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New robustness measure for a kind of event-based PID

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Abstract: In this paper we present a study of the appearance of limit cycles in event-based PID control systems. Our approach is based on the extension of the Tsypkin method, which has been widely used to study the relay control systems. A new margin has been obtained to measure the robustness to limit cycles of continuous controllers when applied on event based control loops. The margin has been calculated to characterize some well known PID controllers tuning methods applied to the control of FOPTD systems with SSOD sampling strategy.

Keywords: event-based-PID, robustness, SSOD, FOPTD

1. INTRODUCTION

Event based PID (EBPID) is a valuable control scheme that conjugates the well known reliability of the PID controllers with the necessity of reducing the data transmission in distributed control systems. The reduction of data traffic minimizes the loss of packages and delays introduced by the network. Furthermore, in the case of wireless networks, the reduction of transmissions increases the lifetime of batteries of self-powered remote sensors (Feeney and Nilsson (2001)).

During the last years some works have been focused on developing new tuning methods for EBPID. In (Beschi et al. (2014)) the design of PI controllers was addressed using a Symmetric Send on Delta (SSOD) sampling strategy for the control of first-order systems with delay. In (Romero et al. (2014)) the authors gave a simple rule for tuning SSOD based PI controllers which were derived by applying the Describing Function (DF) technique. Following the same approach, a complete tuning methodology for this kind of controllers is proposed in (Romero-Pérez and Sanchis (2016)). The extension of the results presented in the before mentioned works to other sampling strategy was presented in (Romero-Pérez and Llopis (2017)), were tuning rules for PI and PID are proposed not only for SSOD but also for Regular Quantization (RQ) sampling.

The main advantage of using the DF is that it allows to build a bridge between the continuous and the event based worlds. Hence, some concepts traditionally used for the analysis and design of continuous control systems in the frequency field can be extended to the study of event-based control systems (EBCS). An example of this are the robustness margins to avoid limit cycle presented in (Romero-Pérez and Llopis (2017)), which are given in terms of the phase and gain margin. It is well known, however, that the validity of the DF depends on the filtering properties of the open loop transfer function: only under certain filtering conditions the higher orders harmonics can be neglected and the DF can be successfully applied. Therefore, low order models, such as first or second order plus time delay (FOPTD,SOPTD), which are commonly used to describe actual industrial processes, are excluded from this approach, and new methods are required if frequency response concepts are wanted to be used.

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In this paper we propose a new robustness measure to prevent limit cycles in SSOD based control systems. Conversely to those presented in (Romero-Pérez and Llopis (2017)), the robustness margins presented here are valid for low order systems. Our proposal is based on the Tsypkin's method (Tsypkin (1984)), which has been widely used to study the relay control systems. The new margins obtained here are applied to study the relative stability to stable oscillations in SSOD-PID control systems when well settled continuous time tuning methods are used for tuning the PID. Concretely we have considered the tuning rules proposed in (Ziegler and Nichols (1942)), (Cohen and Coon (1953)) and (Åström and Hägglund (2004)) for FOPTD systems. The results would shed light on the validity of these methods for tunning SSOD-PID controllers.

2. PROBLEM STATEMENT

Consider the control system shown in Figure 1, which was proposed in (Beschi et al. (2012)), where $C(s)$ and $G(s)$ are the controller and the process transfer functions respectively, y_r is the reference signal to be tracked, y is the controlled output, and p is the disturbance input. It is supposed that the controller is located near the actuator and the sensor sends measurements of process output y (or more precisely of the tracking error e) to the controller through a communication network using the

SSOD strategy. The ZOH block keeps in $\bar{e}$ the last sent value of process output e^* until a new value is transmitted by the SSOD block.

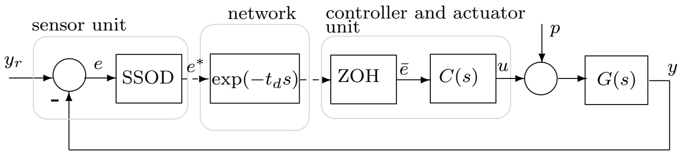

Fig. 1. Networked control system with SSOD sampling strategy. SSOD-$C(s)$ architecture.

