Study on a Kalman Filter based PID Controller

Shin Wakitani * **Hiroki Nakanishi** * **Yoichiro Ashida** * **Toru Yamamoto** *

* *Graduate School of Engineering, Hiroshima University (e-mail: {wakitani/yama}@hiroshima-u.ac.jp)*

Abstract: This study proposes a self-tuning PID controller design method based on a Kalman filter. Recently, data-driven controller tuning methods that can directly tune control parameters by closed-loop data without system models have been received much attention as convenient tuning approaches. On the other hand, in parameter estimation problems, the Kalman filter that can obtain high-precision estimation results has been applied in many research/industrial area. In this paper, a data-driven PID parameters tuning problem that is derived based on a PID control law is resolved as a Kalman filtering problem, and a self-tuning PID controller based on the Kalman filter is proposed. The effectiveness of the proposed method is evaluated by simulation and experimental examples.

Keywords: PID control, self-tuning control, data-driven controller design, extended output, Kalman filter

1. INTRODUCTION

A proportional-integral-derivative (PID) control (Visioli (2006); Vilanova and Visioli (2012)) scheme has been widely applied in the industrial world. PID parameters tuning is very important because tuned PID parameters strongly affect a control performance of a closed-loop system. The most PID controller design scheme is model-based controller design that is based on a mathematical model of a controlled object. However, in recent decades, data-driven controller design schemes represented by the virtual reference feedback tuning (VRFT) (M.C.Campi (2002)) and the fictitious reference iterative tuning (FRIT) (Kaneko (2013)) has been received much attention. These schemes can tune control parameters without any system models by using one-shot operation data obtained by a closed-loop system.

Within this context, a data-driven PID parameter tuning method based on an extended output has been proposed (see Ashida et al. (2016, 2017)), and the effectiveness of the proposed method has been shown by simulation examples and experimental results. According to the method, an extended output which is obtained by a PID control law is firstly defined, and PID parameters are tuned by solving a minimization problem related to the extended output. In this method, the problem can be solved by using one-shot operating data, thus the method does not require any system models. Moreover, the method can be applied the least squares method to solve the minimization problem, and it can easily extend to a data-driven self-tuning controller by the recursive least squares (RLS) method.

On the other hand, lots of parameter estimation methods have been proposed till this day. The Kalman filter is well known as an effective method because the method gives us good estimation results under noisy environments. The Kalman filter has been applied to many industrial/academical area such as aerospace systems, vehicle systems, robots, power prediction, weather forecast etc. (e.g. Jovanovic (2015)). Moreover, Kalman filter algorithms for nonlinear systems have been studied. In the Kalman filter, the Gauss distribution of system noise and observation noise is assumed, and the optimal solution is ensured by giving proper variances of these noises. Thus it is expected to increase the performance of parameter estimation by applying the Kalman filter to the above PID parameter estimation problem. In this paper, a Kalman filter based data-driven control parameter tuning approach is proposed, and the effectiveness of the proposed method is evaluated by simulation and experimental results.

The rest of this paper is organized as follows. In Section 2, it is explained how to derive an extended output form a PID control law, and the relationship between the output and a target tracking problem is explained. In Section 3, Kalman filtering problem is explained, and a PID parameter estimation algorithm based on Kalman filter is described. In Section 4, comparisons with control results between RLS and the proposed method are shown by simulation examples. Finally, Section 5 summarizes the research findings.

2. EXTENDED OUTPUT DERIVATION AND TARGET TRACKING PROBLEM

The following discrete velocity-type PID control law is considered.

$$\Delta u(k) = K_I(k)e(k) - K_P(k)\Delta y(k) - K_D(k)\Delta^2 y(k). \quad (1)$$

$u(k)$ and $y(k)$ are the plant input (the controller output) and the plant output at time k [step]. $e(k)$ expresses the control error given by $e(k) := r(k) - y(k)$, where $r(k)$ is the step-type target value. $K_P(k)$, $K_I(k)$, and

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$K_D(k)$ are the proportional gain, the integral gain, and the derivative gain, respectively. Δ indicates the backward difference operator given by $\Delta := 1 - z^{-1}$. Where z^{-1} is the backward operator that has the following operation: $z^{-1}y(k) = y(k-1)$. By referencing the paper written by Ashida et al. (2017), the following extended output $\phi(k)$ is derived from Eq. (1) under the condition $K_I(k) \neq 0$.

$$r(k) = \phi(k), \tag{2}$$

$$\phi(k) := a_1(k)\Delta u(k) + a_2(k)\{y(k) - y(k-2)\} + a_3(k)\{y(k-1) - y(k-2)\} + y(k-2). \tag{3}$$

Moreover, there are the following relationships among control parameters $a_1(k), a_2(k), a_3(k)$ and PID gains.

