PID controller tuning for integrating processes

Damir Vrančić*, Mikulaš Huba, and Paulo Moura Oliveira*****

** Department of Systems and Control, J. Stefan Institute, Ljubljana, Slovenia (e-mail: damir.vrancic@ijs.si).*
***Slovak University of Technology, Faculty of Electrical engineering and information technology, Ilkovičova 3, 812 19 Bratislava, Slovakia (e-mail:mikulas.huba@stuba.sk)*
**** Department of Engineering, Universidade de Trás-os-Montes e Alto Douro, UTAD, 5001–801 Vila Real, INESC-TEC Technology and Science, Campus da FEUP, 4200 - 465 Porto, Portugal (e-mail: oliveira@utad.pt)*

Abstract: The proposed tuning method for integrating processes, which is based on Magnitude optimum criterion, has been extended to PID types of controllers. The method requires either the process transfer function (in frequency-domain) or the measurement of process steady-state change (in time-domain). The PID controller parameters are calculated analytically by solving fourth-order polynomial. By changing reference-weighting parameter *b*, the user can favour tracking (higher *b*) or control performance (lower *b*). The proposed method has been tested on several process models (lower-order with delay, higher order with delay, and a phase non-minimum process) and the closed-loop responses were relatively fast and non-oscillatory. The comparison with other tuning method based on process step-response data results in favourable tracking and control performance.

Keywords: integrating process, unstable process, Magnitude optimum, PID control.

1. INTRODUCTION

Integrating processes can be found in chemical and process industries, while they are also common in mechanical engineering. Besides pure integrating processes, the processes with one or more large time constants, in comparison to the others, can also be modelled as integrating processes (Åström and Hägglund, 1995).

There are several tuning methods existing for integrating processes and different controller structures, as given in Åström and Hägglund (1995), Panda (2009), Shamsuzzoha and Moonyong (2008), Taguchi and Araki (2000) and Huba (2013).

Until recently, the Magnitude Optimum (MO) method (Whiteley, 1946), which is based on optimisation of the closed-loop amplitude response, was not used on integrating processes. The symmetrical optimum method was used instead, which results in substantial overshoots for set-point reference changes.

In Vrančić and Strmčnik (2011) the MO method has been applied on integrating processes by using 2-degree-of-freedom (2-DOF) PI controller structure. The tuning method could be used either on the general process transfer function (in frequency-domain) with time delay or on process step-response (in time-domain).The tuning results were very good for different types of processes when compared to some other PI controller tuning methods for integrating processes, based on process time-responses.

In this paper the tuning method is extended to PID controller structure. As will be shown in the following sections, the PID controller can be much more efficient for tracking and control when comparing to the PI controller.

The paper is set out as follows. Section 2 derives MO tuning method for 2-DOF PID controller for integrating processes. Section 3 shows the closed-loop responses on reference changes and on process input disturbances on several process models. Results are also compared to other tuning method for integrating processes. Conclusions are provided in section 4.

2. MO TUNING METHOD FOR INTEGRATING PROCESSES

Figure 1 shows the process in a closed-loop configuration with a 2-DOF PID controller, where signals *r*, *u*, *d* and *y* represent a reference, a controller output, an input disturbance and a process output, respectively. Parameters K_P, K_I, K_D, T_F and *b* are proportional gain, integral gain, derivative gain, filter time constant and reference weighting factor, respectively.

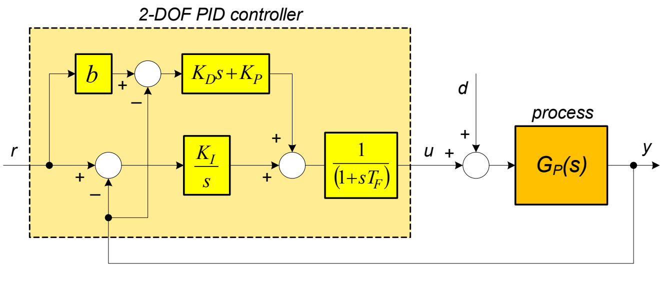

Fig. 1. The closed-loop configuration with integrating using a 2-DOF PID controller.

