Improvement of the Control System Performance based on Fractional-Order PID Controllers and Models with Robustness Considerations

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Abstract: In this paper we assess the performance improvement achievable by using one-degree-of-freedom fractional-order proportional-integral-derivative controllers (FOPI/FOPID) instead of their integer-order counterparts (PI/PID). To this end, we take into account a single-pole fractional-order model, which has the advantage of representing a wide variety of process dynamics, ranging from over-damped (first-order models) to under-damped behaviors, depending on the fractional order α. In the proposed analysis we consider a combined performance index which deals with the trade-off between servo and regulatory control modes. Moreover, the performance of the closed-loop system is optimized subject to a robustness constraint, expressed as a target maximum sensitivity of either 1.4 or 2.0. The obtained performance assessment results are shown for different values of the fractional order α of the model and for different normalized dead times, thus quantitatively evaluating the benefits achievable with fractional controllers on a wide variety of process dynamics.

1. INTRODUCTION

With no doubt, since their introduction in 1940, commercial proportional-integral-derivative (PID) controllers have been the most extensive option that can be found in industrial control applications (Åström and Hägglund, 2006). Their success is mainly due to their simple structure and to the physical meaning of their three parameters (therefore making manual tuning possible and understandable for industrial practitioners). This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. In addition, the PID control algorithm provides satisfactory performance in a wide range of practical situations.

In literature, many techniques to tune PID controllers have been proposed (O'Dwyer, 2009). Many tuning techniques embed in the design process technical aspects that are desirable from an industrial process control point of view, such as speed of response, robustness, noise filtering, etc. Among them, two of the most important objectives are a satisfactory load disturbance rejection (also known as *regulatory control*) and a satisfactory response against changes in the set-point value (also known as *servo control*) (Arrieta and Vilanova, 2007; Alfaro et al., 2009b). Finally, the robustness issue is of paramount importance and is often taken into account by tuning rules (Sánchez et al., 2015). The difficulty in the tuning process is that the three aforementioned aspects in PID control are often competing objectives. Therefore, a PID tuning that considers all these targets should be based on a trade-off among these goals. Nowadays, due to the development of the fractional calculus, many control problems for which the existing theory did not provide satisfactory results can be tackled using the fractional tools (Das, 2011). Despite the extension of advanced methodologies originally developed for integer-order systems to fractional-order ones (Padula and Visioli, 2014; Padula et al., 2013), one of the main applications of fractional calculus in control is the generalization of the classical integer PID, known as fractional-order PID (FOPID) controller which has been firstly proposed in (Podlubny, 1999). The FOPID controller is characterized by two elements: a fractional integrator, whose order is usually denoted as λ, and a fractional differentiator, whose order is usually denoted as μ. Different works (Hui-fang et al., 2015; Padula and Visioli, 2011; Li et al., 2010) have shown that a suitable choice of the fractional orders

μ and λ provides a better performance whether or not robustness conditions are considered. Other studies, see e.g. (Padula and Visioli, 2016), have shown that FOPID controllers are more fragile compared to the standard PID controller, and therefore special care must be used in the tuning procedure. Interestingly, the same need to trade off between the aforementioned different objectives, which is a characteristic feature of PID control, and control in its most broad sense, also affects FOPID controllers (Sánchez et al., 2017).