It is well known that this kind of control loops may have problems of limit cycles (permanent non vanishing oscillations) due to the nonlinearity introduced by the SSOD sampling. The existence of a stable limit cycle depends on the process and the controller. As proved in Romero et al. (2014), a valuable tool for predicting (and avoiding) those limit cycles is the DF technique, whose results are not exact as it relies on the process perfect filtering of the higher order harmonics of the periodic control input signal, resulting in a perfect sinusoid at the output. In fact, there are no processes that produce a perfect filtering, but if the higher order harmonics are reasonably attenuated, the describing function approach can still be used as an approximation. However, for systems with poor filtering response, methods which consider the effect of higher harmonics must be used.

3. TSYPKIN'S METHOD

In order to predict more accurately the existence of limit cycles (and to give some tips to avoid them), we propose a new approach based on the Tsypkin's method (Tsypkin (1984)), which is valid independently of the filtering characteristics of the process transfer function. This approach is especially interesting for systems of low order (e.g. FOPTD), for which the DF technique is very inaccurate and can not be used to predict the limit cycles.

The control loop in Figure 1 can be represented as the non-linear system shown in Figure 2, which could be considered a generalization of a relay control system.

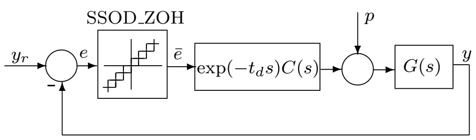

Fig. 2. Non-linear equivalent system to the control systems with SSOD sampling strategy in Figure 1.

According to the Tsypkin's method, the general conditions for oscillation in this system, which are represented in Figure 3 when the rise edge of $\bar{e}$ occurs in $t = 0$, are given by the equations (1a)-(1d).

$$e\left(\frac{T_o^-}{2}\right) = -\delta \tag{1a}$$

$$\frac{de}{dt}\left(\frac{T_o^-}{2}\right) < 0 \tag{1b}$$

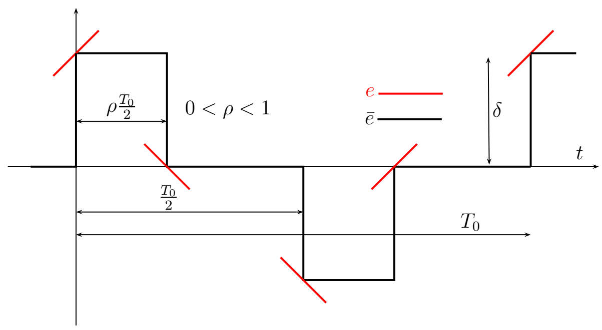

Fig. 3. Oscillations condition in the SSOD-$C(s)$ control systems.

$$e\left(\rho\frac{T_o^-}{2}\right) = 0 \tag{1c}$$

$$\frac{de}{dt}\left(\rho\frac{T_o^-}{2}\right) < 0 \tag{1d}$$

Applying the Fourier transform to signal e and after some calculations, equations (1a)-(1d) can be written in terms of the real and imaginary parts of the open loop transfer function $G_{ol}(j\omega) = exp(-t_d j\omega)C(j\omega)G(j\omega)$ as follows:

$$\sum_{n=1,3..}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi\rho}{2}\right)\cos\left(\frac{n\pi\rho}{2}\right)\Re\{G_{ol}(nj\omega_o)\} + \frac{\pi}{8} = 0 \tag{2a}$$

$$\frac{4\omega_0}{\pi}\sum_{n=1,3..}^{\infty} \sin^2\left(\frac{n\pi\rho}{2}\right)\Re\{G_{ol}(nj\omega_o)\} - \frac{4\omega_0}{\pi}\sum_{n=1,3..}^{\infty} \sin\left(\frac{n\pi\rho}{2}\right)\cos\left(\frac{n\pi\rho}{2}\right)\Im\{G_{ol}(nj\omega_o)\} - \frac{1}{2}\lim_{s\to\infty} sG_{ol}(s) < 0 \tag{2b}$$