One possible controller design objective is to maintain the closed-loop magnitude (amplitude) as flat and as close to unity over as wide a frequency range as possible (Whiteley, 1946; Vrančić *et al.*, 2001). This technique is variously called magnitude optimum, modulus optimum or Betragsoptimum, and results in a fast and non-oscillatory closed-loop time response for a large class of process models.

If $G_{CL}(s)$ is the closed-loop transfer function from the reference (r) to the process output (y):

$$G_{CL}(s)=\frac{Y(s)}{R(s)}=\frac{G_P(s)\left(\dfrac{bK_D s^2+bK_P s+K_I}{s(1+sT_F)}\right)}{1+G_P(s)\left(\dfrac{K_D s^2+K_P s+K_I}{s(1+sT_F)}\right)}, \qquad (1)$$

the controller is determined so as that

$$\begin{aligned}&G_{CL}(0)=1\\ &\lim_{\omega\to 0}\left[\frac{d^{2k}\left|G_{CL}(j\omega)\right|^2}{d\omega^{2k}}\right]=0\,;\ k=1,2,\cdots,k_{\max}\end{aligned} \qquad (2)$$

The upper equation in (2) is simply fulfilled by using a controller structure containing the integral term (under the condition that the closed-loop response is stable), because the steady-state control error is already zero (Vrančić et al., 1999). The number of conditions (k_{max}) in (2) that can be satisfied depends on controller order (number of controller parameters), which is k_{max}=3 in the case for PID controller.

The integrating process is given by the following general rational transfer function with time-delay:

$$G_P(s)=\frac{K_{PR}}{s}\frac{1+b_1 s+b_2 s^2+\cdots+b_m s^m}{1+a_1 s+a_2 s^2+\cdots+a_n s^n}e^{-sT_{del}}. \qquad (3)$$

The PID controller parameters can be calculated by inserting expressions (1) and (3) into expression (2) and by solving equations (2) for k=1, 2 and 3.

<u>Remark 1</u>. The controller filter G_F=1/(1+sT_F) (see Figure 1) should be considered as a part of the process. Therefore, it should be added to expression (3) before calculating controller parameters.

<u>Remark 2</u>. The controller includes integrating term even though the process is of integrating type. Besides other reasons, the integrating term is required for eliminating the control error under process input disturbances.

The whole procedure for the calculation of controller parameters is demanding and time-consuming, since it requires solving the mentioned set of three equations (2). However, the final formula for the PID controller parameters remains analytic, although it requires solving the fourth-order polynomial. The PID controller parameters can be calculated from the following expressions:

$$\begin{gathered}\alpha T_I^4+\beta T_I^3+\gamma T_I^2+\delta T_I+\varepsilon=0\\ \text{where}\\ \alpha=b_1(1+b)^2\left(G_{10}^2 b_1+2bG_{20}\right)\\ \beta=8\left(b_2 G_{10}^3+\left(2b^2+b-1\right)G_{10}G_{20}-b(1+b)G_{30}\right)\\ \gamma=8\begin{pmatrix}-\left(b^2-3\right)G_{10}^4+\left(3b^2+b-6\right)G_{10}^2G_{20}-\\ -2\left(b^2+b-1\right)G_{10}G_{30}+b_1(1+b)G_{20}^2+\\ +b(1+b)G_{40}\end{pmatrix}\\ \delta=16\begin{pmatrix}2G_{10}^5-6G_{10}^3G_{20}+3\left(G_{10}^2G_{30}+G_{10}G_{20}^2\right)-\\ -G_{10}G_{40}-G_{20}G_{30}\end{pmatrix}\\ \varepsilon=16\begin{pmatrix}G_{10}^6-4G_{10}^4G_{20}+2G_{10}^3G_{30}+4G_{10}^2G_{20}^2-\\ -G_{10}^2G_{40}-2G_{10}G_{20}G_{30}-G_{20}^3+G_{20}G_{40}\end{pmatrix}\\ \text{and}\\ b_1=1-b\end{gathered}\ , \qquad (4)$$

