On the equivalence between PD+DOB and PID controllers applied to servo drives

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Abstract: This work shows that a Proportional Derivative controller with weighted Derivative action plus a Disturbance Observer, is equivalent to a Proportional Integral Derivative (PID) controller with weighted Proportional and Derivative actions, when they are applied to servo drives. A byproduct of this equivalence is a tuning rule for the PID controller, called the DOB tuning, which is expressed in terms of the cutoff frequency of the filter employed in the DOB. Experiments in a laboratory testbed allow assessing the performance of a PID controller under the resulting tuning formulae.

Keywords: Disturbance observer, PD control, PID control, controller tuning, servo drive.

1. INTRODUCTION

The Proportional Integral Derivative (PID) control law is without doubt the most employed algorithm for regulating industrial processes Åström and Hägglund (1995), Visioli (2006). It is also employed for controlling servo drives Ellis (2012), Gao et al. (2013), robot manipulators Spong and Vidyasagar (1989), Rocco (1996), and quadrotors Pounds et al. (2012).

On the other hand, the Disturbance Observer (DOB) Ohishi et al. (1988); Ohnishi et al. (1996) relies on input and output measurements and a nominal model of a perturbed plant to estimate the disturbances (see Fig.1). Subsequently, the disturbance estimate is employed to construct an inner feedback controller to counteract the effect of the disturbances. An outer loop controller is then designed based on the nominal model of the plant.

Tuning of PID and DOB-based controllers is a key issue for its practical implementation. The tuning of the PID controllers has been the matter in several works and textbooks Kelly (1995), Skogestad (2001), O'Dwyer (2009), and the Ziegler-Nichols and the Cohen-Coon methods are among the most popular. In the case of DOB-based controllers, their tuning has been accomplished by means of H_∞ techniques Zheng et al. (2017), through the use of binomial filters Lee and Tomizuka (1996), or by solving a mixed sensitivity optimization problem Kim and Chung (2003).

In its worth noting that there exist few works dealing with the relationship between the PID and DOB-based controllers. This issue deserves study because they are linear in nature and incorporate a mechanism for counteracting disturbances. The integral action in a PID controller compensates for constant disturbances, and it has been considered as an implicit disturbance observer Johnson (2008). Besides, the disturbance estimate provided by a DOB-based controller counteracts constant and time-varying disturbances. Interestingly enough, an early work on the DOB Yamada et al. (1997) mentions the equivalence between PID-like and DOB-based controllers for a general class of linear plants. Nevertheless, the authors did not pursue further research on this issue.

The aim of this work is to show that a Proportional Derivative controller with weighted Derivative action plus a Disturbance Observer (PD+DOB), is equivalent to a weighted PID controller. This equivalence assumes that the PD+DOB is designed for a servo drive, and the DOB is fed by velocity measurements. The equivalence provides a tuning rule for the weighted PID controller called the DOB tuning, expressed in terms of the cutoff frequency of the filter used for building the DOB.

The outline of this exposition is as follows. After introducing the basic idea of disturbance observes, the work describes how a PD+DOB controller is applied to a servo drive. Subsequently, the equivalence between the PD+DOB and a PID controller is established. Real-time experiments on a laboratory prototype allow assessing the performance of the servo drive when controlled by the PID controller under the DOB tuning.

2. PRELIMINAIRES ON DISTURBANCE OBSERVERS

Disturbance Observers are employed for rejecting internal and external disturbances acting on a plant. Fig. 1 depicts a block diagram where a Disturbance Observer (DOB) is applied for compensating disturbances in a linear plant. From to this figure, the output of the plant is given by

$$y = P(s)(u + d) \tag{1}$$

The basic idea behind a DOB is to use measurements of the plant input and output to reconstruct the disturbance. The above is accomplished by computing d from (1) and assuming that the plant is minimum phase

$$d = P^{-1}(s)y - u \tag{2}$$

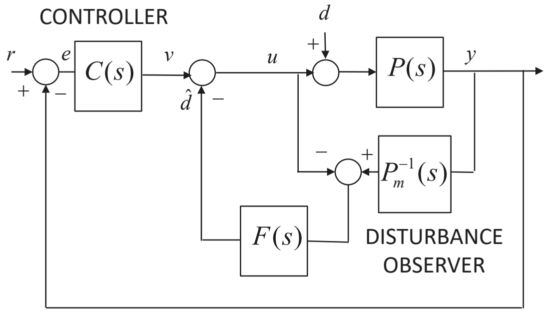

Fig. 1. Disturbance Observer block diagram.

