Fractional Order PID-type Feedback in Fixed Point Transformation-based Adaptive Control of the FitzHugh-Nagumo Neuron Model with Time-delay ⋆

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Abstract: The operation of the nervous system, consequently the various dynamic neuron models, show strong nonlinearities. Their control, that may result in the treatment of various diseases, have to cope with the essential difficulties as the great deviations/uncertainties in the parameters of the available models, and the time-delay related to the observations of the measurable quantities, the computation, and the exertion of the control signal. For tackling model uncertainties a novel, fixed point transformation (FPT)-based adaptive control approach was suggested that generally works by the use of fresh observations on the behavior of the controlled system, therefore its operation may be degraded by time-delay effects. Furthermore, in the practice time-delay effects can be reduced by using model-based extrapolation of the motion of the controlled system for the "dead period" spanned between the observation and the actual appearance of the control action. In the lack of reliable dynamic model such an extrapolation may be questionable. In this research, up to or knowledge, at first time, time-delay effects are studied in the FPT-based adaptive control of the FitzHugh-Nagumo Neuron Model using novel fractional order kinematic feedback terms. This neuron is a relevant paradigm because showing very sharp nonlinearities in its dynamics. It is concluded that the use of an approximation-based extrapolation in a control of this special fractional order PID-type feedback can considerably reduce the consequences of the time-delay problems.

Keywords: Adaptive control, Delay compensation, Fixed point transformation-based adaptive control, Fractional order feedback, Uncertainty, Motion extrapolation.

1. INTRODUCTION

Neurons as "elementary building blocks" of the nervous system produce sharp "spiking" activities, therefore their dynamic models developed for the use in modern life sciences have strongly nonlinear components (e.g. Brodal (2010)). Various modeling efforts can be tracked from the early, relatively simple *"Integrate and Fire Neuron"* by Lapicque (1907) to the very complex *"Hodgkin-Huxley Model"* (see Hodgkin and Huxley (1952)) to describe non-linearities. However, in control technology the "precision" is not the only appreciable virtue of a model. Other properties as simplicity and easy applicability are also important features. This fact lead to the appearance of mathematically simplified models. On this reason FitzHugh (1961) developed the *"Bonhoeffer – van der Pol oscillator"* that is a special version of the original one published by van der Pol (1927). Nagumo et al. (1962) developed an equivalent circuit of this oscillator for use in "experimental" investigations. On similar reason Matsumoto (1984) invented a realization of the Chua circuit. These simplified circuits are appropriate to produce the chaotic phenomena the

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significance of which in the nervous system became clear in the nineties of the past century (e.g. Glass (1995), Rabinovich and Abarbanel (1998), Feudel et al. (2000), Guckenheimer and Oliva (2002), Zhou and Kurths (2003)). Controlling chaotic phenomena naturally obtained great attention e.g. as the synchronization of coupled neurons (Wang et al. (2004)).

The development of the currently prevailing methodology to design adaptive controllers commenced in the early nineties by Slotine and Li (1991). These methods are based on the PhD dissertation by Lyapunov (1892) that was written in Russian, and only later became available for the western world due to its English translation (Lyapunov (1966)).

On the basis of certain critical observations made on the complexity of the Lyapunov function-based design, and on the fact that it requires "too much" because working with *"satisfactory conditions"* instead of *"necessary and satisfactory conditions"* by Tar et al. (2010), elaboration of its alternative was initiated (Tar et al. (2009)) that at first transforms the control task into a fixed point problem then solves it via iteration. The historical root of this problem solving approach arose in the 17th century (Ypma (1995)), and its application area was extended to quite abstract sets (Banach (1922)). For the "problem re-formalization" novel Fixed Point Transformations (FPT) were developed by Dineva et al. (2015). Since this approach, in contrast to the global stability of the Lyapunov function-based design, converges only within a bounded basin, its behavior in the divergent region as well as its stabilization possibilities were extensively investigated (e.g. Kósi et al. (2012b), Kósi et al. (2012a), Várkonyi et al. (2012), Kósi et al. (2013)). For its possible application in neurology the adaptive control of the Chua-Matsumoto circuit was studied by Rudas et al. (2011), and that of the Hodgkin–Huxley neuron was investigated by Bitó and Tar (2015), too. Regarding other fields of life sciences the use of this design method was considered in the adaptive control for treating type I diabetes mellitus (e.g. Eigner et al. (2015)), and in anaesthesia control (e.g. Csanádi and Tar (2016)).