$$\begin{aligned}K_I&=-\frac{G_{00}\left(2G_{10}^2+2G_{10}T_I-2G_{20}-b(1+b)T_I^2\right)}{0.25b_1(1+b)^3T_I^4}\\ K_{DI}&=\frac{0.5b_1(1+b)T_I^2-G_{00}/K_I}{b_1}\end{aligned}. \qquad (5)$$

by making the following transitions:

$$\begin{aligned}K_P&=T_I K_I\\ K_D&=K_{DI}K_I\end{aligned}. \qquad (6)$$

Note that the integral time constant (T_I) in polynomial (4) should be solved first. The result is the real number with the highest value. The calculated value of T_I is used in expressions (5). The final PID controller gains are calculated in (6).

The values G_{00} to G_{40} are the following:

$$\begin{aligned}&G_{00}=A_0^{-1},\ G_{10}=-A_1A_0^{-1},\ G_{20}=A_2A_0^{-1}\\ &G_{30}=-A_3A_0^{-1},\ G_{40}=A_4A_0^{-1}\end{aligned}, \qquad (7)$$

where symbols A_0 to A_4 represent the so-called "characteristic areas" of the process (Vrančić et al., 2001):

$$\begin{gathered}A_0=K_{PR}\\ A_1=K_{PR}\left(a_1-b_1+T_{del}\right)\\ A_2=K_{PR}\left(b_2-a_2-T_{del}b_1+\frac{T_{del}{}^2}{2!}\right)+A_1a_1\\ \vdots\end{gathered}. \qquad (8a)$$

$$A_k = K_{PR}\left(\left(-1\right)^{k+1}\left(a_k - b_k\right) + \sum_{i=1}^{k}\left(-1\right)^{k+i}\frac{T_{del}^i b_{k-i}}{i!}\right) + \\ + \sum_{i=1}^{k-1}\left(-1\right)^{k+i-1} A_i a_{k-i} \quad . \tag{8b}$$

The characteristic areas can also be calculated from time-domain experiment by changing the steady-state of the process. The process input ($u(t)$) and output ($y(t)$) signals can be integrated as given below (Vrančić et al., 2001; Vrančić and Strmčnik, 2011):

$$\begin{aligned} u_0 &= \frac{u(t)-u(0)}{\Delta U} & y_0 &= \frac{\dot{y}(t)-\dot{y}(0)}{\Delta U} \\ I_{U1}(t) &= \int_0^t u_0(\tau)d\tau & I_{Y1}(t) &= \frac{y(t)-y(0)-\dot{y}(0)\cdot t}{\Delta U} \\ I_{U2}(t) &= \int_0^t I_{U1}(\tau)d\tau & I_{Y2}(t) &= \int_0^t I_{Y1}(\tau)d\tau \\ &\vdots & &\vdots \end{aligned} \quad , \tag{9}$$

where

$$\Delta U = u(\infty) - u(0). \tag{10}$$

The characteristic areas can be calculated as follows:

$$\begin{aligned} &A_0 = y_0(\infty) \; ; \; y_1 = A_0 I_{U1}(t) - I_{Y1}(t) \\ &A_1 = y_1(\infty) \; ; \; y_2 = A_1 I_{U1}(t) - A_0 I_{U2}(t) + I_{Y2}(t) \\ &A_2 = y_2(\infty) \\ &y_3 = A_2 I_{U1}(t) - A_1 I_{U2}(t) + A_0 I_{U3}(t) - I_{Y3}(t) \\ &A_3 = y_3(\infty) \\ &\vdots \end{aligned} \quad . \tag{11}$$

As already mentioned in Vrančić and Strmčnik (2011), the process should be in the steady-state before the change of the working point and it is enough to integrate until the transient expressions in (10) and (11) die out.