$$\left.\begin{aligned} K_P(k) &= \frac{2a_2(k) + a_3(k) - 2}{a_1(k)} \\ K_I(k) &= \frac{1}{a_1(k)} \\ K_D(k) &= \frac{1 - a_2(k) - a_3(k)}{a_1(k)} \end{aligned}\right\}. \tag{4}$$

An objective of controller design is making output follow to the reference model output $G_m(z^{-1})r(k)$. In other words, the objective can be written as a minimization problem of the following error $\varepsilon(k)$.

$$\varepsilon(k) = y(k) - G_m(z^{-1})r(k). \tag{5}$$

Where $G_m(z^{-1})$ is the discrete-time transfer function of the reference model given as follows.

$$G_m(z^{-1}) = \frac{z^{-1}P(1)}{P(z^{-1})}, \tag{6}$$

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}. \tag{7}$$

Where,

$$\left.\begin{aligned} p_1 &= -2\exp\left(-\frac{\rho}{2\mu}\right)\cos\left(\frac{\sqrt{4\mu - 1}}{2\mu}\rho\right) \\ p_2 &= \exp\left(-\frac{\rho}{\mu}\right) \\ \rho &:= T_s/\sigma \\ \mu &:= 0.25(1-\delta) + 0.51\delta \end{aligned}\right\}. \tag{8}$$

In the above equation, T_s $[s]$ is the sampling interval. σ [s] and δ $(0 \leq \delta \leq 2.0)$ indicate desired rise time and damping property of a closed-loop system, respectively. The Eq. (5) can be written as the following equation by utilizing the relationship in Eq. (2).

$$\varepsilon(k) = y(k) - G_m(z^{-1})\boldsymbol{\phi}(k) \tag{9}$$

$$= y(k) - G_m(z^{-1})y(k-2) - G_m(z^{-1})\bar{\boldsymbol{\phi}}(k). \tag{10}$$

Where,

$$\bar{\boldsymbol{\phi}}(k) = a_1(k)\Delta u(k) + a_2(k)\{y(k) - y(k-2)\} + a_3(k)\{y(k-1) - y(k-2)\}. \tag{11}$$

In this paper, relation ships of Eqs. (10) and (11) will be applied to a Kalman filtering problem explained in the next section.

3. SELF-TUNING CONTROLLER DESIGN BASED ON KALMAN FILTER

3.1 Kalman Filter

It is assumed that a system model can be given as the following SISO state-space model.

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{b}(k)\{\tilde{u}(k) + \xi_v(t)\} \tag{12}$$

$$\tilde{y}(k) = \boldsymbol{c}(k)\boldsymbol{x}(k) + \xi_w(t) \tag{13}$$

$\tilde{u}(k)$ and $\tilde{y}(k)$ are the input and the output of the state-space model. $\boldsymbol{x}(k) \in \Re^{l\times 1}$ expresses the state variables vector that has l elements. $\boldsymbol{A} \in \Re^{l\times l}, \boldsymbol{b} \in \Re^{l\times 1}, \boldsymbol{c} \in \Re^{1\times l}$ expresses the system matrix/vector. $\xi_v(t)$ and $\xi_w(t)$ are the system noise and the observation noise, respectively, and it is assumed that each noise is the independent white Gaussian noise. Moreover, each noises' mean and variances are $N(0, \sigma_v^2)$ and $N(0, \sigma_w^2)$, respectively. The Kalman filter is a filter that can give an algorithm to be obtained $\boldsymbol{x}^*(k)$ which minimizes the following minimum mean square error to the state-space system by using the observed data $\{\tilde{y}(k),\ k = 1, 2, \ldots, N\}$.

$$\boldsymbol{x}^*(k) = \arg\min_{\hat{\boldsymbol{x}}(k)} J(\hat{\boldsymbol{x}}(k)), \tag{14}$$

$$J(\hat{\boldsymbol{x}}(k)) = E[(\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k))^T(\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k))]. \tag{15}$$

Where $\boldsymbol{x}(k)$ indicates the true state variables vectors. In the Kalman filter algorithm, the estimated state variables are recursively updated by the following prediction step and filtering step.

- **Prediction step**

$$\hat{\boldsymbol{x}}^-(k) = \boldsymbol{A}(k-1)\hat{\boldsymbol{x}}(k-1) + \boldsymbol{b}(k)u(k-1), \tag{16}$$

$$\boldsymbol{P}^-(k) = \boldsymbol{A}(k-1)\boldsymbol{P}(k-1)\boldsymbol{A}^T(k-1) + \sigma_v^2(k-1)\boldsymbol{b}(k-1)\boldsymbol{b}^T(k-1). \tag{17}$$

  Where $\hat{\boldsymbol{x}}^-(k)$ and $\boldsymbol{P}^{-1}(k)$ are the priori state estimate and the priori estimate covariance, respectively.

