Analysis of Effects due to Right Half Plane Zeros in PI Controller based Hydro Turbine ⋆

Soumyadeep Bose * **Yogesh V. Hote** * **Sandeep D. Hanwate** *

* *Department of Electrical Engineering, Indian Institute of Technology Roorkee, Roorkee-247667, India(e-mail: soumyadeepb9@gmail.com, yhotefee@iitr.ac.in).*

Abstract: In this paper, some detrimental time-domain characteristics (zero-crossings, overshoot(due to zeros) and initial undershoot) are analyzed using the system transfer function model, for identifying the presence of these effects in step response of non-minimum phase linear systems. Moreover, regarding initial undershoot, a theorem for detection using the state-space model, without the need to obtain transfer function, is proposed. In addition to detection, a theorem for estimating the percentage of initial undershoots is also proposed. Applications of these theorems are carried out on load frequency control of hydro-electric power plant which employs hydro-turbine, a non-minimum phase system. Moreover, the effect of a PI controller on initial undershoot is discussed for a general non-minimum phase linear system and illustrated for load frequency control of hydro-electric power plant.

Keywords: Initial undershoot, load frequency control, Markov parameters, overshoot, step response, zero-crossings

1. INTRODUCTION

Control system engineering has undergone extensive research since 1950s, and can be classified in numerous ways. In terms of system properties, a major class of systems are known as Non-Minimum Phase (NMP)systems. Such systems, when formulated in linear terms, contain at least one pole or zero on the Right-Half-Plane (RHP) of the s-plane, as defined in Dorf and Bishop (2014). From asymptotic stability point of view, asymptotically stable systems whose inverse gives unstable impulse response are the stable NMP systems. NMP systems have some drawbacks in contrast to Minimum Phase (MP) systems. For instance, because of an unstable zero, additional phase lead or lag is introduced in frequency domain, causing the closed-loop system become prone to instability. From the controller perspective, presence of a negative term in the denominator of the closed-loop system reduces the stability boundary for tuning the control parameters, thus posing limitations on system performance. Moreover, Qiu and Davison (1993) mentioned that MP systems are advantageous over NMP ones, for instance, regarding the accuracy in regulation, tracking and robustness is assured by MP systems possessing the property of right invertability. Authors of this paper also mentioned that some NMP systems perform almost as good as an MP system while some others are impossible to control to give desired response. Although not so prevalent, many practical systems such as drum boiler performance reported by Åström and Bell (2000), work of Su and Khorasani (2001) on single link flexible manipulator, small-signal performance of a SEPIC inverter in Hegde and Izadian (2013), vertical take-off and landing system of an aircraft in Boekfah (2017) etc. exist in daily life.

Constricting the discussion to the main context of this paper, RHP zeros, besides the effects discussed so far, also impose some unique drawbacks when it comes to the step response of these systems. These effects are zero-crossings, overshoot (due to zeros) and initial undershoot, which are formally defined and described in Hoagg and Bernstein (2007). While zero-crossings occur only for NMP systems' responses, presence of overshoot (due to zeros) and initial undershoot are indirectly linked to NMP systems. Multiple number of zero-crossings with respect to the initial and steady-state values introduce undesirable ringing and sluggishness in the system's response, which are often very difficult to nullify. Hence, analysis on the occurrence and tackling these effects is important to improve NMP system's performance.

In this paper, theorems to identify presence of the discussed effects in step response of Continuous-Time, Linear Time-Invariant (CTLTI) NMP systems (using respective transfer functions) are analyzed. The main contribution of this paper is the proposal of theorems to identify and estimate initial undershoot in step response of NMP systems. The major advantage of the proposed theorems is this, that the analysis can be done using both the state-space and transfer function form of the system's model. This is in contrast to the reported literature on identification of this effect, where the theorems are limited to the transfer function model only. The theorems are used to analyze a practical problem of Load Frequency Control (LFC) of a hydro-electric plant, which involves an NMP

⋆ This work was not supported by any organization.

system-hydro-turbine, is carried out on the basis of these theorems. It is also described how different water starting times can affect the initial undershoot in the system's response. In addition to this, analysis of the effect of a classical Proportional Integral (PI) controller on initial undershoot of a generalized system, as well as for the LFC involving hydro-turbine is done using the described theorems, in order to provide a guidance for power system engineers while designing PI control for this and other control engineering problems such as voltage regulation of a system employing DC-DC boost converter, stabilitzation of cart pendulum system, etc.