In these early investigations the problem of time-delay effects was not considered as a central point. Only the initial steps were done in controlling Classical Mechanical systems (e.g. Redjimi and Tar (2018)). The aim of the present paper is to report the first preliminary result in life sciences by considering the control of the FitzHugh – Nagumo neuron model with a novel fractional order feedback.

2. ON THE FRACTIONAL ORDER PID-TYPE FEEDBACK SUGGESTED

Historically the "systematic PID control" was invented in the forties of the past century, and it was applied for the control of ships (e.g. Bennett (1993)). Based on the simple property of the differential equation $\dot{x}(t) = -\Lambda x(t)$, i.e. that for a *constant parameter* $\Lambda > 0$ its solution, $x(t) = x(t_0)\exp\left(-\Lambda(t-t_0)\right) \to 0$ as $t \to \infty$, for a *nominal trajectory to be tracked* $q^N(t)$, and the *actually realized one* $q(t)$, for *kinematically prescribing* the wished tracking error relaxation, multiple integrals of the tracking error can be introduced as

$$
\begin{aligned}
&e_0(t) = q^N(t) - q(t),\\
&e_1(t) \overset{def}{=} \int_{t_0}^{t} e_0(\xi)\mathrm{d}\xi, \dots\\
&e_{n+1}(t) \overset{def}{=} \int_{t_0}^{t} e_n(\xi)\mathrm{d}\xi,\ n \in \mathbb{N}, \text{ etc.}
\end{aligned}
\tag{1a}
$$

For the control of an *order* $K \in \mathbb{N}$ *system* the requirement $\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^{K+L} e_L(t) \equiv 0$ can be prescribed, that, if it is realized, will drive the tracking error and its appropriate integrals to zero, because it contains the order K time-derivative of the given coordinate of the system $q^{(K)}(t)$ that immediately can be set by the exerted control "force". According to Munkhammar (2004), perhaps it is the simplest way to the introduction of the idea of the fractional order feedback if we realize that the multiple error integrals can be expressed as *Riemann-Liouville n-fold integrals* defined as

$$
F_n(t) = \frac{1}{(n-1)!}\int_{t_0}^{t} f(\xi)(t-\xi)^{n-1}\mathrm{d}\xi \text{ for } n \in \mathbb{N} \ , \tag{2}
$$

that mathematically can be extended to complex numbers as $n \equiv \alpha \in \mathbb{C}$ as a *fractional order integral* defined as

$$
I_a^{\alpha} f(x) \overset{def}{=} \frac{1}{\Gamma(\alpha)}\int_a^x f(\xi)(x-\xi)^{\alpha-1}\mathrm{d}\xi \ . \tag{3}
$$

This extension is based on the properties of *Euler's* Γ *function*, and normally, according to Munkhammar, all the important proofs in the subject area are based on the particular features of the Γ and $\mathcal{B}$ functions. With the exception of the non-positive integers as $\alpha \in \{0, -1, -2, \dots\}$ the extended function is finite, otherwise it is divergent. The *"Fractional Order Derivative"* as oder α is defined in (4) for $x > a$ on the reason that the fractional order integral for the same order parameter $\alpha \in (0,1)$ yields the original function, i.e. $D_a^{\alpha} I_a^{\alpha} f(t) = f(t)$.

$$
D_a^{\alpha} f(x) \overset{def}{=} \frac{1}{\Gamma(1-\alpha)}\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x f(\xi)(x-\xi)^{-\alpha}\mathrm{d}\xi \ . \tag{4}
$$

Though countless possibilities are available for the elaboration of various, different concepts for the definition of fractional order derivatives, perhaps the version given in (4), i.e. the *Riemann-Liouville definition* is the most frequently used one in control technology. For instance Deutschmann et al. (2017) elaborated a *"Linear Time-Invariant"* LTI-type model for a "soft" robotic system that contains tendons, and applied a PD^{α}-type controller. Also, various fractional-order feedback terms can be invented by replacing the integer order derivatives or integrals in the PID control by fractional order ones as $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ (e.g. Dumlu and Erenturk (2014)). It is also possible to introduce additional feedback terms in the form $\mathrm{PDD}^{1/2}$ (e.g. Bruzzone and Fanghella (2014)). For instance, by Muresan et al. (2015) fractional order control of unstable processes, the magnetic levitation was studied. Also, the fractional order control of a visual servoing system is an interesting example (Copot et al. (2013)).