- **Filtering step**

$$\boldsymbol{g}(k) = \frac{\boldsymbol{P}^-(k)\boldsymbol{c}(k)}{\boldsymbol{c}^T(k)\boldsymbol{P}^-(k)\boldsymbol{c}(k) + \sigma_w^2(k)}, \tag{18}$$

$$\hat{x}(k) = \hat{\boldsymbol{x}}^-(k) + \boldsymbol{g}(k)(\tilde{y}(k) - \boldsymbol{c}(k)^T\hat{\boldsymbol{x}}^-(k)), \tag{19}$$

$$\boldsymbol{P}(k) = (\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{c}^T(k))\boldsymbol{P}^-(k). \tag{20}$$

Where $\boldsymbol{g}(k)$ is the Kalman gain, $(\tilde{y}(k) - \boldsymbol{c}(k)^T\hat{\boldsymbol{x}}^-(k))$ in Eq. (19) is called the innovation. $\hat{x}(k)$ is the posteriori state estimate that is obtained by the priori state estimate, the Kalman gain and the innovation. Moreover, $\boldsymbol{P}(k)$ is the posteriori estimate covariance.

3.2 Proposed Method

A Kalman filtering problem related to the control parameter estimation problem in Section 2 can be formulated as follows.

$$\boldsymbol{\theta}^*(k) = \arg\min_{\hat{\boldsymbol{\theta}}(k)} J(\hat{\boldsymbol{\theta}}(k)), \tag{21}$$

$$J(\hat{\boldsymbol{\theta}}(k)) = E[(\boldsymbol{\theta}(k) - \hat{\boldsymbol{\theta}}(k))^T(\boldsymbol{\theta}(k) - \hat{\boldsymbol{\theta}}(k))]. \tag{22}$$

Where $\boldsymbol{\theta}(k)$ expresses ideal control parameters $\boldsymbol{\theta}(k) = [a_1(k)\ a_2(k)\ a_3(k)]^T$. A state-space for the control parameter estimation problem is introduced as follows.

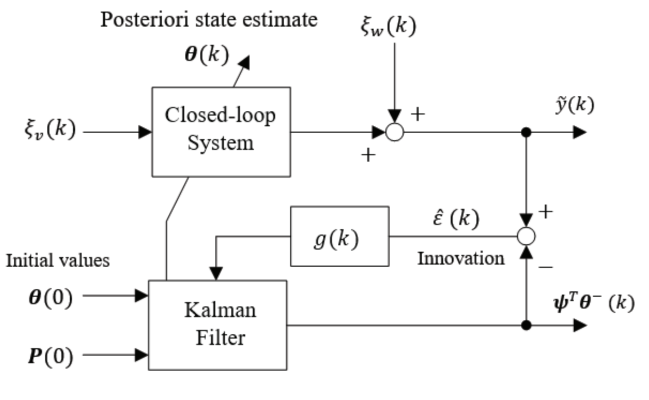

Fig. 1. Block diagram of proposed parameter estimator based on Kalman filter.

$$\boldsymbol{\theta}(k+1) = \boldsymbol{\theta}(k) + \boldsymbol{b}\xi_v(k), \tag{23}$$

$$\tilde{y}(k) = G_m(z^{-1})\boldsymbol{\psi}^T(k)\boldsymbol{\theta}(k) + \xi_w(k). \tag{24}$$

Where,

$$\tilde{y}(k) = y(k) - G_m(z^{-1})y(k-2), \tag{25}$$

$$\boldsymbol{\psi}(k) = [\Delta u(k)\ y(k) - y(k-2)\ y(k-1) - y(k-2)]. \tag{26}$$

By comparing with the above state space model to Eqs. (12) and (13), the control parameter estimation problem can be resolved as Kalman filtering problem under the conditions $\boldsymbol{A}(k) = \boldsymbol{I} \in \Re^{3\times3}$, $\boldsymbol{b}(k) = [1\ 1\ 1]^T$ and $\boldsymbol{c}(k) = \boldsymbol{\psi}(k)$. The block diagram of the proposed control parameter estimation structure based on the Kalman filter is shown in Fig. 1. The figure is shown that the tracking error in Eq. (10) is utilized as the innovation in the Kalman filter.

3.3 Control Parameter Estimation Algorithm

Firstly, the initial vales of the control parameters vector $\boldsymbol{\theta}(0)$ and the priori estimate covariance $\boldsymbol{P}(0) \in \Re^{3\times3}$ are given. Note that the control parameter $a_1(0)$ must satisfy the $a_1(0) \neq 0$. Secondly, the variances of the system noise σ_v^2 and the observation noise σ_w^2 are given as setting parameters. After the above preparations, the control parameters at each time step are estimated by following steps.