The rest of the paper is organized as follows. First, the characteristics are briefly discussed in Section 2, along with the theorems (both existing and proposed). Next, the analysis of change in load frequency corresponding to change in water valve opening of a hydro-turbine based power plant is carried out in Section 3 using the theorems mentioned in Section 2. Along with this, analysis of the same in closed-loop form is carried out with a classical PI controller. Besides this, effects of changing controller gains on the percentage of initial undershoot in step response of the system is also obtained. Finally, conclusion on the work and future aspects of the proposed techniques are given in Section 4.

2. THEOREMS OF ANALYSIS

Let a single valued, continuous, infinite times differentiable at $t=0$ function $y(t)$ represent the step response of a CTLTI system whose transfer function is $G(s)$. Let the Region Of Convergence (ROC) of $G(s)$ is defined as: $\{s_r \in \mathbb{C} \mid \mathbb{R}(s) > s_r\}$

2.1 Brief explanation:

Zero-crossings: Zero-crossing is a phenomenon when a function, single or multi-valued, crosses the line(s) of independent variable(s). In case of $y(t)$, if, for some instants or continuous intervals of time, $y(t)$ remains zero, then the function is said to exhibit zero-crossing. The former is termed as point zero-crossing while the latter interval zero-crossing.

Overshoot(due to zeros): Overshoot is defined as the maximum positive deviation of the system response w.r.t. the desired steady-state value. Although it generally occurs due to complex poles of the system, overshoot can also be present in over-damped systems because of zeros.

Initial undershoot: The progression of the response in 'wrong' direction for a while, after the initiation of the process is termed as 'initial undershoot'. Hoagg and Bernstein (2007) mentions about two types of undershoot, where they mentioned initial undershoot as Type-A undershoot too.

2.2 Existing theorems of analysis:

Theorem 1: According to Damm and Muhirwa (2014), if G(s) has n_1 number of positive real zeros and n_2 number of zeros at $s=0$, then its step response $y(t)$ has at least :

- n_1 number of zero-crossings if $n_2 < 2$.
- $n-1$ number of zero-crossings if $n_2 \geq 2$ (where n is the number of zeros of $G(s)$).

Proof: For proof, refer to Damm and Muhirwa (2014).

Assumption:

- $G(0) < \infty$
- $G(0) \neq G(\infty)$

Theorem 2: According to Hoagg and Bernstein (2007), if $G(s) - G(0)$ has at least one positive real zero, then $y(t)$ has overshoot.
Proof: Proof is given in Hoagg and Bernstein (2007).

2.3 Proposed theorems of analysis:

Assumption: $G(s)$ is output-controllable.

Theorem 3: If the first non-zero Markov parameter of $G(s)$ has opposite sign to that of $G(0)-y(0)$, then $y(t)$ has initial undershoot.
Proof: The condition for the presence of initial undershoot, as given in Hoagg and Bernstein (2007), Hoagg et al. (2007) and Damm and Muhirwa (2014), is as follows:

$$sgn[y^{(\mu)}(0)] \neq sgn[G(0) - G(\infty)] \tag{1}$$

where $y^{\mu}(0)$ is the μth derivative of $y(t)$ such that $y^{(\mu-1)}(0) = y^{(\mu-2)}(0) = \ldots = y^{'}(0) = 0$, and μ is a positive integer greater than zero. Then μ represents the rank of the system $G(s)$.
A CTLTI system represented in state-space form as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

By suitable computations and using Laplace transform, the transfer function for the above system is given by $G(s)$ as follows:

$$G(s) = C(sI - A)^{-1}B + D \tag{3}$$

Treating $Y(s) \equiv \mathcal{L}[y(t)]$, with step input, $Y(s)$ is written as:

$$Y(s) = \frac{C(sI - A)^{-1}B + D}{s} \tag{4}$$

Taking Inverse Laplace Transform of (4), the general expression for $y(t)$ is given below:

$$y(t) = \int_0^t Ce^{At}Bdt + D \tag{5}$$

In (5), $e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$ is known as the state transition matrix of the system.
Differentiating (5) w.r.t. time t for μ number of times,

$$y^{(\mu)}(t) = CA^{\mu-1}e^{At}B \tag{6}$$

Hence, at $t = 0$, the first non-zero derivative of $y(t)$ is given by $CA^{\mu-1}B$ which is the μth Markov parameter of $G(s)$. This expression for Markov parameter is given in Hassiotis (2000).