The PI controller parameters can be calculated by fixing K_D=0 and solving only the first two derivatives in (2) (for k=1 and 2). The following PI controller parameters are obtained (Vrančić and Strmčnik, 2011):

$$\begin{aligned} K_P &= \frac{-A_1 + \sqrt{A_1^2 + \xi}}{\xi} \\ K_i &= 0.5 A_0 \left(1-b^2\right) K_P^2 \end{aligned} \quad , \tag{12}$$

where

$$\xi = \left(1-b^2\right)\left(A_0 A_2 - A_1^2\right). \tag{13}$$

Remark 3. Note that weighting factor b<1. If b=1, the integral gain approaches zero (Vrančić and Strmčnik, 2011). In practice the values 0<b<0.9 should be used.

The PID controller tuning proceeds as follows:

a) Select PID controller filter time constant T_F.
b) Calculate characteristic areas A_0 to A_4 (in frequency-domain) from expression (8) or from expressions (9) to (11) (in time-domain) by changing the steady-state of the process. According to Remark 1, add the controller filter to the process transfer function (3) before.
c) Set reference weighting factor b. According to Remark 2, recommended values of b are below 1 (e.g. between 0 and 0.9).
d) Calculate the PID controller parameters from expression (6).

Matlab files, performing entire tuning of PID controller parameters from the process model or the process time-responses is available on-line (Vrančić, 2018).

Illustrative example

Let us calculate the PID controller parameters for the following integrating process:

$$G_P(s) = \frac{e^{-0.5s}}{s(1+2s)(1+s)} \tag{14}$$

with the a-priori chosen controller filter time constant T_F=0.1s.

The characteristic areas are calculated from expression (8), after adding the controller filter to the process transfer function:

$$\begin{aligned} &A_0 = 1, \; A_1 = 3.6, \; A_2 = 8.99 \\ &A_3 = 19.79, \; A_4 = 41.42 \end{aligned} \quad . \tag{15}$$

The PID controller parameters are calculated from expression (6) by choosing different values of reference weighting factor b. The controller parameters are given in Table 1. The closed-loop responses for reference change and for input disturbance (d=0.1) are shown in Figure 2. It can be seen that tracking performance increases by increasing factor b. On the other hand, disturbance rejection improves by decreasing factor b. A good compromise between tracking and control performance seems to be at b=0.5 (see dashed lines in Fig. 2).

Table 1. The PID controller parameters for different values of factor *b*.

b	K_P	K_I	K_D
0	0.60	0.095	0.88
0.5	0.54	0.076	0.86
0.9	0.39	0.0137	0.81

The Bode plots of the closed-loop amplitude (gain) from the reference to the process output, for all three values of b are shown in Figure 3. It is obvious that the Bode plots correspond to the MO criteria in expression (2) and that the closed-loop bandwidth increases by increasing factor b (the tracking speed improves by increasing b).

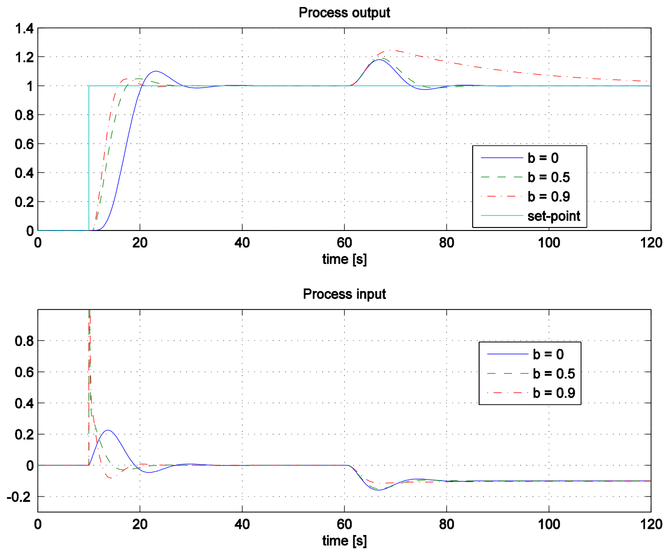

Fig. 2. The closed-loop responses on the process G_P for different values of b.