Fractional calculus finds another interesting application in the dual aspect of the controller design, which is the process modeling. In particular, a generalized model structure which comprises a single fractional pole plus a dead time, hereafter referred to as Fractional First Order Plus Dead Time (FFOPDT) model, has been extensively investigated (Li and Chen, 2014; Luo and Chen, 2010; Das et al., 2011a; Tavakoli et al., 2010). The FFOPDT can capture both the well-known first-order-plus-dead-time (FOPDT) model dynamics and the under-damped dynamics, which is traditionally described via a second-order-plus-dead-time (SOPDT) model (Das et al., 2011b). This property is highly desirable in practice because it does not require to modify the model structure according to the dynamics of the process to be modeled, thus greatly simplifying next step, which is the controller tuning. Specials efforts have been made to find the suitable four parameters that compose this model (static gain K, the time constant T, the dead time L and the fractional order, denoted as α) to match a given response. In particular, researches have focused on the fractional order α, that plays a major role in changing the dynamic behavior of the model, (Tavakoli et al., 2010). Note that the FFOPDT model represents a genuine generalization of the FOPDT model, which is re-obtained when the fractional order α is set to 1. It is worth mentioning that other more complex fractional models can be found in the literature. In (Das et al., 2011b) a fractional model that has two fractional orders has been proposed and (Tavakoli et al., 2010) proposes a model with five parameters, among which only one is of fractional order. However, because of their simplicity and their capability of representing several dynamics in a satisfactory way, in the present work we consider FFOPDT models, along the lines of (Guevara et al., 2015).

In this paper, we will show the advantages of using FOPID controllers, and the subclass of FOPI controllers, against their integer counterparts, namely PID an PI controllers. In particular, we propose an optimization-based tuning procedure, where the optimal trade-off between the servo and the regulatory mode is found by using an optimization functional that combines both objectives. The third goal, which is the robustness of the closed-loop system, is taken into account by constraining the maximum sensitivity M_S to either 1.4 (robust tuning) or 2.0 (aggressive tuning). The aforementioned constrained optimization problem is solved for different normalized dead times and for different fractional orders of the FFOPDT model, thus providing a comprehensive quantitative performance assessment. Simulation examples show the effectiveness of the proposed tuning approach and the advantage of using FFOPDT models and fractional controllers.

The paper is organized as follows. Section 2 presents the control system configuration and concepts used to tackle the performance and robustness issue. Section 3 describes the results obtained from the comparison of the control systems performance due to the use of integer and fractional PID controllers. In Section 4 there is an example of the use of the fractional calculus in the design of control systems and the paper ends with conclusions in Section 5.

2. MATERIALS AND METHODS

Consider the closed-loop control system shown in Fig. 1, where $P(s)$ is the controlled-process model and $C(s)$ the controller to be tuned. In this system, $r(s)$, $u(s)$, $d(s)$, and $y(s)$, are the set-point value, the controller output, the load-disturbance, and the controlled variable, respectively.

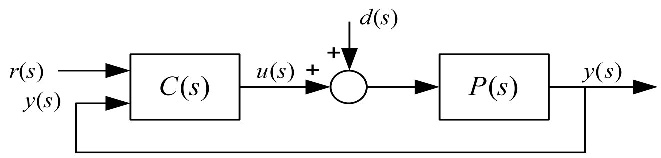

Figure 1. Closed-loop control system

2.1 Controlled Process Models

The controlled process $P(s)$ is considered here a FFOPDT model, whose dynamics is described by the following transfer function

$$P(s) = \frac{K\mathrm{e}^{-Ls}}{Ts^{\alpha}+1}. \tag{1}$$

where K is the static gain, T is the time constant, α is the fractional order, and L is the dead time. The FFOPDT dynamics can be completely characterized by using two dimensionless parameters: the fractional order α and the fractional normalized dead time $\tau_o = \frac{L}{T^{\frac{1}{\alpha}}}$. The class of models (1) represents a wide range of processes, from non-oscillatory processes, including first-order and over-damped processes, to processes with an oscillatory dynamics. In this paper we consider $1 \leq \alpha \leq 2$. Note that when $\alpha = 1$ a classical FOPDT model is obtained and when $\alpha = 2$ a pair of pure imaginary poles are obtained (undamped oscillatory behavior).