According to this equation the disturbance d may be interpreted as an input-output error modeling term. A problem with this estimation scheme is that the plant inverse $P^{-1}(s)$ is not proper and would require measurements of the time derivatives of the output y that may be not available in practice. On the other hand, in many practical applications only a nominal plant $P_m(s)$ is known *a priori* . In order to circumvent the above problems the disturbance estimation is performed as follows

$$\hat{d} = \left[P_m^{-1}(s)y - u\right] F(s) \tag{3}$$

The transfer function $F(s)$ is a strictly proper stable filter with characteristic polynomial of degree $n_f \geq n$ where n corresponds to the degree of the characteristic polynomial of $P_m(s)$. The above condition guarantees a proper or strictly proper transfer function $P_m^{-1}(s)F(s)$. It is also worth noting that this estimation scheme assumes that the bandwidth of the filter is equal or greater that the bandwidth of the disturbance d. Note also that the disturbance estimate $\hat{d}$ is injected to the plant input to counteract the effects of the real disturbance d.

Assuming an ideal disturbance compensation, i.e. when the effects of d are completely compensated, a controller is designed in order to obtain a closed-loop system fulfilling some design criteria.

3. DISTURBANCE OBSERVER APPLIED TO A SERVO DRIVE

Consider a servo drive composed of a servomotor, an angular position sensor, and a power amplifier working in current mode. A model of this system is

$$J\ddot{q}(t) + F(\dot{q}) = ku + \bar{d}_m \tag{4}$$

Variables q, $\dot{q}$ and $\ddot{q}$ are respectively the angular position, velocity and acceleration of the servo drive, u is the control input voltage, J the servomotor and load lumped inertia, $f(\dot{q})$ is a nonlinear friction term that may include viscous and Coulomb friction torques, k is a parameter related to the amplifier gain and to the motor torque constant, and the term $\bar{d}_m$ is an external disturbance.

Model (4) has the next alternative writing

$$\ddot{q}(t) = -f(\dot{q}) + bu + \bar{d} \tag{5}$$

where $b = k/J$, $f(\dot{q}) = F(\dot{q})/J$ and $\bar{d} = \bar{d}_m/J$. If the friction torques are unknown, then they are lumped with the disturbance $\bar{d}$. This remark allows writing (5) as follows

$$\ddot{q}(t) = bu + d \tag{6}$$

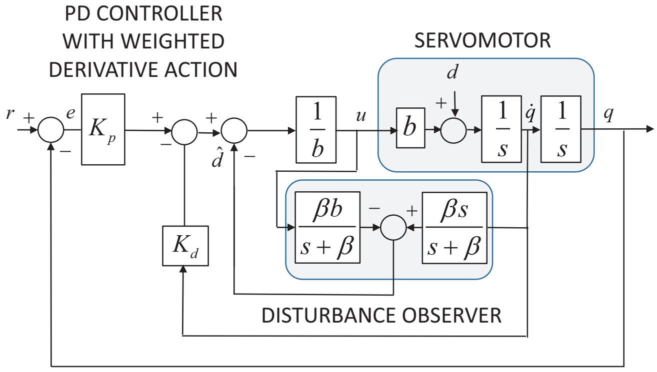

Fig. 2. PD+DOB controller applied to a servo drive.

and $d = \bar{d} - f(\dot{q})$. Assume that the Laplace transform for d exists, then it permits obtaining the Laplace transform of (6)

$$s^2Q(s) = bU(s) + D(s) \tag{7}$$

with $Q(s) = \mathcal{L}\ \{q\}$, $D(s) = \mathcal{L}\ \{d\}$ and $U(s) = \mathcal{L}\ \{u\}$. The notation $\mathcal{L}\ \{\bullet\}$ stands for the Laplace operator

Based on the original DOB, Fig. 2 depicts the block diagram of a Proportional with weighted Derivative action controller plus a Disturbance Observer (PD+DOB) applied to a servo drive. The DOB filter is defined as

$$F(s) = \frac{\beta}{s + \beta} \tag{8}$$

with cutoff frequency $\beta > 0$. The proposed PD+DOB controller relies on angular velocity measurements, and assumes knowledge of the servo drive input gain b.