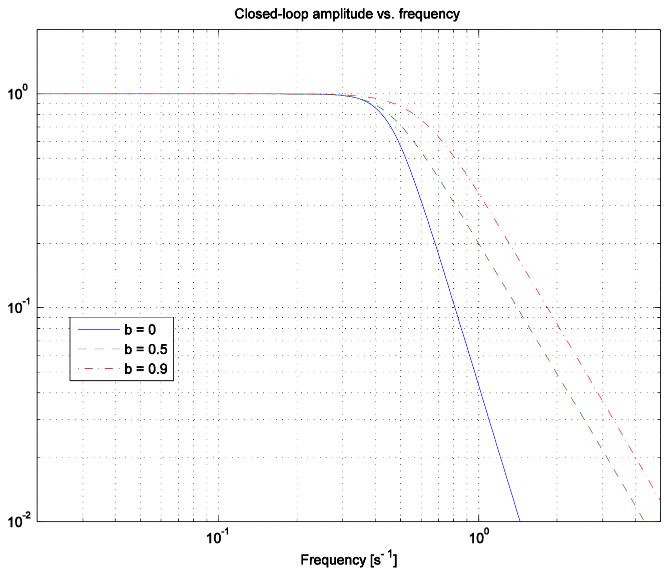

Fig. 3. Bode plots for different values of factor b.

The PI controller parameters, for b=0.5, are calculated from expression (12):

$$\begin{aligned} K_P &= 0.15 \\ K_I &= 0.0086 \end{aligned} . \tag{16}$$

The comparison between the PI and the PID controller at b=0.5 (see Figure 4) shows much better tracking and control performance of the PID controller.

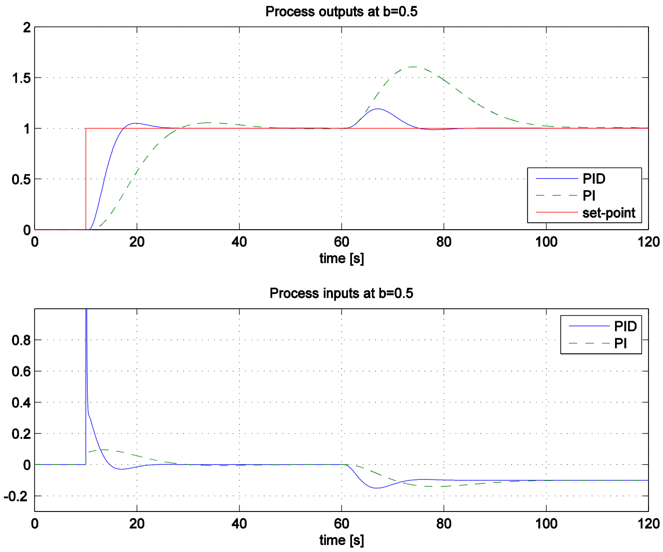

Fig. 4. Comparison between the PI and the PID controller closed-loop responses.

3. EXAMPLES

The method has been tested on some other types of integrating processes (the second-order with delay, the fourth-order with delay and the phase non-minimum processes) and compared to the method proposed by Åström and Hägglund (1995). In the final version of the paper, the comparison to some other tuning methods are foreseen. In all the cases the a-priori chosen controller filter time constant is T_F=0.1s and process input disturbance d=0.1 appears at t=60s in all examples.