Thus, combining (6) and (1), it can be concluded that if the signs of first non-zero Markov parameter and $[G(0) - y(0)]$ are opposite, then $y(t)$ has initial undershoot.

Corollary: For an asymptotically stable CTLTI system having a strictly proper transfer function $G(s)$, if the product of the leading and trailing coefficients of $\mathcal{N}(s)$ ($\mathcal{N}(s)$ is the numerator of $G(s)$) is negative, then initial undershoot is present in its step response $y(t)$.

Proof: Expanding (3) in negative powers of s as follows:

$$G(s) = D + CBs^{-1} + CABs^{-2} + \ldots + CA^{\mu-1}B + \ldots \tag{7}$$

Taking limiting values of the following, w.r.t. $s \to \infty$,

$$\lim_{s\to\infty} G(s) = D$$
$$\lim_{s\to\infty} s\{G(s) - D\} = CB$$
$$\lim_{s\to\infty} [s^2\{G(s) - D\} - s(CB)] = CAB$$
$$\vdots$$
$$\lim_{s\to\infty} \left[s^\mu\{G(s) - D\} - \sum_{j=1}^{\mu-1} s^{\mu-j} CA^{\mu-j-1}B \right] = CA^{\mu-1}B \tag{8}$$

Writing $G(\infty) = D$ in (8), the value of μth Markov parameter (if $G(s)$ is represented in transfer function form) can be obtained from the following equation,

$$M_\mu = \lim_{s\to\infty} \left[s^\mu\{G(s) - G(\infty)\} - \sum_{j=1}^{\mu-1} s^{\mu-j} M_{\mu-j} \right] \tag{9}$$

Let $G(s)$ is represented as,

$$\begin{aligned} G(s) &= \frac{\mathcal{N}(s)}{\mathcal{D}(s)} \\ &= \frac{\alpha_0 s^m + \alpha_1 s^{m-1} + \ldots + \alpha_m}{\beta_0 s^n + \beta_1 s^{n-1} + \ldots + \beta_n} \; (m < n) \end{aligned} \tag{10}$$

where $\alpha = \{\alpha_0, \alpha_1, \ldots, \alpha_m\}$ and $\beta = \{\beta_0, \beta_1, \ldots, \beta_n\}$ are sets of real numbers. From (10), rank of the system is $\mu = n - m$. Hence the first non-zero Markov parameter is:

$$\begin{aligned} M_\mu &= \lim_{s\to\infty} [s^\mu\{G(s) - G(\infty)\}] \\ &= \frac{\alpha_0}{\beta_0} \end{aligned} \tag{11}$$

Value of $G(0)$ is α_m/β_n and $y(0)$ is zero, since $G(s)$ is strictly proper. As it is also asymptotically stable, both b_0 and b_n are positive. Hence, for the following equation to be negative,

$$M_\mu\{G(0) - y(0)\} = \frac{\alpha_0 \alpha_m}{\beta_0 \beta_n} \tag{12}$$

either of the two, α_0 and α_m, has to have opposite sign to that of the other.

The percentage of initial undershoot (T_μ) in the step response of a CTLTI NMP system is defined as the maximum deviation of $y(t)$ with respect to $y(0)$, in the opposite direction of $G(0)$, taken as percentage of $y(\infty)$, that is,

$$T_\mu = \frac{y_\mu - y(0)}{y(\infty)} \times 100\%$$

where y_μ is the maximum value achieved by $y(t)$ in the direction opposite to that of $y(\infty)$. The relation between the percentage of initial undershoot and the Markov parameters of the system is given in the following theorem. *Theorem 4:* If the step response of a CTLTI NMP system $G(s)$, having rank μ, has initial undershoot, then the percentage of initial undershoot(T_u) is related to the first and second non-zero Markov parameters of the system as follows:

$$T_u \propto \left| \frac{M_\mu}{G(0) M_{\mu+1}} \right| \tag{13}$$

Proof: From (6), value of μth derivative of $y(t)$ at $t = 0$ is same as the μth Markov parameter of the system.