$$\sum_{n=1,3..}^{\infty} \frac{1}{n}\sin^2\left(\frac{n\pi\rho}{2}\right)\Im\{G_{ol}(nj\omega_o)\} + \frac{\pi}{8} = 0 \tag{2c}$$

$$\frac{4\omega_0}{\pi}\sum_{n=1,3..}^{\infty} \sin^2\left(\frac{n\pi\rho}{2}\right)\Re\{G_{ol}(nj\omega_o)\} + \frac{4\omega_0}{\pi}\sum_{n=1,3..}^{\infty} \sin\left(\frac{n\pi\rho}{2}\right)\cos\left(\frac{n\pi\rho}{2}\right)\Im\{G_{ol}(nj\omega_o)\} - \frac{1}{2}\lim_{s\to\infty} sG_{ol}(s) < 0 \tag{2d}$$

It is worth noticing that these conditions are sufficient and necessary for the existence of limit cycles. Consequently, if any of the previous equations is not fulfilled, then steady state oscillations do not appear in the system. As can be noted, the equality conditions for oscillation given by equations (2a) and (2c) are simpler than those given by equations (2b) and (2d). Therefore, taking into account that the objective when tuning a PID controller is to avoid limit cycles, the equations (2a) and (2c) can be used to define a new robustness index to prevent oscillations.

The evaluation of left members of equations (2a) and (2c) in ranges of ω and ρ define a locus in the plane, which we named the Tsypkin Locus (TL). If the point $[0, 0]$ is contained in one of the trajectories defined by the TL, and the conditions (2b) and (2d) are fulfilled, the system will oscillate.

Instead of representing the TL in a plane defined by conditions (2a) and (2c), an alternative representation in the Nyquist plot is presented in the next section, which can sound more familiar in the context of control systems analysis and design.

3.1 Tsypkin's Bands

In order to represent the TL together with the Nyquist plot of G_{ol}, the equality conditions (equations (2a) and (2c)) are transformed to equations (3) and (4), which determine the set of points in which the system will oscillate with a given frequency (ω_o) and pulse amplitude (ρ). Detailed calculation has been omitted for the sake of brevity.

$$\Re\{G_{ol}(\omega_o)\} = \frac{-\frac{\pi}{4} - \sum_{n=3,5..}^{\infty} \frac{1}{n} \sin(n\pi\rho)\, \Re\{G_{ol}(nj\omega_o)\}}{\sin(\pi\rho)} \tag{3}$$

$$\Im\{G_{ol}(\omega_o)\} = \frac{-\frac{\pi}{8} - \sum_{n=3,5..}^{\infty} \frac{1}{n} \sin^2\left(\frac{n\pi\rho}{2}\right) \Im\{G_{ol}(nj\omega_o)\}}{\sin^2\left(\frac{\pi\rho}{2}\right)} \tag{4}$$

The region in the Nyquist plot is obtained by evaluating the right hand members of these expressions for values of ρ and ω. According to Figure 3 the values of ρ must lay between 0 and 1. The set of points obtained are organized in the form of bands that we call Tsypkin's bands. The band is formed by branches, each one of which corresponds to a frequency value and $\rho \in]0, 1[$. Regarding the range of ω, it has been considered from 0 to the gain margin crossover frequency (where phase crosses -180 degrees). This is a reasonable choice because intersections between the Tsypkin's bands and the $G_{ol}(j\omega)$ are more probable to take place for this range of frequency, that is, in the lower-left quadrant.

Two examples of the Tsypkin's bands are given in Figures 4 and 5 for FOPTD systems with time delays of 1.2 sec and 0.4 sec respectively. Unitary time constant and static gain have been considered. In both cases the controller has been tuned with the Ziegler-Nichols method. The branch that belong to each frequency is represented in a different color. In order to make the figure more readable, a limited number of branches has been represented.