Dealing with a fractional-order v requires the application of a method for the rational approximation of the non integer term. In this work, the integer representation of the fractional part can be obtained by applying the so-called CRONE approach (Oustaloup et al., 2000), defined as:

$$s^v \to s^v_{[wl,wh]} \approx C_o \prod_{k=1}^{N} \frac{1+\frac{s}{w_{z,k}}}{1+\frac{s}{w_{p,k}}}, v > 0, \tag{2}$$

where it is necessary to choose the frequency range $[\omega l, \omega h]$ (selected as $[0.001, 1000]$ in this work) where the approximation is valid. Additionally, the term C_o, is adjusted so that the approximation has unit gain at the gain crossover frequency. Furthermore, the parameter N in (2) (in this case $N = 8$) can be used to select the number of poles and zeros of the transfer function that approximates the fractional term.

2.2 1DoF PID Controller Equation

The output signal of a one-degree-of-freedom (1DoF) PID controller is

$$u(s) = K_p \{e_p(s) + e_i(s) + e_d(s)\}, \tag{3}$$

with

$$e_p(s) = r(s) - y(s), \tag{4}$$

$$e_i(s) = \frac{1}{T_i s}[r(s) - y(s)], \tag{5}$$

$$e_d(s) = -\frac{T_d s}{\frac{T_d}{\zeta} s + 1} y(s), \tag{6}$$

where K_p is the controller proportional gain, T_i the integral time, T_d the derivative time and $\frac{T_d}{\zeta}$ is the derivative filter time constant (traditionally selected by fixing $\zeta = 10$ (Visioli, 2006)). As it is shown in (6), the derivative action is only applied to the feedback signal, in order to avoid extreme instantaneous changes in the controller output signal, known as derivative kick, when a set-point step change occurs (Åström and Hägglund, 2006).

2.3 1DoF FOPID Controller Equation

For the FOPID controller, the output signal is

$$u(s) = K_p \{e_{pf}(s) + e_{if}(s) + e_{df}(s)\}, \tag{7}$$

with

$$e_{pf}(s) = r(s) - y(s), \tag{8}$$

$$e_{if}(s) = \frac{1}{T_i s^{\lambda}}[r(s) - y(s)], \tag{9}$$

$$e_{df}(s) = -\frac{T_d s^{\mu}}{\frac{Td}{\zeta} s + 1} y(s), \tag{10}$$

where λ and μ are the fractional orders of the integral and derivative part, respectively. Moreover ζ is defined as

$$\zeta = 10 T_d^{\frac{\mu - 1}{\mu}}, \tag{11}$$

with the purpose of locating the filter pole cut-off frequency one decade above the fractional derivative gain crossover frequency, thus generalizing the classical rule $\zeta = 10$.

2.4 Performance and Robustness

The tuning methods applied to obtain the optimal parameters consider the trade-off between performance and robustness along the line of Alfaro et al. (2009a); Vilanova et al. (2012). The functional to be optimized is a multiobjective performance index given by

$$J_e = J_{er} + J_{ed}, \tag{12}$$

where J_{er} quantifies the set-point tracking performance and J_{ed} measures the load disturbance rejection performance. Both indexes are computed as the integral of the absolute value of the error, given by

$$\int_0^{\infty} |e(t)|\, \mathrm{d}t = \int_0^{\infty} |r(t) - y(t)|\, \mathrm{d}t. \tag{13}$$

It is well known that robustness issue should also be taken into account for the control system. Indeed, the design procedure is usually based on low-order linear model identified at the closed-loop operating point and possible nonlinearities and uncertainties found in most of the real-world industrial processes can change the dynamics. To this end, it is necessary to consider certain stability margins, or robustness requirements, for the control system. In this paper, as an indicator of the closed-loop system robustness, the maximum value of the sensitivity function will be used. It is defined as

$$M_S = \max_{\omega} |S(\mathrm{j}\omega)| = \max_{\omega} \frac{1}{|1 + C(\mathrm{j}\omega)P(\mathrm{j}\omega)|}. \tag{14}$$

The M_S value should remain, at least for stable processes, in the range $1.4 \le M_S \le 2.0$ (Åström and Hägglund, 2006). We consider here the limits of this range, i.e. $M_S = 1.4$ and $M_S = 2.0$.