Table 1. Identification of curve shape by derivatives

First Derivative	Second Derivative	Shape
Positive	Positive	Increasing and concave up
Positive	Negative	Increasing and concave down
Negative	Positive	Decreasing and concave up
Negative	Negative	Decreasing and concave down

Table 1 lists the indications about the shape of a smooth curve at a particular point, based on the signs of its first and second derivatives at that point. Hence, as first and second non-zero derivatives at $t = 0$ of step response of a system are same as the first and second Markov parameters, using the concept of curve tracing using derivatives, it can be anticipated that the percentage of initial undershoot can be related to the ratio between the first and second non-zero Markov parameters.

3. ANALYSIS OF HYDRO-TURBINE BASED LOAD FREQUENCY CONTROL

From Hammid et al. (2016), the general block diagram of Automatic Load Frequency Control (ALFC) of a power plant is as follows:

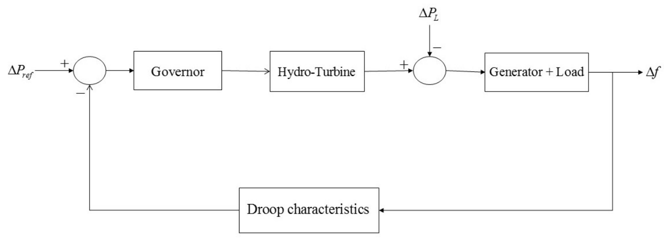

Fig. 1. Block Diagram of ALFC

The transfer function between the change in frequency($\triangle f$) and change in water valve opening $\triangle P_{ref}$, is NMP in nature (neglecting droop characteristic), which is given in the following equation:

$$G_{hyd}(s) = G_g(s) G_t(s) G_p(s) \tag{14}$$

where $G_g(s), G_t(s)$ and $G_p(s)$ are the transfer functions of the governor, hydro-turbine and the generator and load taken together, respectively. These are defined as follows:

$$G_g = \frac{1}{T_g s + 1};\; G_t = \frac{1 - T_w s}{1 + 0.5 T_w s};\; G_p = \frac{K_p}{T_p s + 1}$$

The individual systems $G_g(s)$,$G_h(s)$ and $G_p(s)$, along with the droop characteristics R are interconnected as shown in Fig. 2.

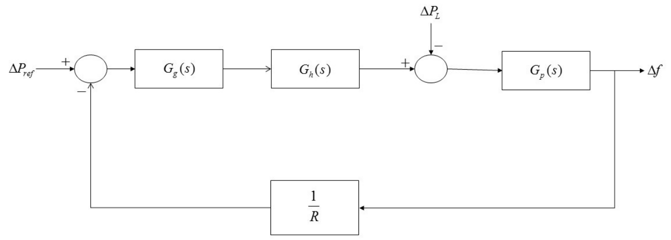

Fig. 2. Generalized plant model for ALFC

3.1 Open-loop analysis

For plant model without droop characteristics, substituting $K_p = 1, T_p = 6, T_w = 4, T_g = 0.2$, as used by Tan (2010), in (14), the resultant transfer function is as follows:

$$G_{hyd}(s) = \frac{1-4s}{7.2s^3 + 16.8s^2 + 8.6s + 1} \tag{15}$$

Analyzing the above transfer function using the theorems, the following results are obtained regarding the response of $\triangle f$ for unit step change in water valve opening:

- $G_{hyd}(s)$ has only one positive zero at $s = 0.25$, hence at least one zero-crossing will be present in $\triangle f(t)$.
- $G_{hyd}(s) - G_{hyd}(0)$
$$= \frac{-s(s^2 + 2.33s + 1.75)}{(s+1.667)(s+0.5)(s+0.1667)} \tag{16}$$
Neither the above equation has any positive zero, nor $G_{hyd}(s)$ has any complex poles, implying absence of overshoot.
- Since the first and last terms of the numerator of $G_{hyd}(s)$ has opposite signs, hence the step response will have initial undershoot.