4. EQUIVALENCE BETWEEN PD+DOB AND PID CONTROLLERS

Fig. 2 defines the PD+DOB control law

$$u = \frac{1}{b}\left[K_pe - K_d\dot{q} - \hat{d}\right] \tag{9}$$

The proportional and derivative gains K_p and K_d are positive constants, $e = r - q$, r is a constant desired reference, and $\hat{d}$ is an estimate of the disturbance d. According to Åström and Hägglund (1995) and Visioli (2006), a weighted derivative action uses an error e_d where a weight is imposed to the reference r, i.e. $e_d = \bar{c}r - y$ being $\bar{c} \geq 0$ a constant weight, and y the measured variable. In the case of the controller (9) $\bar{c}$ is set to zero in the Derivative action, which produces the well-known Proportional controller plus velocity feedback.

Applying the Laplace transform to (9) leads to

$$U(s) = \frac{1}{b}\left[K_pE(s) - K_dsQ(s) - \hat{D}(s)\right] \tag{10}$$

with $E(s) = \mathcal{L}\ \{e\}$, $E(s) = R(s) - Y(s)$, $R(s) = \mathcal{L}\ \{r\}$, $Q(s) = \mathcal{L}\ \{q\}$, and $\hat{D}(s) = \mathcal{L}\ \{\hat{d}\}$.

On the other hand, the next expression describes the dynamics of the DOB in Fig. 2

$$\dot{\hat{d}} = -\beta\hat{d} + \beta[\ddot{q} - bu] \tag{11}$$

Substituting (9) into (11) yields

$$\dot{\hat{d}} = \beta[\ddot{q} - K_pe + K_d\dot{q}] \tag{12}$$

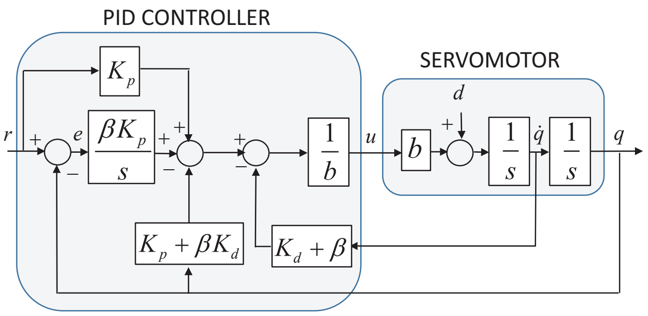

Fig. 3. Weighted PID controller applied to a servo drive.

whose Laplace transform is given by

$$\hat{D}(s) = \beta K_d Q(s) - \beta K_p \frac{1}{s} E(s) + \beta s Q(s) \tag{13}$$

Substituting (13) into (10) boils down to

$$\begin{aligned} U(s) = & \frac{1}{b}[K_p E(s) - \beta K_d Q(s) \\ & + \beta K_p \frac{1}{s} E(s) - (K_d + \beta) s Q(s)] \end{aligned} \tag{14}$$

Equation (14) corresponds to a weighted PID controller. Fig.3 depicts a block diagram of this controller, and its alternative writing in terms of weighted errors is

$$U(s) = \frac{1}{b}\left[\bar{K}_p E_p(s) + \bar{K}_i \frac{1}{s} E(s) + \bar{K}_d s E_d(s)\right] \tag{15}$$

where

$$E_p(s) = \bar{b} R(s) - Q(s) \tag{16}$$
$$E(s) = R(s) - Q(s) \tag{17}$$
$$E_d(s) = \bar{c} R(s) - Q(s) \tag{18}$$
$$\bar{b} = \frac{K_p}{K_p + \beta K_d} \tag{19}$$
$$\bar{c} = 0 \tag{20}$$

The terms $E_p(s) = \mathcal{L}\ \{e_p\}$ and $E_d(s) = \mathcal{L}\ \{e_d\}$ are respectively the weighted errors used in the Proportional and Derivative actions, and the weights are $\bar{b}$ and $\bar{c}$. Note also that the error used in the integral action is not weighted. On the other hand, the gains in (15) have the following expressions

$$\begin{aligned} \bar{K}_p &= K_p + \beta K_d \\ \bar{K}_i &= \beta K_p \\ \bar{K}_d &= K_d + \beta \end{aligned} \tag{21}$$