According to the meaning of this branches, a system will oscillate if a point of its open loop transfer function in the Nyquist plot also belongs to the Tsypkin branch of its frequency. In the figures this means that, in order for the system to oscillate, a point of a specific color must also belong to the branch of its color.

Thus, it can be seen in Figure 4 that this system will not oscillate at the frequencies where the Tsypkin branches have been evaluated. Additionally, in this figure, we have represented graphically a new measure of robustness to oscillations, that we named Tsypkin's Margin M_T, which is the minimum distance between the points of G_{ol} and their respective Tsypkin branch.

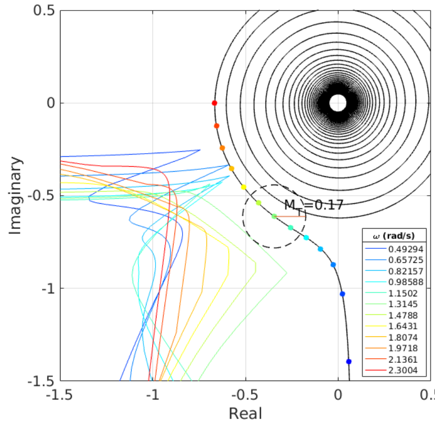

Fig. 4. Tsypkin band of a non-oscillating system in Nyquist diagram.

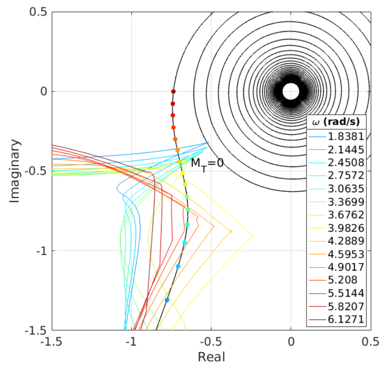

Fig. 5. Tsypkin band of an oscillating system in Nyquist diagram.

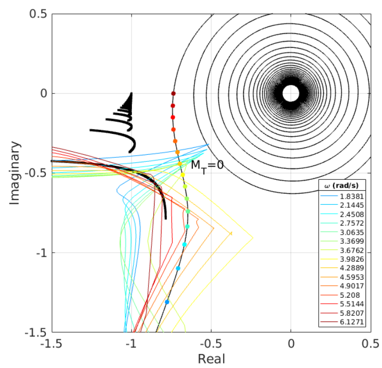

Fig. 6. Comparison between the Tsypkin method and DF methods in predicting oscillations.

On the other hand, in Figure 5, even when the exact branch where oscillation takes place does not coincide with the represented branches, it can be seen that there exists a point of G_{ol} that will also belong to the corresponding branch of the Tsypkin band, and, therefore, the system will oscillate.

Additionally, the Figure 6 has been included in order to compare the Tsypkin method and the DF method in predicting the oscillations. While, according to the

DF method, this system will not oscillate because there is no intersection between its traces and the G_{ol}, the Tsypkin method reveals a steady state oscillation with an intersection in one of its branches.

It is worth to remark that the distance M_T presented above, only offers information about the robustness against oscillation appearance. Conversely to other robustness indexes, as gain and phase margin, it is not directly related with the performance of the close loop system, as this margin is not necessarily placed in the same region of the Nyquist plot. In the following sections this measure M_T will be evaluated in detail for different systems and tuning methods.

4. DIMENSIONLESS CHARACTERIZATION OF FOPTD SYSTEMS AND CONTROLLERS

In order to generalize the results for any FOPTD system, its equations can be expressed in a dimensionless way using the transformation $\bar{s} = Ls$. Owing to this variable change, the analysis of the behavior becomes easier due to the appearance of the ratio L/τ which will be of interest. Using this transformation on the general expression of a FOPTD system the resulting equation become:

$$G(s) = \frac{Ke^{-Ls}}{\tau s + 1} \xrightarrow{s=\frac{\bar{s}}{L}} G(\bar{s}) = \frac{Ke^{-\bar{s}}}{\frac{\tau}{L}\bar{s} + 1} \qquad (5)$$