The constant T_w, as given in Kundur et al. (1994), is known as 'water starting time', and plays an important role in the transient characteristics of the turbine. Considering two different values of $T_w = 2$ s and $T_w = 4$ s in the following two cases:

$$G_{hyd1}(s) = \frac{(1-4s)}{(1+0.2s)(1+2s)(1+6s)}$$
$$M_{2_1} = -1.667,\ M_{3_1} = 9.8611$$
$$r_1 = |M_{2_1}/M_{3_1}| = 0.1690$$
$$G_{hyd2}(s) = \frac{(1-2s)}{(1+0.2s)(1+2s)(1+6s)}$$
$$M_{2_2} = -1.667,\ M_{3_2} = 11.1111$$
$$r_2 = |M_{2_2}/M_{3_2}| = 0.1500$$

Therefore, from above analysis, since $r_2 < r_1$, percentage of initial undershoot is lesser with $T_w = 2$ s than with $T_w = 4$ s. Thus, percentage of initial undershoot in $\triangle f(t)$ decreases with decrease in water starting time. To get a glimpse of how much change takes place due to change in parameters, the values of r_1 and r_2 can be looked upon. However, in this problem, r_1 and r_2 are close enough, as M_{2_1} and M_{2_2} are same. So, instead of the ratios, if the focus is drawn towards the third Markov parameter's value, then it can be seen that M_{3_1} is significantly lesser than M_{3_2} as compared to r_1 greater than r_2, which reveals how much the change in initial undershoot can take place by changing T_w from 4 seconds to 2 seconds.

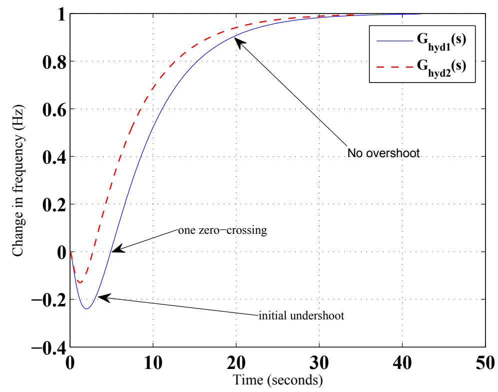

Fig. 3. Step response of change in frequency

In Fig. 3, the response with solid line correspond to the step response of $G_{hyd1}(s)$ and the other of $G_{hyd2}(s)$. Looking at the response of $G_{hyd1}(s)$, it is clear that the response has one zero-crossing, initial undershoot and no overshoot. Moreover, the percentage of initial undershoot is more in the response of $G_{hyd1}(s)$ than that of $G_{hyd2}(s)$, thus confirming all the transfer function based analyses.

3.2 Closed-loop analysis

Using a classical PI controller with the hydro-turbine based hydro-electric power plant for the LFC caused by small change in water valve opening, the resultant closed-loop system is as follows:

$$G_{cl}(s) = \frac{G_c(s)G_{hyd}(s)}{s + G_c(s)G_{hyd}(s)} \tag{17}$$

where $G_c(s) = (K_p s + K_i)/s$ and $G_{hyd}(s)$ is given in (14). Substituting $G_{hyd}(s)$ with (15), the following equation is obtained:

$$G_{cl}(s) = \frac{(K_p s + K_i)(1-4s)}{\Delta(s)} \tag{18}$$

where $\Delta(s) = 7.2s^4 + 16.8s^3 + (8.6 - 4K_p)s^2 + (1 - 4K_i)s + K_i$.

Taking the controller parameter values to be $(K_p, K_i) = (0.725, 0.12)$ in (18), the resultant system transfer function is given below:

$$G_{cl}(s) = \frac{1.2083(0.25-s)(s+0.1685)}{(s+5.266)(s+0.1642)(s^2+0.2368s+0.05783)} \tag{19}$$

Analyzing (19) with the discussed theorems as follows:

(1) Equation (19) has one positive real zero at $s = 0.25$, so its step response will have at least one zero-crossing.

(2)
$$G_{cl}(s) - G_{cl}(0) = \frac{-s(s+5)}{(s+5.266)(s+0.1642)} \times \frac{(s+0.5)(s+0.1667)}{(s^2+0.2368s+0.05783)} \tag{20}$$
In the above equation, no positive real zero lies on the RHP of s-plane. However, $G_{cl}(s)$ is under-damped. Hence its step response will experience overshoot but not due to zeros.

(3) The first non-zero Markov parameter of (19) is $-0.56K_p$ which is negative for $K_p > 0$, whereas $G_{cl}(0) = 1$, which is positive. Hence step response of $G_{cl}(s)$ will experience initial undershoot.