Case 1. The following second-order integrating process with delay is chosen:

$$G_{P1}(s) = \frac{e^{-s}}{s(1+s)^2} . \tag{17}$$

The characteristic areas, when taking into account the a-priori chosen filter T_F=0.1s, are calculated from expression (8):

$$\begin{aligned} &A_0 = 1,\ A_1 = 3.1,\ A_2 = 5.81, \\ &A_3 = 8.75,\ A_4 = 11.75 \end{aligned} . \tag{18}$$

The PID controller parameters, for b=0 and b=0.5 (denoted as MO00 and MO05 in the following text), are calculated from expression (6):

$$\begin{aligned} &b = 0,\ K_P = 0.49,\ K_I = 0.0766,\ K_D = 0.567 \\ &b = 0.5,\ K_P = 0.44,\ K_I = 0.0575,\ K_D = 0.54 \end{aligned} . \tag{19}$$

The closed-loop responses for reference change and for input disturbance are shown in Figure 5. It can be seen that the closed-loop response is relatively fast and stable without oscillations, all according to the MO criteria. The results have been compared to the method proposed by Åström and Hägglund (1995), which is based on choosing maximum sensitivity value to either M_S=1.4 (in subsequent text Ms14) or M_S=2.0 (in subsequent text Ms20). However, the actual maximum sensitivity still depends on the process. The tracking and control performance has been evaluated by integral of square error for reference changes (ISEr) and for input disturbance with amplitude 0.1 (ISEu). The measured Ms values as well as the ISEr and ISEu values are given in Table 2.

Table 2. The ISE and Ms values for example 1

	MO00	*MO05*	*Ms14*	*Ms20*
ISEr	5.73	3.19	6.85	4.96
ISEu	0.258	0.32	4.85	0.44
Ms	2.71	2.25	1.32	1.78

According to ISE values in Table 2, the proposed method MO05 gives better tracking and control performance than Ms14 and Ms20. The best control performance is achieved with MO00, at the cost of slower tracking performance.

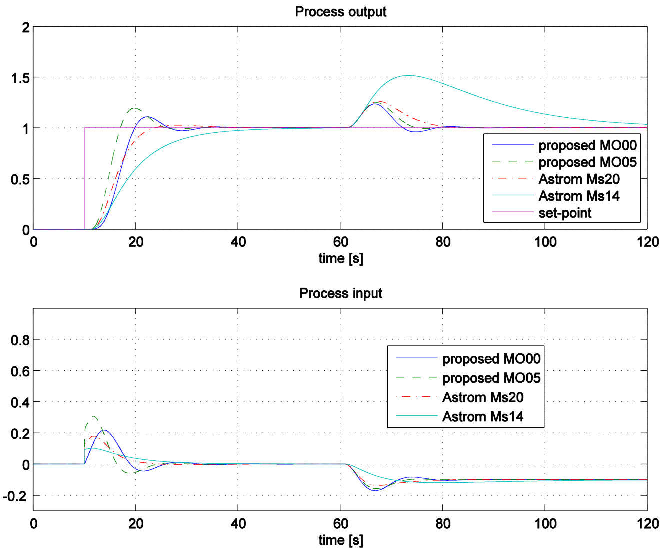

Fig. 5. The closed-loop responses on the process G_{P1}.

Case 2. The following fourth-order integrating process is chosen:

$$G_{P2}(s)=\frac{e^{-0.5s}}{s(1+s)^4}. \tag{20}$$

The characteristic areas, when taking into account the controller filter T_F=0.1s, are the following:

$$\begin{aligned}&A_0=1,\ A_1=4.6,\ A_2=12.59\\&A_3=26.78,\ A_4=49.01\end{aligned}. \tag{21}$$

The PID controller parameters, for b=0 and b=0.5, are:

$$\begin{aligned}&b=0,\ K_P=0.309,\ K_I=0.0314,\ K_D=0.517\\&b=0.5,\ K_P=0.279,\ K_I=0.0235,\ K_D=0.493\end{aligned}. \tag{22}$$

The closed-loop responses for the reference change and for input disturbance are shown in Figure 6. Again, the MO tuning method for integrating processes gives stable response without oscillations and with small overshoots, all according to the MO criterion. The measured Ms values as well as the ISEr and ISEu values are given in Table 3.