For any PID tuning method, the controller parameters can be defined as functions of system's ones. Depending on these functions, the tuning rules are called homogeneous or non-homogeneous. The homogeneous tuning rules in term of the FOPTD system's parameters are defined as follows, (Balaguer et al. (2013)):

$$K_c = K^{-1}\phi_1; T_i = L\phi_2; T_d = L\phi_3; \qquad (6)$$

where ϕ_i are functions of the ratio L/τ. Substituting these equations in the controller's expression and making the variable change:

$$C(s) = K_c \left(\frac{T_i T_d s^2 + T_i s + 1}{T_i s} \right)$$
$$C(\bar{s}) = K^{-1}\phi_1 \left(\frac{\phi_2\phi_3\bar{s}^2 + \phi_2\bar{s} + 1}{\phi_2\bar{s}} \right) \qquad (7)$$

The dimensionless open-loop transfer function can now be obtained as:

$$G_{ol}(\bar{s}) = C(\bar{s})G(\bar{s}) = \frac{\phi_1}{\phi_2} \frac{\phi_2\phi_3\bar{s}^2 + \phi_2\bar{s} + 1}{\bar{s}\left(\frac{\tau}{L}\bar{s} + 1\right)} e^{-\bar{s}} \qquad (8)$$

The resulting open-loop transfer function depends only on the dimensionless ratio L/τ and the tuning functions which also depend on this ratio. In consequence, the robustness margin M_T defined in the previous section, for FOPTD systems, will only depend on the ratio L/τ.

5. ANALYSIS OF THE RESULTS

In this section, the Tsypkin margins will be obtained for FOPTD systems and controllers using the dimensionless approach. PI and PID controllers will be tuned using different methods, namely Ziegler-Nichols, Cohen-Coon and AMIGO.

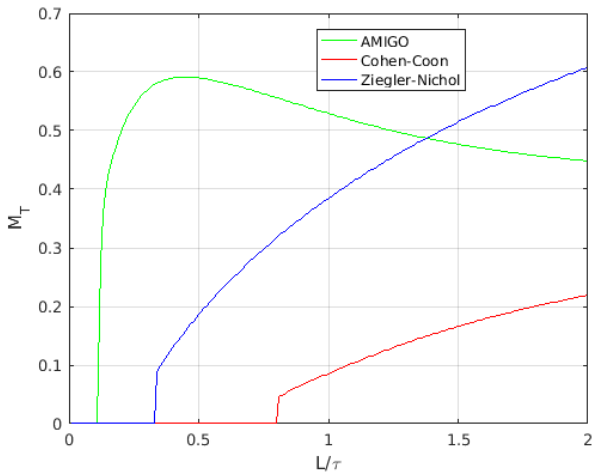

Fig. 7. Tsypkin Margin, M_T, for the different PI tuning methods as a function of L/τ.

5.1 *PI tuning*

In the Figure 7, the Tsypkin Margin, M_T, is plotted as a function of L/τ, for the different PI tuning methods. This parameter, M_T, defines the robustness margin to limit cycle oscillations. Comparing the different tuning methods, the AMIGO method leads to higher values of M_T than the other two methods, while the Cohen-Coon results in the lowest values. Ziegler-Nichols method gives intermediate results.

Another interesting point is the relation of the robustness to oscillations to the ratio L/τ. In general, the higher this ratio, the higher the value of M_T. This means that the problems of oscillations are more likely to occur if the behavior of the system is more similar to a pure first order system (L/τ small). In fact, for each tuning method there is a critical value L/τ below which the resulting $M_T = 0$, what means that the closed loop has a non vanishing oscillation. This critical value depends on the tuning method.

The existence of the limit cycle does not guarantee that the oscillation will appear, as this also depends on the size of the attraction region of that limit cycle. If the attraction regions is small, then the limit cycle only appears for certain specific initial conditions or disturbances. In order to illustrate this fact an example is provided.

