamma} + \sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q} \frac{2\zeta_m^2 - 1}{\omega_{n_m}^2}} - 2\sum_{m=1}^{q} \frac{\zeta_m}{\omega_{n_m}} = 1.735,$$

and therefore, system (29) can be stabilized by a PI_f controller. Following the methodology proposed in this work, in order to obtain the tuning parameters of the controller given by (2), the first step is to consider the cutoff frequency of the filter in the PI_f controller as $\phi = -7.992$. After that, from equation (24), the range of values for the stabilizing gain $\bar{k}_f$ is $-0.240 < \bar{k}_f < 0.884$. Considering $\bar{k}_f = 0.4$, the gain $\bar{k}_i$ must be small enough to satisfy the conditions (12) and (14). For this case, $\bar{k}_i = 0.022$. Solving (25) allows obtaining the range of stabilizing gain given by $1.37 < \bar{k}_p < 14.06$. Considering $\bar{k}_p = 2$, and finally using the definition given in (8), the proposed PI_f controller is represented as:

$$PI_f = 2\left(1 + \frac{0.0068}{s} + \frac{-5.496}{s + 7.99}\right).$$

Fig. 3 illustrates the control strategy performance regarding the reactor temperature when input reference equal to $50^\circ F$ is considered. In the same figure is show how the proposed controller provides a more appropriate transients response than the classical PI and PID controllers with a filtered derivative action (PI_{fda} and PID_{fda} respectively) since not only eliminates the noise but also provides a smoother response which is a desirable feature in any controller. To evaluate the performance of the three used controllers, the Integral Squared Error (ISE), as an indicator of efficiency, was used (illustrated in Fig. 4). The results show that the PI_f has better performance than the PI_{fda} and the PID_{fda}, with the PI_{fda} being the controller with the worst performance.

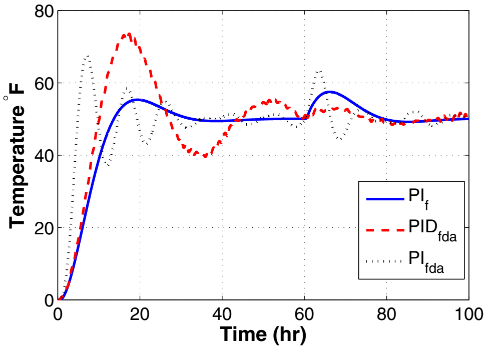

Fig. 3. PI_f controller performance.

Notice that the cutoff frequency of the PI_f controller is directly related to the stability ranges required to deal with unstable time delayed systems. While, by the other hand, the cutoff frequency of PI_{fda} and PID_{fda} controllers do not have any relation with stability tuning. For this reason, and for comparison purposes, in this work, tuning parameters used for PI_{fda} and PID_{fda} controllers are empirically proposed, without guaranteeing stability for delays of larger magnitude.

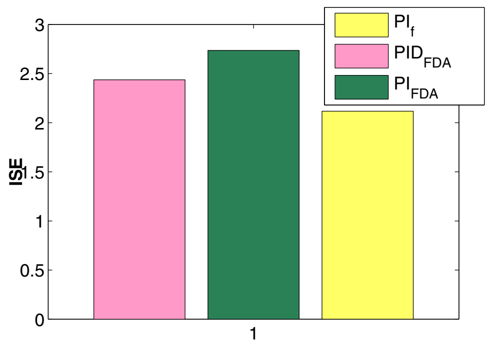

Fig. 4. Performance indicator (Integral Squared Error (ISE)).