Therefore, the PD+DOB controller of Fig. 2 is equivalent to a standard PID controller with weighted Proportional and Derivative actions. This equivalence is true if velocity measurements are available. However, in practice velocity sensors are not available and then some kind of velocity estimation will be needed. This issue will produce some discrepancies between the responses of both controllers as it will be shown in the experiments. Note also that the tuning rules (21) of the weighted PID controller (15) depends on the gains of the PD controller and on the cutoff frequency of the filter used to construct the DOB. In the sequel, this tuning will be called the DOB tuning.

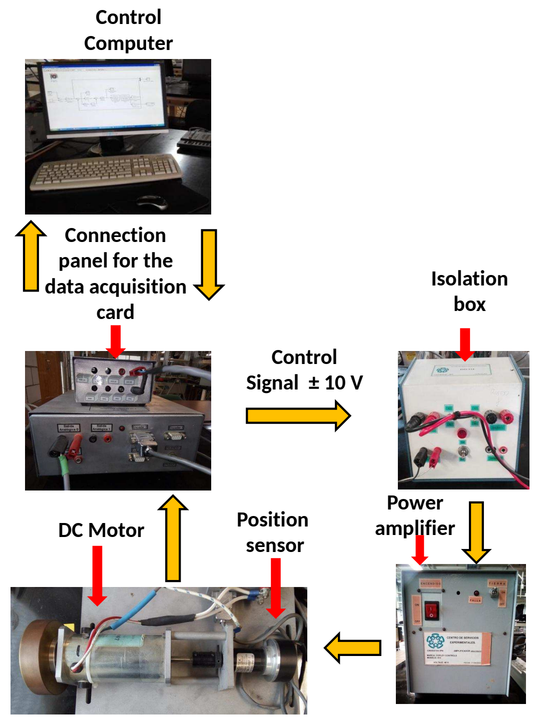

Fig. 4. Experimental setup.

5. THE WEIGHTED PID CONTROLLER UNDER THE DOB TUNING.

The next equation corresponds to the transfer function of the plant (7) in closed loop with the controller (14) without considering disturbances

$$G(s) = \frac{Q(s)}{U(s)} = \frac{N(s)}{D(s)} \tag{22}$$

The polynomials in (22) are defined as

$$\begin{aligned} D(s) &= s^3 + (K_d + \beta)s^2 + (K_p + \beta K_d)s + \beta K_p \\ &= (s + \beta)(s^2 + K_d s + K_p) \end{aligned} \tag{23}$$

and

$$\begin{aligned} N(s) &= K_p s + \beta K_p \\ &= K_p(s + \beta) \end{aligned} \tag{24}$$

The fact that K_p, K_d and β are positive constants guarantees that the poles of $G(s)$ are stable when applying the DOB tuning to the weighted PID controller. The cutoff frequency β defines one pole of the closed loop system, and the proportional and derivative gains K_p and K_d sets the other two poles independently of β. Hence, the desired transient response profile of the closed loop system, i.e. if the response is underdamped, critically damped or overdamped, is solely determined by K_p and K_d. On the other hand, the numerator and denominator of $G(s)$ have a common root at $s = -\beta$, therefore, a minimum representation of $G(s)$ does not contain this root.

Table 1. Experimental results for the weighted PID and PD+DOB controllers.

Experiment	K_p	K_d	β	ISE
1 PID	400	80	20	101.7836
2 PD+DOB	400	80	20	96.7817

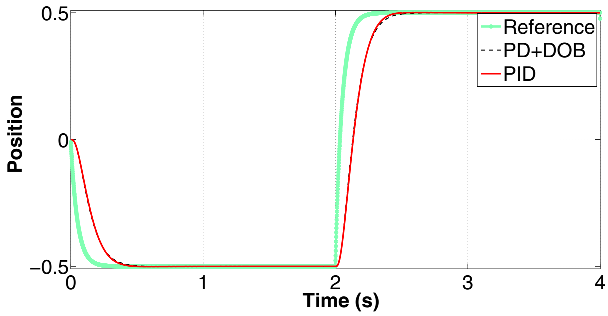

Fig. 5. Responses of the weighted PID and PD+DOB controllers.