2.5 Improvement provided by FOPI(D) controllers

Aimed at quantifying the performance improvement provided by FOPI(D) controllers, the indexes $J_{n(PI)}$ and $J_{n(PID)}$ will be employed. These are defined as:

$$J_{n(PI)} = \frac{J_{e(FOPI)}}{J_{e(PI)}}, \tag{15}$$

for FOPI/PI controller and as

$$J_{n(PID)} = \frac{J_{e(FOPID)}}{J_{e(PID)}}, \tag{16}$$

for FOPID/PID controllers.
As can be seen in (15) and (16), the improvement in the performance will be measured considering two different cases, depending on the use (or not) of the derivative action.

3. PERFORMANCE ASSESSMENT

In this section, optimal values of the cost function (12) are provided for different values of τ_0, ranging from 0.1 to 2.0, and also for several values of the fractional order model α. The rationale is to study a whole family of processes and to examine a variety of dynamics. In this framework, PI, PID, FOPI and FOPID controllers are tuned by minimizing the cost function (12) with a constraint on the maximum sensitivity M_S.

3.1 PI vs FOPI controllers

First, consider the case when $M_S = 1.4$ is selected. PI and FOPI controllers were tuned by considering the fractional order α in the range $1.0 \le \alpha \le 1.7$, because outside this interval, the normalized proportional gain κ_p attains extremely low values. Note that this is in line with the long-held perception that PI regulators are unsuitable to control highly underdamped processes. Therefore, considering more oscillatory dynamics in the process than the ones obtained with $\alpha = 1.7$ does not seem meaningful. In Fig. 2 the results on how the use of integer and fractional controllers affect the performance of the control loop for different values of α and τ_0 are shown. It can be appreciated that an increase of the fractional order α, which entails higher overshoots and more oscillatory transient responses, produces a more appreciable effect on the integrator fractional order λ. In Fig. 3 is shown how the integrator fractional order λ varies according to the values of α and τ_0. It is interesting to note that, when $\alpha = 1$,

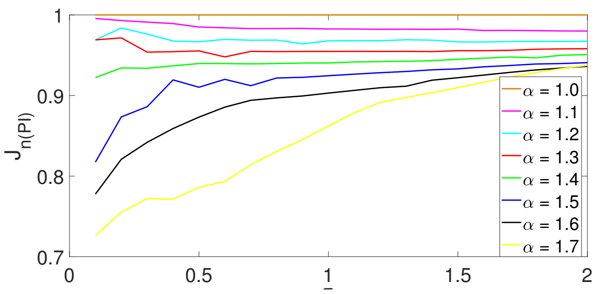

Figure 2. $J_{n(PI)}$ behavior for PI and FOPI controllers with $M_S = 1.4$

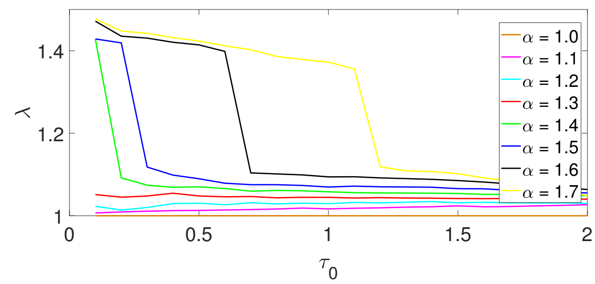

Figure 3. λ behavior for FOPI controller with $M_S = 1.4$

there is no point in using a FOPI controller, because the fractional integrator does not improve the performance. These results are in line with the ones presented in (Padula and Visioli, 2011), where it is shown that the optimal value of λ is always unitary when controlling first-order processes. On another side, it can be seen that the effect of the fractional parameter λ tends to be bigger when the normalized dead time is short.