Observing the fact that in (4) the integrand at the upper limit of the integration is singular, following the idea proposed by Redjimi and Tar (2018) to avoid numerical problems, we apply only the integer order integral of (4). So the differentiation of the "delicate integral" can be evaded. Supposing that $f(\xi)$ in (4) is *uniformly continuous*, by the use of a discrete time-grid it can be assumed that $f(\xi)$ varies slowly between the grid-points, and a simple approximation that keeps $f(\xi)$ constant between the grid-points can be done, the so obtained integral can be exactly calculated over the grid. Due to the "forgetting nature" of the integral in (4) it is not needed to complete it over the whole interval $[t, t_0]$: it can be truncated and restricted to $[t, t-T]$ with a *finite memory length* T. Accordingly, with this approach, the replacement of the integer order derivative as in (5a) was suggested

$$\dot{e}(t) = \dot{e}(t_0) + \int_{t_0}^{t} \ddot{e}(\xi)\mathrm{d}\xi \qquad \text{to be replaced by}$$

$$\int_{t_0}^{t} \left[D_{t_0}^{\alpha}\dot{e}(\xi)\right]\mathrm{d}\xi \ , \alpha \in (0,1) \ . \tag{5a}$$

Accordingly, with this approach an approximate "Fractional Order PID" controller can be defined by the use of the desired tracking error relaxation for a 2nd order system at time instant t as

$$\ddot{q}^D = \ddot{q}^N + \Lambda_1^3 e_1 + 3\Lambda_1^2 e_0 + 3\Lambda_2 \int_{t_0}^{t} \left[D_{t_0}^{\alpha}\dot{e}(\xi)\right]\mathrm{d}\xi \ , \tag{6}$$

in which the ratio of the constants Λ_1 and Λ_2 can be appropriately set. By studying the response given for a step function when the exact realization of $\ddot{q}^D$ is assumed, according to Fig. 1, in the forthcoming simulations the settings $\delta t = 10^{-3}$ s, $\alpha = 0.7$, grid length 150 steps, and $\Lambda_2/\Lambda_1 = -2.25$ were used. It can well be seen that the multiple integral-based solution provides great fluctuations, and that the FPD controller yields better tracking properties than the PD controller. In similar manner the FPID seems to be better than the PID controller.

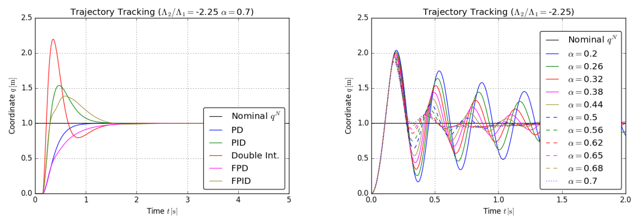

Fig. 1. Comparison of the classic PD, PID, FPD, FPID, and double integral-based solution in tracking a step function (grid width $\delta t = 10^{-3}$ s, $\alpha = 0.7$, grid length 150 steps, and $\Lambda_2/\Lambda_1 = -2.25$), and a fine resolution graph for various α values

3. THE FIXED POINT TRANSFORMATION-BASED ADAPTIVE APPROACH AND POSSIBLE EXTRAPOLATION

In this paper the neuron model also investigated by Dineva et al. (2015) was applied as

$$\frac{\mathrm{d}v}{\mathrm{d}t} = v - \frac{v^3}{3} - w + I_{ext} + \mu I_{Ctrl} \tag{7a}$$

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{v + a - bw}{\tau} \tag{7b}$$

in which I_{Ctrl} denotes the control signal, and parameter μ denotes the sensitivity of the system to the control current.

Three different settings were used in the simulations: one was used for the generation of the nominal trajectory to be tracked, the other set was available for the controller as a priori known approximate model data, and the third one represented the properties of the actual system under control (Table 1). While Dineva et al. investigated the control of a 1st order system dynamically coupled to some "parasite dynamics" (i.e. the control of $v(t)$ was considered by the use of the control signal I_{Crtl}), here we consider the control of an underactuated system in which $w(t)$ is controlled by this current. This task has the *relative order* 2, since only $\ddot{w}$ can be directly controlled by I_{Crtl}, and the motion of $v(t)$ happens accordingly. By the use of the *approximate model parameters* in the time-derivative of (7b), that is in $\ddot{w}$ the derivative $\dot{v}$ appears that through (7a) is in direct relationship with I_{Crtl}.