Table 3. The ISE and Ms values for example 2

	MO00	*MO05*	*Ms14*	*Ms20*
ISEr	8.79	5.01	6.85	7.47
ISEu	1.04	1.28	4.85	1.47
Ms	2.72	2.25	1.32	1.85

According to ISE values in Table 3, the proposed method MO05 again gives better tracking and control performance than Ms14 and Ms20. The best control performance is achieved with MO00, at the cost of slower tracking performance.

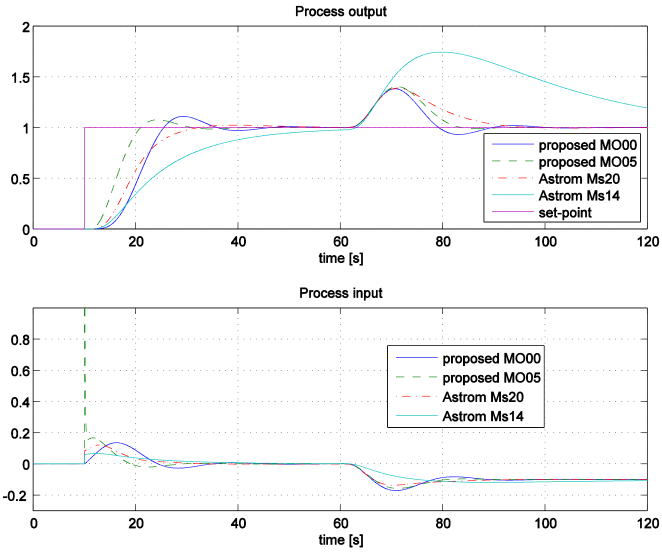

Fig. 6. The closed-loop responses on the process G_{P2}.

Case 3. The following non-minimum phase integrating process is chosen:

$$G_{P3}(s)=\frac{1-2s}{s(1+s)^2}. \tag{23}$$

The characteristic areas, when taking into account the controller filter T_F=0.1s, are calculated from expression (8):

$$A_0 = 1,\ A_1 = 4.1,\ A_2 = 7.41,\ A_3 = 10.74,\ A_4 = 14.07\,. \quad (24)$$

The PID controller parameters, for b=0 and b=0.5, are calculated from expressions (4):

$$\begin{aligned} &b = 0, K_P = 0.32, K_I = 0.0367, K_D = 0.388 \\ &b = 0.5, K_P = 0.27, K_I = 0.0233, K_D = 0.34 \end{aligned} \quad (25)$$

The measured maximum sensitivity (M_S) for b=0 is M_S=3.70, for b=0.5 is M_S=2.61, for Ms14 is M_S=1.42, and for Ms20 is M_S=2.35.

The closed-loop responses for reference change and for input disturbance are shown in Figure 7. The measured Ms values as well as the ISEr and ISEu values are given in Table 4.

Table 4. The ISE and Ms values for example 3

	***MO*00**	***MO*05**	***Ms*14**	***Ms*20**
ISEr	7.99	5.24	8.75	7.12
ISEu	1.14	1.46	22.84	1.58
Ms	3.70	2.61	1.43	2.35

According to ISE values in Table 4, the proposed method MO05 again gives better tracking and control performance than Ms14 and Ms20. As in the previous two experiments, the best control performance is achieved with MO00, at the cost of slower tracking performance.

4. CONCLUSIONS

The paper presented novel PID tuning method for integrating processes, which was based on MO criterion. It was shown that the PID controller parameters, when using 2-DOF configuration, can be analytically calculated.

The proposed method has been tested on 4 different process models, including higher-order process models with delays and non-minimum phase process models. The closed-loop responses were stable and relatively fast for all tested process models.