For $M_S = 2.0$, the fractional order α also was considered in the range $1.0 \leq \alpha \leq 1.7$ in the tuning of PI and FOPI controllers due to the same reasons mentioned above.

Fig. 4, shows how both types of controllers modify the performance of the closed-loop system. It can be appreciated again how an increase of the fractional order α allows a greater action of the fractional integrator order λ, as well as, its effect over the performance tends to be less when the normalized dead time increase. Through a comparison

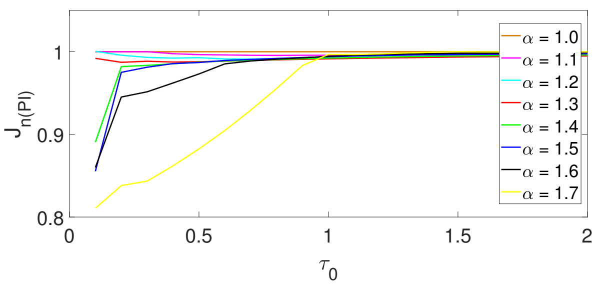

Figure 4. $J_{n(PI)}$ behavior for PI and FOPI controllers with $M_S = 2.0$

of the results obtained for $M_S = 1.4$ and $M_S = 2.0$, it can be seen how there is a bigger distance between the performance reached with PI and FOPI controllers when the constraint in the robustness is given by a $M_S = 1.4$. It means that the fractional integrator order λ is useful in the control of processes when a high robustness is required.

3.2 PID vs FOPID controllers

In this section, the performance assessment for FOPID/PID controllers is presented. We consider first the $M_S = 1.4$ case. For both types of controllers, the fractional order of the process model is selected in the range $1.0 \leq \alpha \leq 1.9$. Values of α greater than 1.9 were not taken into account because the process becomes practically an undamped second-order system, which is unlikely to be encountered in practical applications.

The results obtained using both types of controllers are shown in Fig. 5 and in Fig. 6.

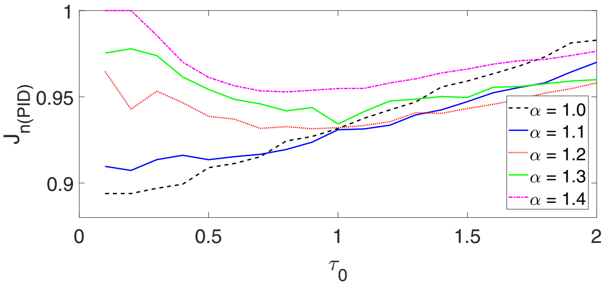

Figure 5. $J_{n(PID)}$ behavior for PID and FOPID controllers with $M_S = 1.4$ and $1.0 \leq \alpha \leq 1.4$

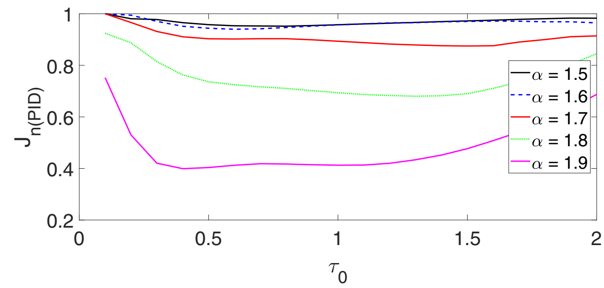

Figure 6. $J_{n(PID)}$ behavior for PID and FOPID controllers with $M_S = 1.4$ and $1.5 \leq \alpha \leq 1.9$

It can be seen in Fig. 5 that when $M_S = 1.4$, for processes with $1.0 \leq \alpha \leq 1.1$ (this range includes overdamped and critically-overdamped processes), the control system performance improvement can be up to 11%, thanks to the use of FOPID controllers. Again, the value of the $J_{n(PID)}$ index tends to increase when the normalized dead time increases.