To compare the effect imposed by different K_p and K_i values on initial undershoot, the following analysis is done with the help of Theorem 4.

- Only K_p changed:
  - $K_p = 0.725, K_i = 0.12$
    $M_2 = -1.2083, M_3 = 6.9493$
    $r_1 = 0.0365$
  - $K_p = 0.4, K_i = 0.12$
    $M_2 = -0.6667, M_3 = 3.7444$
    $r_2 = 0.0211$

  Since r_2 is lesser than r_1, hence according to Theorem 4, the percentage of initial undershoot can be reduced by decreasing the value of K_p. In Fig. 4, the step responses for two different K_p values are plotted. The figure clearly implies that the percentage of initial undershoot is reduced in case of lowered K_p, while keeping K_i fixed. Other time-domain effects brought in by doing this can be observed in the increment in both peak overshoot and peak time, thus indicating slowdown of the process.

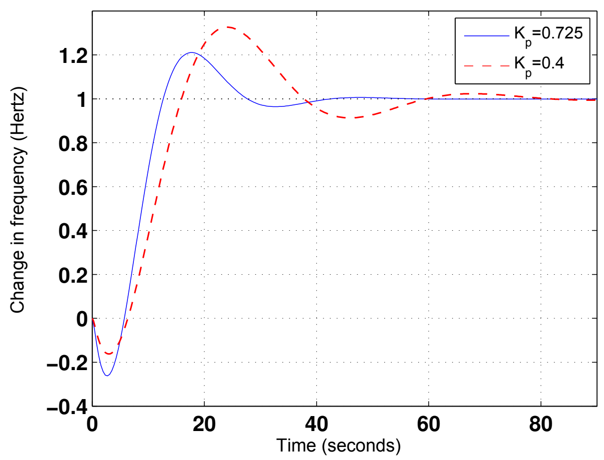

Fig. 4. Effect of changing K_p only

- Only K_i is changed: Keeping K_p fixed and increasing K_i, the comparison is as follows:
  - $K_p = 0.725, K_i = 0.12$
    $M_2 = -1.2083, M_3 = 6.9493$
    $r_1 = 0.0365$
  - $K_p = 0.725, K_i = 0.2$
    $M_2 = -1.2083, M_3 = 6.8160$
    $r_3 = 0.0380$

  Thus from this analysis, it can be said that increase in K_i (while K_p remains unchanged) increases the percentage of initial undershoot, which is undesirable. Fig. 5 depicts the step response curves pertaining to case 2, that is, with K_i varied only. Besides increasing the percentage of initial undershoot, increasing K_i also made the system more under-damped, and thus resulting in a more sluggish system response.

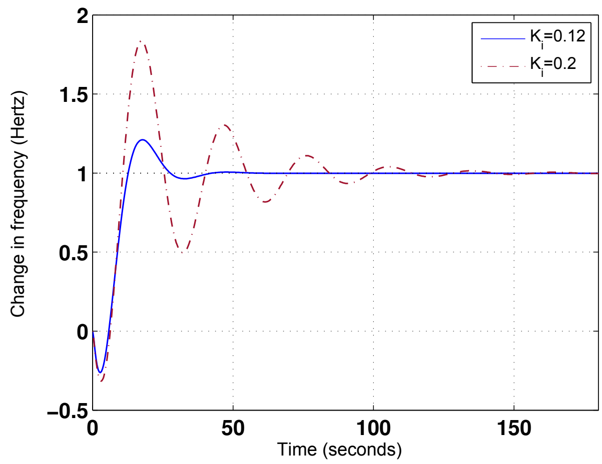

Fig. 5. Effect of changing K_i only

To summarize the above discussion, it can be inferred that, while tuning the PI controller, if K_p is lowered and K_i as well be decreased, then the percentage of initial undershoot can be reduced. It should be kept in mind that it is not important how the values of K_p and K_i are obtained. Any method meant for PI controller design, be it classical or modern, can be used to determine the parameter values. The proposed theorem can then be used to assess the effect of changing the PI controller values (to improve other time-domain characteristics) on initial undershoot.

The effects on initial undershoot and other time-domain characteristics with the above three (K_p, K_i) pairs can be tallied from Table 2.