The closed-loop tracking and control responses, when compared to the method proposed by Åström and Hägglund (1995), were comparable or better. Note that another advantage of the proposed method is that the controller parameters can be tuned from the given process transfer function or directly from the process time-response during the steady-state change.

In the future we are planning to compare the proposed method to some other tuning methods, to derive tuning formulas for different PID controller structures or other types of controllers, and to evaluate impact of the filter order on the measurement noise attenuation (Huba, 2015; Huba et al., 2016).

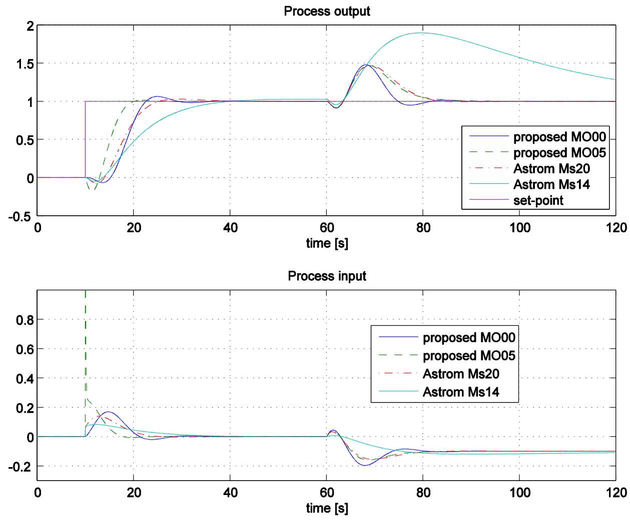

Fig. 7. Closed-loop responses of the process G_{P3}.

REFERENCES

Åström K. J., and Hägglund, T., (1995). “PID controllers: Theory, design and tuning”, 2nd Ed., Instrument Society of America, USA.

Gorez, R., (1997). “A survey of PID auto-tuning methods”, In: Journal A, Vol. 38, No. 1, pp. 3-10.

Huba, M. (2013). “Performance measures, performance limits and optimal PI control for the IPDT plant”, Journal of Process Control, 23 (4), pp. 500-515.

Huba, M. (2015) “Filter choice for an effective measurement noise attenuation in PI and PID controllers”, IEEE International Conference on Mechatronics, ICM 2015, pp. 46-51.

Huba, M., Bisták, P., and Huba, T. (2016) “Filtered PI and PID control of an Arduino based thermal plant”, IFAC-PapersOnLine, 49 (25), pp. 336-341.

Panda, R. C., (2009) “Synthesis of PID controller for unstable and integrating processes”, In: Chemical Engineering Science, Vol. 64, pp. 2807-2816.

Shamsuzzoha, M. and Moonyong, L., (2008). “Analytical design of enhanced PID filter controller for integrating and first order unstable processes with time delay”, In: Chemical engineering science, Vol. 63, pp. 2717-2731.

Taguchi, H. and Araki, M., (2000). “Two-degree-of-freedom PID controllers – their functions and optimal tuning”, In: Proceedings of the IFAC Workshop on Digital Control (PID’00), Terrassa, 2000. pp. 95-100.

Vrančić, D., Strmčnik, S. and Juričić, Đ., (2001). “A magnitude optimum multiple integration method for filtered PID controller”, In: Automatica. Vol. 37, pp. 1473-1479.

Vrančić, D., and S. Strmčnik, (2011). “Design of 2-DOF PI controller for integrating processes”, In: Proceedings of the 8th Asian Control Conference, Kaoshiung, Taiwan, May 15-18, 2011, pp. 1135-1140.

Vrančić, D. (2018). “Matlab and Simulink files for PID controller tuning for integrating processes”, [online]. Available at: http://dsc.ijs.si/damir.vrancic/tools.html

Whiteley, A. L., (1946). “Theory of servo systems, with particular reference to stabilization”, The Journal of IEE, Part II, 93(34), pp. 353-372.