As it can be appreciated in Fig. 5 and Fig. 6, when the fractional order α increases from 1.0 to 1.5, the $J_{n(PID)}$ index grows, but from 1.5 to 1.9, the $J_{n(PID)}$ index tends to decrease, which is an indicator of a remarkable performance in the control loop provided by the use of the fractional order μ associated to the derivative action. For example, it can be seen that for a fractional order $\alpha = 1.9$, the improvement in the performance of the closed-loop system can reach values up to 60% due to the use of FOPID controllers.

It is important to mention that the greatest difference in the performance provided by PID and FOPID controllers is obtained in the range $0.5 \leq \tau_0 \leq 1.5$ and when the fractional order α varies from 1.2 to 1.9. This also can be appreciated in Fig. 7 and Fig. 8 that show how in a general way the highest values of the derivative fractional order μ are located in that range.

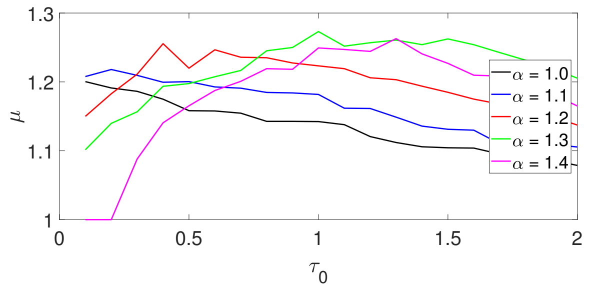

Figure 7. μ behavior for FOPID controller with $M_S = 1.4$ and $1.0 \leq \alpha \leq 1.4$

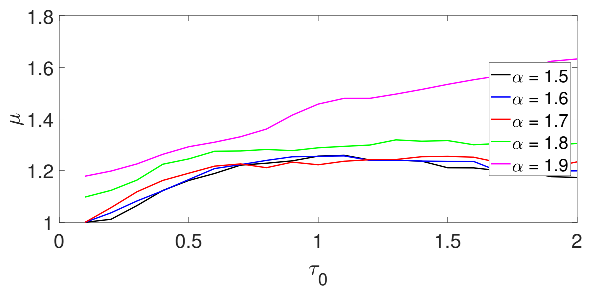

Figure 8. μ behavior for FOPID controller with $M_S = 1.4$ and $1.5 \leq \alpha \leq 1.9$

The results of the performance of the control systems obtained using PID and FOPID controllers tuned for $M_S = 2.0$ are shown in Fig. 9 and Fig. 10.
For this case, again, Fig. 9 and Fig. 10 show how the $J_{n(PID)}$ index increases in general when the fractional order α goes from 1.0 to 1.5, but from this last value to $\alpha = 1.9$ it decreases and therefore a better performance in the closed-loop is reached using FOPID controllers.
Moreover, can be seen how the best performance through FOPID controllers is achieved, for the most of the combinations between the fractional order α and the normalized dead time τ_0, when τ_0 is selected in the range $0.5 \leq \tau_0 \leq 1.5$.

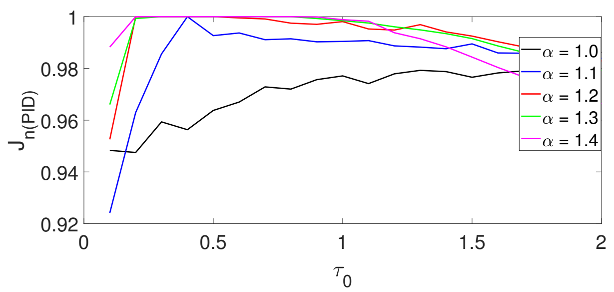

Figure 9. $J_{n(PID)}$ behavior for PID and FOPID controllers with $M_S = 2.0$ and $1.0 \leq \alpha \leq 1.4$

4. SIMULATION EXAMPLE

As an illustrative example, consider an oscillatory process studied in (Das et al., 2018) and defined by the transfer function