According to Figure 7, a FOPTD system with a ratio $L/\tau = 0.1$ will oscillate with any tuning method. For this example the AMIGO method has been used. The system to be controlled is a FOPTD with $K = 1$, $L = 1$ and, to accomplish the ratio, $\tau = 10$. The controller's parameters for this system using the AMIGO PI method have been obtained as ($K_c = 2.8236$ and $Ti = 5.5038$).

Two simulations are performed. In the first one, the disturbance changes from 0 to 3δ when the system is stabilized, and in the second one the disturbance changes from 0 to 2δ. The rest of parameters remain unchanged.

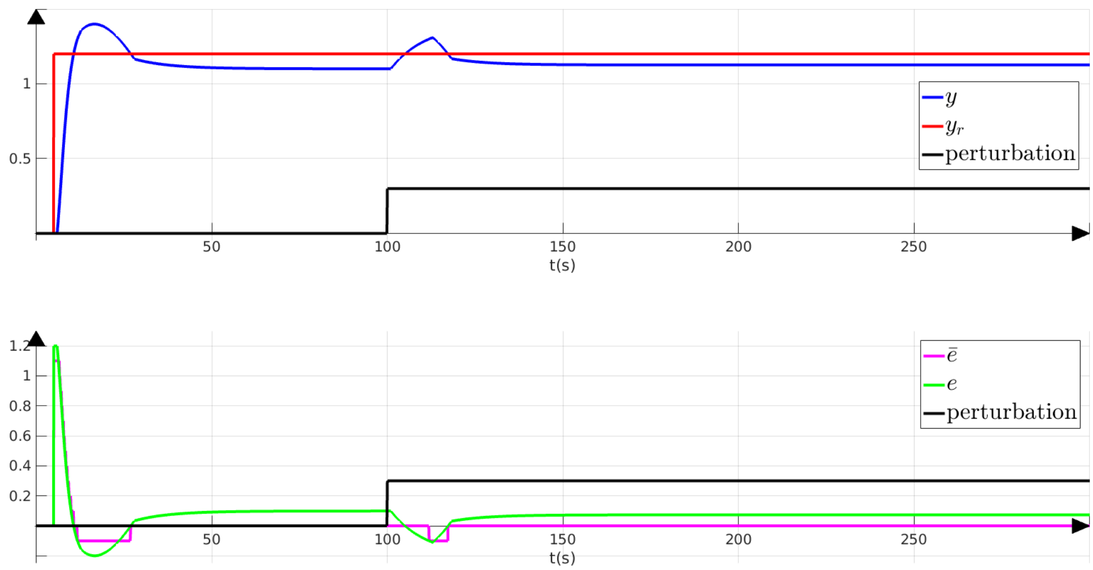

Fig. 8. Results of the simulation with $p = 3\delta$. In the upper figure, reference, system's response and disturbance signal are shown. In the bottom figure, error, SSOD quantified error and disturbance signal are shown.