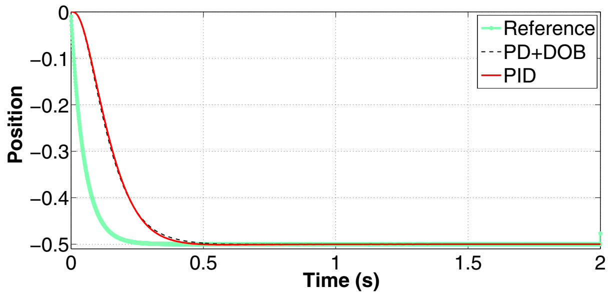

Fig. 6. Closer look to the responses of the weighted PID and PD+DOB controllers.

6. EXPERIMENTS

6.1 Experimental setup

Fig. 4 depicts the setup used in the experiments. It consists of a brushed DC motor JDTH-2250-DQ-1C from Clifton Precision driving a brass disk, a Servotek optical encoder SA-7388-1 that measures the servomotor angular position, a power amplifier Copley Controls 413 working in current mode, and a box that galvanically isolates a data acquisition card from the power amplifier . A Servotogo card mounted inside a personal computer with a Intel Core 2 quad processor is used for performing data acquisition. All the programming is done using The Mathworks Matlab/Simulink software together with the real-time software WINCON from Quanser Consulting. The Simulink diagrams use a sampling period of 1 ms and the Euler-ode1 integration method. The value of the input gain in model (6) is $b = 51.49$. The servomotor angular velocity $\dot{q}$ is estimated from position measurements through the next filter

$$G_v(s) = \frac{300s}{s+300}\frac{400}{s+400} \tag{25}$$

6.2 Comparative study using the weighted PID and PD+DOB controllers

The following experiments show the performance of the PD+DOb and the PID controllers applied to a servo drive. The PID controller is tuned according to the DOB tuning (21). The reference is a filtered step of 0.5 servomotor shaft revolutions. In order to evaluate large position errors and excessive oscillatory responses, the performance is measured using the Integral Squared Error (ISE) index

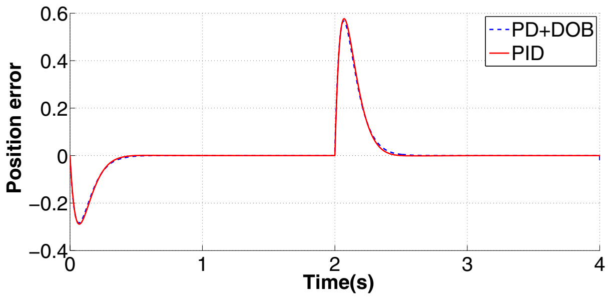

Fig. 7. Error signals for the weighted PID and PD+DOB controllers.

$$ISE = \int_0^T 100\,[e(t)]^2\,dt \tag{26}$$

which is evaluated at $T = 2$s. Table 1 resumes the outcomes of this experiments. Note that the PD+DOB controller produces a slightly smaller value of the ISE performance index. This result agrees with the step responses in Fig.5 and 6, and the position error graphs in Fig.7 , which show that the PD+DOB controller produces a faster response than the one provided by the PID controller. Interestingly enough, numerical simulations not reported here for the sake of space, do not exhibit differences in the response of these controllers when they are simulated without velocity estimators. Therefore, the dynamics of the filter (25) used to estimate the servomotor angular velocity seems to affect in a different way the behavior of both controllers. It is worth remarking that in the PD+DOB controller the velocity estimate produced by the filter is used to compute the disturbance estimate whereas in the case of the PID controller the velocity estimate only feeds the Derivative action. It is reasonable to assume that other kinds of velocity estimators like Luenberger observers and finite differences-based estimators would also produce different outcomes in these controllers. Fig.8 shows the signals for both controllers. They are almost the same but the peak in the PD+DOB controller signal is slightly higher than the one observed in the PID controller signal. This outcome also agrees with the quicker response observed with the PD+DOB controller, i.e. it produces more torque than the PID controller, which translates into a faster response.