Table 1. The parameters of the trajectory generator, that of the "approximate model" and the actually controlled "Actual/Exact" neurons

Parameter	Ideal	Approximate	Actual/Exact
a	$a_i = 0.75$	$a_a = 0.8$	$a_e = 0.7$
τ	$\tau_i = 11$	$\tau_a = 10$	$\tau_e = 12.5$
b	$b_i = 0.55$	$b_a = 0.6$	$b_e = 0.5$
I_{ext}	$I_{ext_i} = 0.45$	$I_{ext_a} = 0.4$	$I_{ext_e} = 0.5$
μ	$\mu_i = 1.0$	$\mu_a = 1.0$	$\mu_e = 1.0$

Since the FPT-based design tries to realize an arbitrary desired $\ddot{w}^D$ 2nd time-derivative, in its kinematic block the FPID design detailed in (6) can be applied (Fig. 2). If our *digital controller* has δt discrete time-resolution, and $\ddot{w}$ varies only slowly, this schema generates a *sequence of deformed control signals* by a three-variable function G as $\ddot{w}_{n+1}^{Def} = G\left(\ddot{w}_n^{Def}, \ddot{w}_n, \ddot{w}_{n+1}^{D}\right)$ that, in the case of convergence, corresponds to *adaptive learning*: the control signal in cycle $n+1$ depends on the desired 2nd time-derivative in cycle $n+1$, on the deformed control signal and the observed response in the previous cycle, n. For the *"Adaptive Deformation"* various fixed point transformations can be used. For multiple variable cases the single variable *monotonic, smooth scalar function* with the attractive fixed point $x_\star$ $F : \mathbb{R} \mapsto \mathbb{R}$ $F(x) \stackrel{def}{=} x/2 + D$ stands with $D = 0.6$, and the *response error in the cycle i* $h_i \stackrel{def}{=} f(r_i^{Def}) - r_{i+1}^{Des} \in \mathbb{R}^m$ can be used in the FPT as

$$r_{i+1}^{Def} = G\left(r_i^{Def}, f\left(r_i^{Def}\right), r_{i+1}^{D}\right) \stackrel{def}{=} \begin{cases} r_i^{Def} \text{ if } h_i < \epsilon = 10^{-10} \ , \\ \left[F(A\|h_i\| + x_\star) - x_\star\right]\dfrac{h_i}{\|h_i\|} + r_i^{Def} \text{ otherwise} \ . \end{cases} \tag{8a}$$

Its convergence properties were discussed by Dineva et al. (2015). In our special case the system's response is $r \equiv \ddot{w} \in \mathbb{R}$.

This structure is evidently flexible enough for the introduction of various delay times if the observation on the "realized response" becomes available only later for the controller in the *"Adaptive Deformation" function*. However, the "obsolence" of the available observations can degrade the operation of the controller.

Regarding the extrapolation possibilities in our case the following scenario is assumed: the sensor signals and the control commands have the delay time $T_D\delta t$ ($T_D \in \mathbb{N}$ measures the delay time in δt units). If the controlled process is commenced at time t_0, any actuation at time t can be realized only if $t - 2T_D\delta t > t_0$ because the sensor data originating from the measurement at discrete time $i - 2T_D$ becomes available for the controller at the time $i - T_D$ for the calculation of the "deformed control signal" as $\ddot{q}^{Des}_{i-T_D}$. Then $\ddot{q}^{Def}_{i-T_D}$ is taken as $r^{Def}_{i-T_D} = G\left(r^{Def}_{i-T_D-1}, f^{Obs}_{i-1}, r^{Des}_{i-T_D}\right)$, and finally the control forces calculated from it will be exerted on the controlled system at time i. (The controller's signal calculated at the discrete time instant $i - T_D - 1$ causes observable response effect f^{Obs}_{i-1} at time $i - 1$.) In the simulations the observed data f_{i-T_D} can be stored in the memory. As time elapses, i.e. the discrete index i increases, the stored value later becomes available for the controller as f^{Obs}_{i-1}.