Table 2. Information about time-domain characteristics

K_p, K_i	$(0.725, 0.12)$	$(0.4, 0.12)$	$(0.725, 0.2)$
Rise Time	$5.3027s$	$7.3592s$	$3.5296s$
Peak Time	$17.8908s$	$23.9840s$	$17.4711s$
Overshoot	21.0484%	32.6012%	83.6377%
Undershoot	26.2232%	16.2288%	31.5683%
Settling Time	$36.5077s$	$67.8409s$	$109.6069s$

4. CONCLUSION

In this paper, study on identification of certain time-domain characteristics such as zero-crossings, overshoot (due to zeros) and initial undershoot, caused by the presence of RHP zeros in step response of NMP systems, is done. This involves brief mathematical analysis and theorems of identification. Among these theorems, two are reported earlier, while the theorems for initial undershoot is proposed in this paper. The theorems for initial undershoot include both identification of its presence as well as comparison of the same in two or more different NMP systems' step responses, to aid the control engineers while designing suitable controllers for these systems. Application of these theorems is done on a practical NMP system problem, which is the control of variations in load frequency due to small changes in water valve opening of a hydro-turbine based hydro-electric plant. Identification of these characteristics in open-loop system is done as well as effect of different water storing time of the hydro-turbine on percentage of initial undershoot is done using the theorems. Similar analysis is done with the closed-loop model, where a classical PI controller is employed for the load frequency control. Application of the theorem pertaining to estimation of percentage of initial undershoot is done to observe the effect of changing PI controller parameters on initial undershoot, so as to provide directions while carrying out the tuning process.

REFERENCES

Åström, K.J. and Bell, R.D. (2000). Drum-boiler dynamics. *Automatica*, 36(3), 363–378.

Boekfah, A. (2017). Nonlinear output-transition control for nonminimum-phase systems: The vtol example. *2017 2nd International Conference on Control and Robotics Engineering (ICCRE)*, 97–101. doi: 10.1109/ICCRE.2017.7935050.

Damm, T. and Muhirwa, L.N. (2014). Zero crossings, overshoot and initial undershoot in the step and impulse responses of linear systems. *IEEE Transactions on Automatic Control*, 59(7), 1925–1929. doi: 10.1109/TAC.2013.2294616.

Dorf, R. and Bishop, R. (2014). *Modern Control Systems*. Pearson Prentice Hall.

Hammid, A.T., Hojabri, M., Sulaiman, M.H., Abdalla, A.N., and Kadhim, A.A. (2016). Load frequency control for hydropower plants using pid controller. *Journal of Telecommunication, Electronic and Computer Engineering (JTEC)*, 8(10), 47–51.

Hassiotis, S. (2000). Identification of damage using natural frequencies and markov parameters. *Computers & Structures*, 74(3), 365 – 373. doi: https://doi.org/10.1016/S0045-7949(99)00034-6.

Hegde, S. and Izadian, A. (2013). A new sepic inverter: Small signal modeling. *IECON 2013 - 39th Annual Conference of the IEEE Industrial Electronics Society*, 240–245. doi:10.1109/IECON.2013.6699142.

Hoagg, J.B. and Bernstein, D.S. (2007). Nonminimum-phase zeros - much to do about nothing - classical control - revisited part ii. *IEEE Control Systems*, 27(3), 45–57. doi:10.1109/MCS.2007.365003.

Hoagg, J., Chandrasekar, J., and Bernstein, D. (2007). On the zeros, initial undershoot, and relative degree of collinear lumped-parameter structures. *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME*, 129(4), 493–502. doi: 10.1115/1.2719764.

Kundur, P., Balu, N.J., and Lauby, M.G. (1994). *Power system stability and control*, volume 7. McGraw-hill New York.

Qiu, L. and Davison, E.J. (1993). Performance limitations of non-minimum phase systems in the servomechanism problem. *Automatica*, 29(2), 337–349.

Su, Z. and Khorasani, K. (2001). A neural-network-based controller for a single-link flexible manipulator using the inverse dynamics approach. *IEEE Transactions on Industrial Electronics*, 48(6), 1074–1086. doi: 10.1109/41.969386.

Tan, W. (2010). Unified tuning of pid load frequency controller for power systems via imc. *IEEE Transactions on power systems*, 25(1), 341–350.