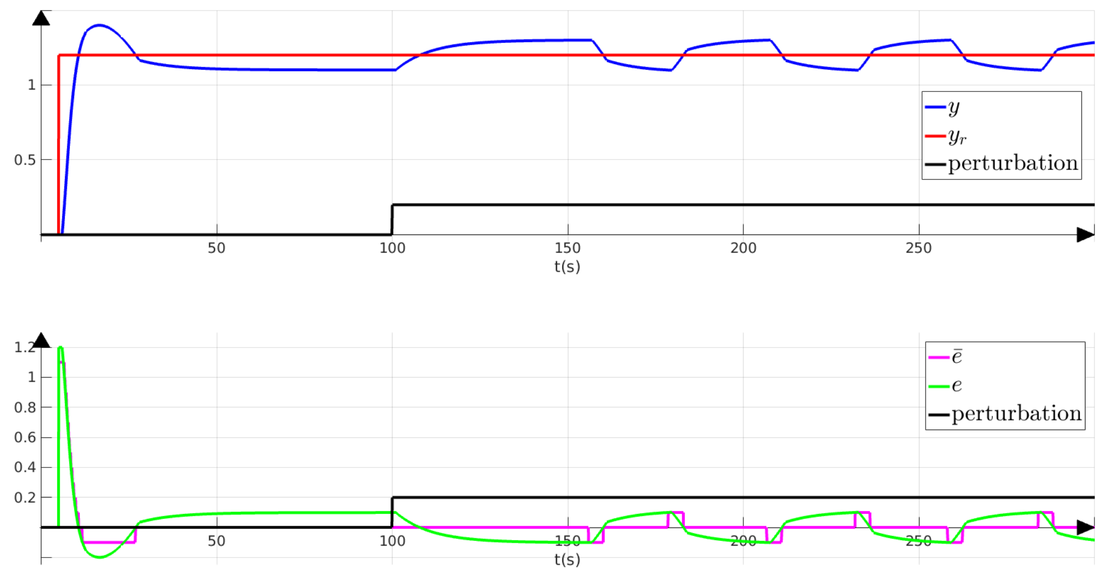

Fig. 9. Results of the simulation with $p = 2\delta$. In the upper figure, reference, system's response and disturbance signal are shown. In the bottom figure, error, SSOD quantified error and disturbance signal are shown.

The results of these simulations can be seen in Figures 8 and 9 for the changes of 3δ and 2δ respectively. In the first case, the system stabilizes, managing to reject the disturbance without oscillations. It can be seen that the error does not reach the superior bound $+\delta$. In contrast, in the second one, with a lower disturbance, the system reaches the limit cycle, switching from $+\delta$ to $-\delta$ periodically. Similar results have been observed changing the set-point and without any disturbance on the system and for Ziegler-Nichols and Cohen-Coon methods. Consequently, the study of the attraction region must be addressed in future works.

5.2 PID tuning

This method can also be applied to PID controllers. In Figure 10, Tsypkin Margin M_T, is plotted as a function of L/τ, for PI and PID tuning using the AMIGO method.

An interesting feature is that the PI controller is more robust to oscillations than the PID. This is not obvious, because the design of the PI and PID controllers are based on a similar robustness to instability. The reason is that with the derivative term, the Nyquist plot tends to move, for low frequencies, to the region where its Tsypkin bands are located. This is because of the phase characteristics of the derivative term, that adds phase for low frequencies. Therefore, if we compare a PI and a PID with the same phase margin, then the phase of the PID, for frequencies below the gain crossover frequency, is more negative.

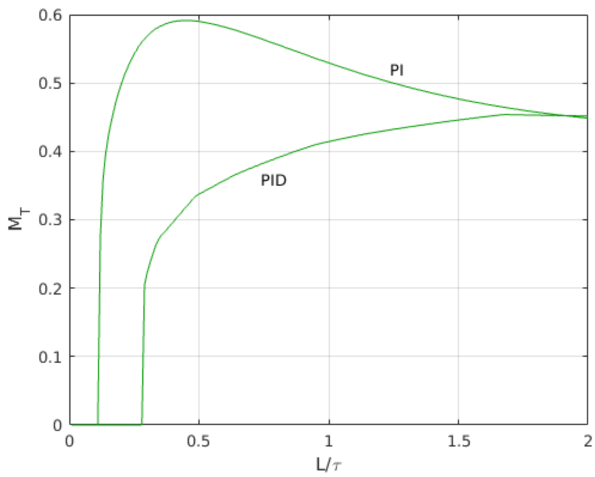

Fig. 10. M_T for PI and PID with AMIGO method.