Remark 2. The considered PID_{fda} and PI_{fda} controllers were designed as (30) and (31) respectively,

$$H_1(s) = k_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right) \tag{30}$$

$$H_2(s) = k_p \left(1 + \frac{1}{T_i s}\right) \frac{1}{1 + \frac{1}{N} s}, \tag{31}$$

where the the tuning parameters for the PID_{fda} are given as $k_p = 5$, $T_i = 20$, $T_d = 1.2$ and $N = 9.5$ for a $\tau = .25$ while for the PI_{fda} are given by $k_p = 5$, $T_i = 45$, $N = 9.5$ with a $\tau = 2.5$ but with a maximum delay $\tau_{max} = 0.49$ (smaller than the one supported by the PI_f).

5. CONCLUSION

This paper presents necessary and sufficient condition for the stabilization of a class of high-order unstable delayed linear systems with one unstable pole, $q-1$ possible complex conjugate poles and one minimum phase zero by using a novel control law called the PI_f controller. The proposed controller is based on an standard PI controller plus a first order low-pass filter. Additionally, the procedure to determine the parameters for the stabilizing gains k_p, k_i and k_f are given in order to provide an accurate PI_f controller tuning. It is worth noting that the proposed PI_f controller not only maintains the basic properties of conventional PI/PID controllers regarding constant disturbance rejection and tracking of step references; besides, it exhibits two advantages. First, the stability condition is improved by the term $\sqrt{\frac{1}{\gamma^2} + 2\sum_{m=1}^{q} \frac{2\zeta_m^2 - 1}{\omega_{nm}^2}}$, compared with P or PI controllers, i.e., the PI_f allows stabilizing systems with larger delays than the ones allowed by the conventional P/PI controllers. A second advantage is that the PI_f controller does not resort on derivative terms; the above feature eases its practical implementation and also, this fact allows to reduce the noise caused by the derivative term. Aditionally, the PI_f controller is able to stabilize systems with one minimum phase zero, a problem that has not been addressed in past work by means of $P/PI/PID$ controllers. Finally, a numerical example applied on chemical process-propylene glycol production was used to verify the performance of the proposed strategy using a numerical simulation.

REFERENCES

Bequette, B.W. (2003). *Process Control. Modelling, Design and Simulation.* Prentice-Hall.

Hernandez-Perez, M.A., del Muro-Cuellar, B., and Velasco-Villa, M. (2015). PID for the stabilization of high-order unstable delayed systems with possible complex conjugate poles. *Asian-Pacific Journal of Chemical Engineering*, 10(5), 687–699.

Hwang, C. and Hwang, J. (2004). Stabilization of first-order plus dead-time unstable processes using PID controllers. *IEE Proc. Control Theory Appl.*, 151, 89–94.

Lee, S.C., Wang, Q.G., and Xiang, C. (2010). Stabilization of all-pole unstable delay processes by simple controllers. *Journal of Process Control*, 20(2), 235 – 239.

Niculescu, S.I. (2001). Delay effects on stability. a robust control approach. *Springer-Verlag.*

Novella, D.F., del Muro, B., Marquez, J., Hernandez, M., and Velasco, M. (2017). PD-PID controller for delayed systems with two unstable poles: a frequency domain approach. *International Journal of Control*, 0(0), 1–13.

Ou, L., Zhang, W., and Gu, D. (2006). Sets of stabilizing PID controllers for second-order integrating process with time-delay. *IEEE Pro. Control Theory and Applications*, 153(5), 607–614.

Pierdomenico, P. and Ciccio, M.P.D. (2011). Iss feedback control redesign for continuous stirred tank reactors. *International Journal of Robust and Nonlinear Control*, 21(17).

Pontryagin, I.S. (1995). On the zeros of some elementary transcendental function. *Amer. Math. Soc. Transl.*, 2, 95–110.

Silva, G.J., Datta, A., and Bhattacharyya, S.P. (2004). *PID controllers for time-delay systems.* Springer, London.

Sipahi, R., i. Niculescu, S., Abdallah, C.T., Michiels, W., and Gu, K. (2011). Stability and stabilization of systems with time delay. *IEEE Control Systems*, 31(1), 38–65.

Zhong, Q. (2006). Robust control of time delay system. *Springer-Verlag.*