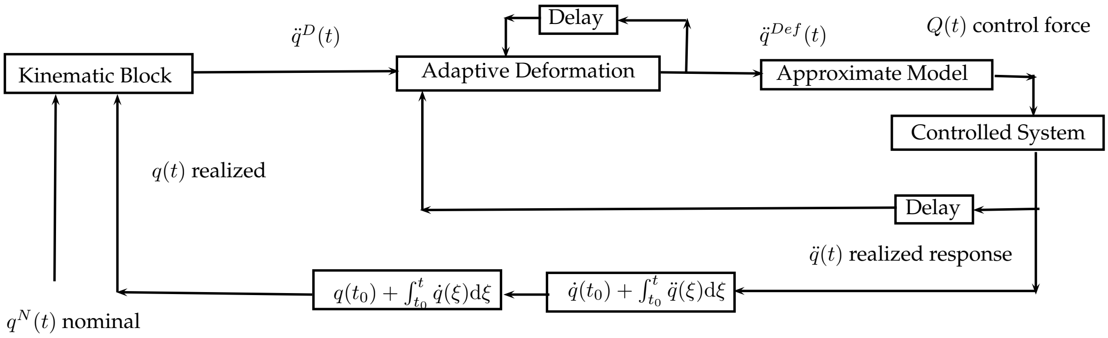

Fig. 2. Schematic structure of the "Fixed Point Transformation-based Adaptive Controller" taken from Redjimi and Tar (2017)

For dealing with the "dead zone" $i \in [i - 2T_D, i]$ various options are available. If precise models are available for LTI systems the "Smith Predictor" invented in 1957 (see e.g. Warwick and Rees (1988)) can be used for the estimation of the state after the observation to decrease the obsolence of the observed data. However, in our case no precise model is availabe for extrapolation. Since any kind of "fancying" for the system's behavior in the dead zone *within the controller design* does not mean contradiction with the principle of causality, we have various options. In the present paper we applied some "extrapolation" for the period $[i - 2T_D, i - T_D]$ to feed the kinematic block with "less obsolete *extrapolated* data". We applied some "refreshments" in the arguments of G by assuming the observation of the responses as $r^{Def}_{i-2T_D+k} = G\left(r^{Def}_{i-2T_D+k-1}, f_{i-2T_D+k-1}, r^{Des}_{i-2T_D+k}\right)$ with $k \in \{1, 2, \ldots, T_D\}$ in which in the place of the *observable response* the response of the approximate model was placed.

4. SIMULATION RESULTS

In the simulations the *adaptive parameter* in (8) was $A = -1$, the discrete time-resolution was $\delta t = 10^{-3}$s, the feedback constant was $\Lambda_1 = 0.5\,\mathrm{s}^{-1}$, and the fractional order derivative was calculated for $n = 150$ steps. For $T_D = 120$ without adaptation and extrapolation the resulst are given in Fig. 3. It can be seen that by switching on the adaptation the tracking error for the controlled variable $w(t)$ slightly decreased (Fig. 4). Finally, by switching on the adaptation and extrapolation in Fig. 5 the situation was drastically improved.

To reveal the significance of adaptivity, in Fig. 6 the variation of the control current I_{Ctrl} and the "Nominal", "Desired", "Deformed", and the "Realized" $\ddot{w}$ values are described. It can well be seen that the "Desired" and the

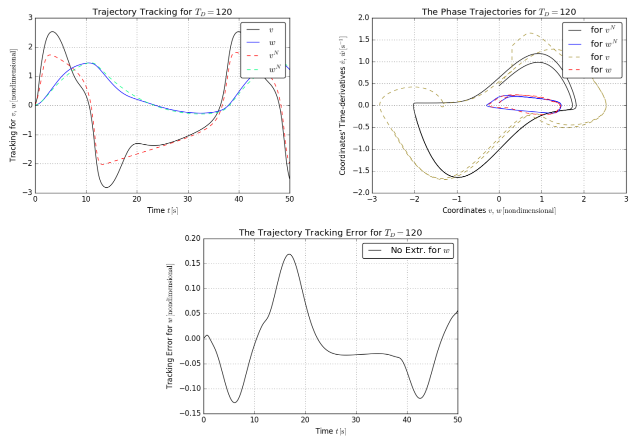

Fig. 3. Tracking properties for $T_D = 120$ without adaptation and extrapolation

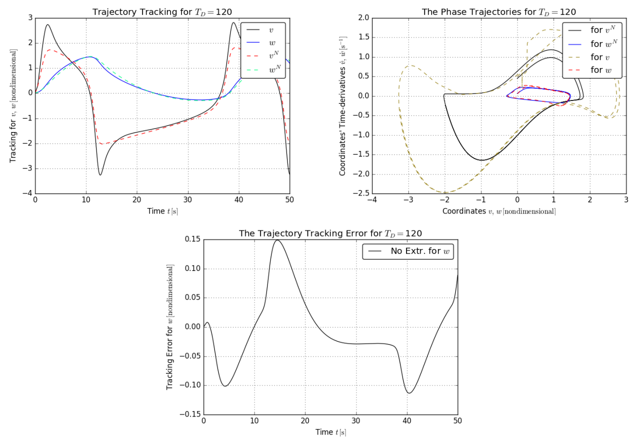

Fig. 4. Tracking properties for $T_D = 120$ with adaptation and without extrapolation