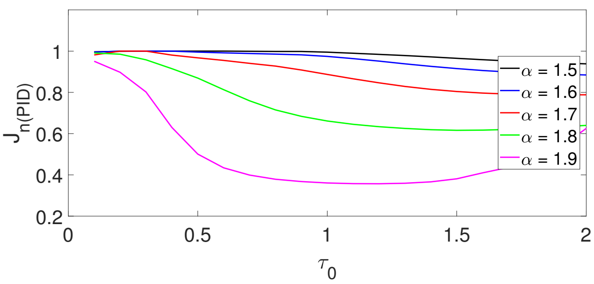

Figure 10. $J_{n(PID)}$ behavior for PID and FOPID controllers with $M_S = 2.0$ and $1.5 \leq \alpha \leq 1.9$

$$\frac{e^{-s}}{9s^2 + 2.4s + 1}. \tag{17}$$

Using the program *IDFOM* devised in (Guevara et al., 2015), the fractional-order model

$$\frac{e^{-2.2594s}}{4.8270s^{1.4504} + 1}. \tag{18}$$

has been identified. The controllers PI, FOPI, PID and FOPID have been tuned for $M_S = 1.4$. The responses of the designed control systems against step changes in the set-point value and in load disturbance signal are shown in Fig. 11. As expected, fractional controllers improves significantly the performance achieved by using integer controllers. In Table 1 the parameters of the different

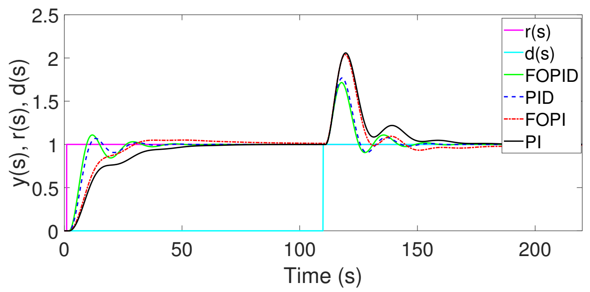

Figure 11. Controlled output using PI, FOPI, PID and FOPID controllers for $M_S = 1.4$

controllers as well as the performance index J_e are shown. From these results, the index $J_{n(PI)}$ exhibits an improvement of the performance in the closed-loop system of around 11.3% and the index $J_{n(PID)}$ an improvement of approximately 2.6%, both due to the contribution of the fractional parameters of the FOPI/FOPID controllers.

Table 1. Controller parameters for $M_S = 1.4$ and J_e index

	PI	FOPI	PID	FOPID
J_e	29.19	25.88	14.21	13.84
K_p	0.1466	0.2185	0.4559	0.6705
T_i	2.1406	3.6846	2.8384	3.9211
λ	–	1.1440	–	1.0000
T_d	–	–	2.1791	1.9053
μ	–	–	–	1.2006

5. CONCLUSIONS

This paper quantitatively evaluates the advantages achievable in a process control context due to the combined

use of FOPID controllers and fractional models. On the one side, the fractional model allows the representation of different dynamics without the need to change the structure of the model, as happens when integer models are employed. On the other side, it is demonstrated through the J_{nPI} and J_{nPID} performance indexes that the use of fractional controllers provides a better performance when also robustness considerations are taken into account.
The optimization functional proposed in this paper provides an optimal trade-off between set-point tracking and load disturbance rejection performance. Finally, the robustness issue is explicitly considered by constraining the maximum sensitivity. For a practical implementation in industry, a controller should meet all the aforementioned objectives.
The several results shown in Section 3 allow the deduction that the fractional derivative order μ has a most notable impact on the closed-loop system performance with respect to the fractional integrator order λ.
It is believed that the proposed results provide the elements for the potential user to decide whether it is worth using a (more complex) fractional controller for a real-world application.

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The financial support from the University of Costa Rica, under the grant 731-B4-213, is greatly appreciated. Also, this work has received financial support from the Spanish CICYT program under grant DPI2016-77271-R and by the Australian Research Council under the grant DP160104994.

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