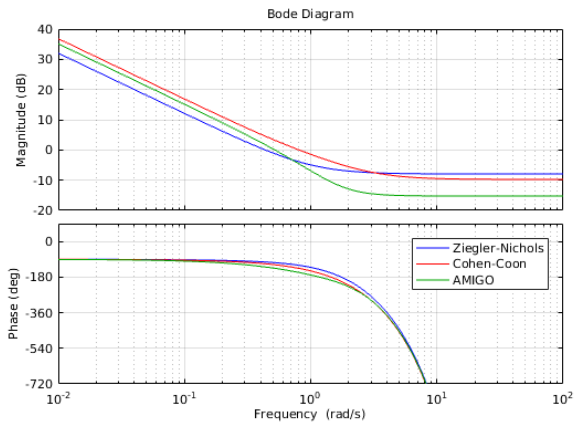

Fig. 11. Bode diagram of the resulting G_{ol} for the same system using different tuning methods.

The M_T was also calculated for PID with Ziegler-Nichols and Cohen-Coon methods, but due to the PID high frequency gain, the calculation is very sensible to the number of harmonics considered, and, in order to obtain an accurate value for the M_T margin, a high number of harmonics must be taken into account. This increases very significantly the computation time needed, compared to the case of the AMIGO PID method. This can be observed in Figure 11, where the bode diagram of G_{ol} is shown for a given system with three different controllers, one for each method. It is clear that the AMIGO controller filters the high order harmonics better than Ziegler-Nichols and Cohen-Coon controllers.

6. CONCLUSION

In this paper the robustness to oscillations of PI and PID controllers with a SSOD sampling strategy has been evaluated. To this end, Tsypkin's method has been used for developing a new measure of robustness against limit cycles in this type of systems. Due to Tsypkins's method, the measure offers accurate results for systems with poor high frequency filtering, as the FOPTD systems.

With regard to PI controllers, a comparative of its robustness to oscillations has been presented for Ziegler-Nichols, Cohen-Coon and AMIGO methods. The AMIGO obtains the best results in terms of robustness and range of systems covered.

On the other hand, with respect to PID controllers, only AMIGO controller has been evaluated due to the high dependency on high order harmonics of the other methods. Interestingly enough, the PI presents better results (regarding robustness to oscillations) than the PID, even though both are designed with a similar robustness to instability criterion.

REFERENCES

Åström, K. and Hägglund, T. (2004). Revisiting the Ziegler-Nichols step response method for PID control. *Journal of Process Control*, 14(6), 635–650.

Balaguer, P., Ibeas, A., Pedret, C., and Alcntara, S. (2013). *Homogeneity of PID tuning rules*, chapter 6, 79–94. Number 90 in IET CONTROL ENGINEERING. The Institution of Engineering and Technology, London, United Kingdom.

Beschi, M., Dormido, S., Sánchez, J., and Visioli, A. (2014). Tuning of symmetric send-on-delta proportional-integral controllers. *IET Control Theory & Applications*, 8, 248–259(11).

Beschi, M., Dormido, S., Sánchez, J., and Visioli, A. (2012). Characterization of symmetric send-on-delta PI controllers. *Journal of Process Control*, 22(10), 1930–1945.

Cohen, G.H. and Coon, G.A. (1953). Theoretical consideration of retarded control. *Transactions of the ASME*, (75), 827–834.

Feeney, L. and Nilsson, M. (2001). Investigating the energy consumption of a wireless network interface in an ad hoc networking environment. In *INFOCOM 2001. Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, volume 3, 1548–1557 vol.3.

Romero, J.A., Sanchis, R., and Penarrocha, I. (2014). A simple rule for tuning event-based pid controllers with symmetric send-on-delta sampling strategy. In *Proceedings of the 2014 IEEE Emerging Technology and Factory Automation (ETFA)*, 1–8.

Romero-Pérez, J.A. and Llopis, R.S. (2017). Tuning and robustness analysis of event-based pid controllers under different event generation strategies. *International Journal of Control*, 0(ja), 1–38.

Romero-Pérez, J.A. and Sanchis, R. (2016). A new method for tuning PI controllers with symmetric send-on-delta sampling strategy. *ISA Transactions*, 64, 161 – 173.

Tsypkin, Y.Z. (1984). *Relay control systems*. Cambridge University Press.

Ziegler, J.G. and Nichols, N.B. (1942). Optimum settings for automatic controllers. *trans. ASME*, 64(11).