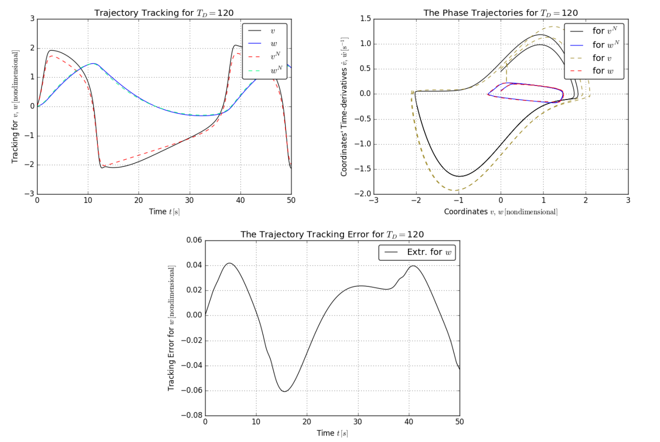

Fig. 5. Tracking properties for $T_D = 120$ with adaptation and extrapolation

"Realized" values are in each other's close vicinity, and that they considerably differ from the "Deformed" value. The considerable difference between the "Nominal" and the "Desired" values makes evident the significance of the FPID corrections in the kinematic design.

To reveal the significance of the "long delay", in Fig. 7 the tracking error for variable $w(t)$ and the control currents $I_{Ctrl}(t)$ are described for a "short delay" $T_D = 20$.

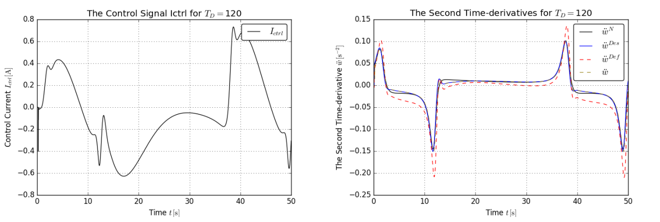

Fig. 6. The control current I_{Ctrl} and the various $\ddot{w}$ values for $T_D = 120$ with adaptation and extrapolation

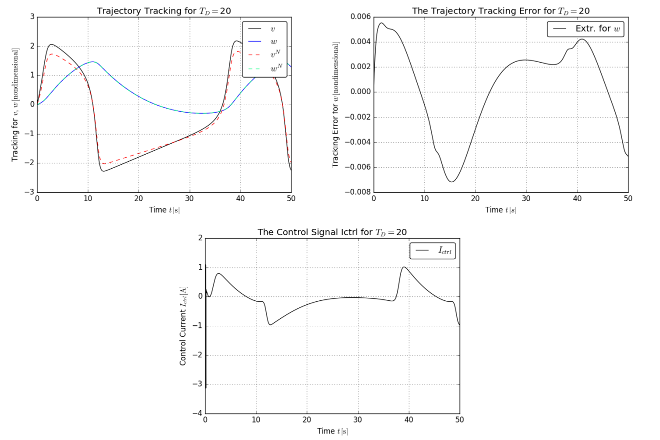

Fig. 7. The trajectory tracking, the tracking error for variable w, and the control current I_{Ctrl} for $T_D = 20$ with adaptation and extrapolation

5. CONCLUSION

In this research the combination of the FPT-based adaptive control with a particular, also FPT-based extrapolation technique was found successful in numerical simulations for a novel fractional order feedback-based adaptive control of the "underactuated" FitzHugh – Nagumo neuron with time-delay.

In the suggested method integer order integral of the fractional derivatives is applied to evade numerical difficulties. This approach allows simple approximate numerical calculation that can be accepted for uniformly continuous integrands. The approximation also truncates the fractional order-based long memory of the control.

The main finding is that in spite of being in the lack of a reliable system model, that normally would be required for making the necessary extrapolation for the stabilization of the controller burdened by time-delay problems, some extrapolation can be done that considerably can improve the tracking properties of the controller.

In further research we plan to check this approach in the control of various, strongly nonlinear systems also burdened by time delay effects.

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