6.3 Experimental results with the weighted PID controller using different values of the parameter β

The PID controller using the DOB tuning is evaluated with respect to different values of the parameter β. In addition to the ISE index (26), performance is assessed through the Integral of the Absolute value of the Control (IAC) and the Integral of the Absolute value of the Control Variation (IACV) indexes defined as

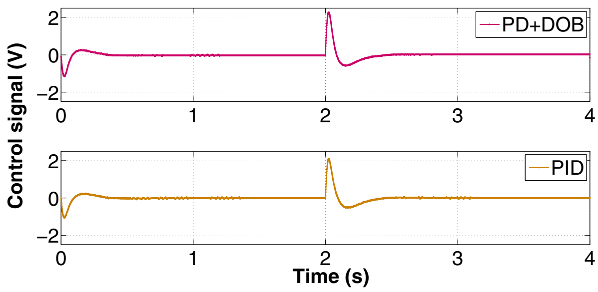

Fig. 8. Control signals for the weighted PID and PD+DOB controllers.

Table 2. Experimental Results for the weighted PID controller using the DOB tuning.

Experiment	K_p	K_d	β	ISE	IAVC	IAC
1	400	80	0	100.9910	6.2421	0.1566
2	400	80	10	101.632	8.5889	0.1185
3	400	80	20	101.80848	9.1970	0.1354
4	400	80	30	101.9634	9.8852	0.140

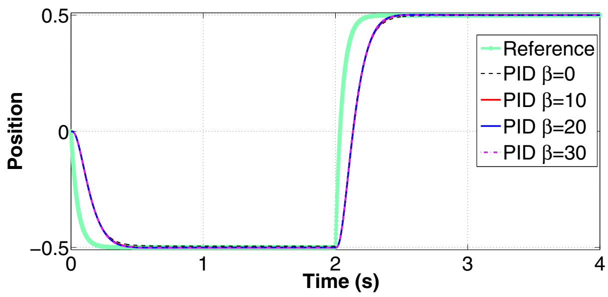

Fig. 9. Step response for the weighted PID controller under the DOB tuning.

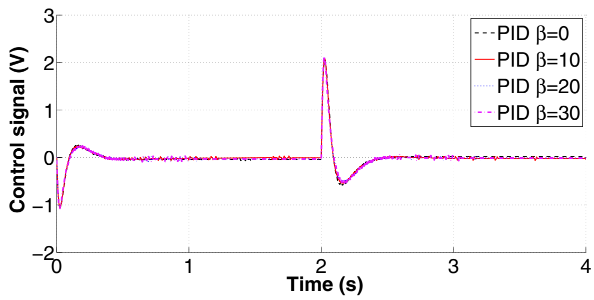

Fig. 10. Control signals for the weighted PID controller under the DOB tuning.

$$IACV = \int_0^T |u(t)|\, dt \tag{27}$$

$$IACV = \int_0^T \left|\frac{du(t)}{dt}\right| dt \tag{28}$$

where $T = 2$s. The filtered step response employed in the first experiment is also applied here. Table 2 displays the values of the performance indexes for several values of β maintaining the same values of the proportional and derivative gains K_p and K_d. The values of the ISE index remain almost constant for all the tested values of β. On the other hand, when $\beta = 0$, i.e., without integral action the IAC index is higher than in the case of non-zero values of β. If β increases so do the IAC and IACV indexes. Fig.9 and 10 depict the responses of the servo drive as well as the respective control signals. No significant change in response is observed when β is increased. These outcomes show that low values of β, which translates into low values of the PID controller gains, produce reasonable performance.

7. CONCLUSIONS

This work shows preliminary results on a tuning rule called de DOB tuning, for weighted PID controllers applied to servo drives. This rule is obtained by showing the equivalence between a PD controller endowed with a disturbance observer (DOB), called the PD+DOB controller, and a weighted PID. This equivalence is true under the assumption that velocity measurements are available. The experiments show the following results. Both controllers produce smooth responses without overshoot and display essentially the same performance in terms of the Integral Squared Error (ISE) index, and slight discrepancies exist due to the effect of the filter used to estimate the servomotor angular velocity. Moreover, increasing the value of the β term used in the DOB tuning, which corresponds to the cutoff frequency of the DOB in the PD+DOB controller, does not significantly improves closed-loop performance. Therefore, large gains in the weighted PID controller are not necessary to obtain reasonable performance. Future work includes using the weighted PID controller under the DOB tuning when the servomotor is affected by more complex disturbances. The effect on closed-loop performance of other velocity estimators would be worth studying.

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