- **Prediction step**

$$\hat{\boldsymbol{\theta}}^-(k) = \hat{\boldsymbol{\theta}}(k-1), \tag{27}$$

$$\boldsymbol{P}^-(k) = \boldsymbol{P}(k-1) + \sigma_v^2\boldsymbol{b}\boldsymbol{b}^T. \tag{28}$$

- **Filtering step**

$$\boldsymbol{g}(k) = \frac{\boldsymbol{P}^-(k)\boldsymbol{\psi}(k)}{\boldsymbol{\psi}^T(k)\boldsymbol{P}^-(k)\boldsymbol{\psi}(k) + \sigma_w^2}, \tag{29}$$

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}^-(k) + \boldsymbol{g}(k)\hat{\varepsilon}(k), \tag{30}$$

$$\hat{\varepsilon}(k) = \tilde{y}(k) - \boldsymbol{\psi}(k)^T\hat{\boldsymbol{\theta}}^-(k), \tag{31}$$

$$\boldsymbol{P}(k) = (\boldsymbol{I} - \boldsymbol{g}(k)\boldsymbol{\psi}^T(k))\boldsymbol{P}^-(k). \tag{32}$$

4. NUMERICAL EXAMPLE

The effectiveness of the proposed method is verified by simulation examples. The following 2nd-order lag system is considered as the controlled object.

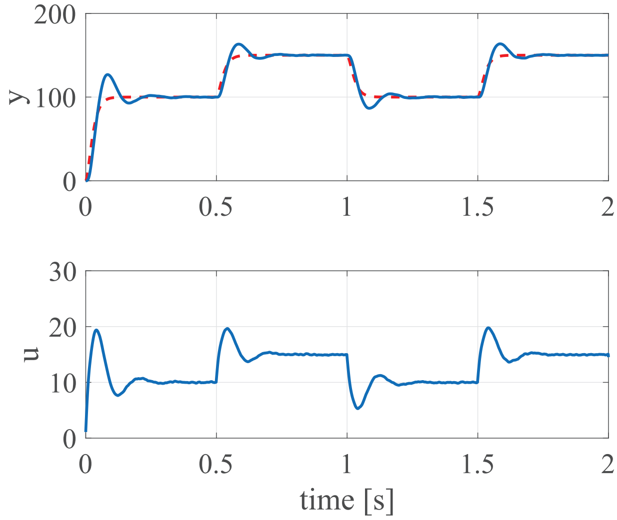

Fig. 2. Control result obtained by RLSM.

$$G(s) = \frac{K\omega_n^2}{(1+Ts)(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \tag{33}$$

In the simulation, the above system is discretized as the following ARX model with the white Gaussian noise $\xi(k)$ discretized by $T_s = 1$ ms. The mean and the variance of the noise are $N(0, 1\times10^{-5})$.

$$A(z^{-1})y(k) = z^{-1}B(z^{-1})u(k) + \xi(k). \tag{34}$$

System parameters $A(z^{-1})$ and $B(z^{-1})$ are calculated by the 'c2d' command in MATLAB. In this simulation a reference signal $r(k)$ is given as follows.

$$r(k) = \begin{cases} 100 & (0\text{ s} \le t < 0.5\text{ s},\ 1\ \le t < 1.5\text{ s}) \\ 150 & (0.5\text{ s} \le t < 1\text{ s},\ 1.5\text{ s}\ \le t < 2\text{ s}) \end{cases} \tag{35}$$

Where t indicates the time in continuous time domain.

Firstly, the fixed PID control gains were calculated based on the pole-assignment method (Wakitani et al. (2011)). Where, in order to uniquely determine the PID gains by the pole-assignment method, the above system was approximated by the following equation.

$$G'(s) := \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{36}$$

Moreover, setting parameters of a reference model $G_m(z^{-1})$ that were required to calculate PID gains were set to $\sigma = 30$ ms and $\delta = 0$. The calculated PID gains are shown below.

$$K_P = 0.144,\ K_I = 0.0109,\ K_D = 1.93. \tag{37}$$

The control result by using the above fixed PID gains are shown in Fig. 2. The figure shows that the stable control result can be obtained by using the controller. However, the overshoot is observed every set point change.

Secondly, the recursive PID gain tuning algorithm by RLS based on the extended output was applied in order to compare to the proposed method. The initial control parameters vector $\boldsymbol{\theta}(0)$ were set as follows based on PID gains in Eq. (37).

$$\boldsymbol{\theta}(0) = [91.34\ 190.4\ \ -365.6]^T \tag{38}$$

Moreover, the initial covariance matrix was set to $\boldsymbol{P}(0) = \mathrm{diag}\,[100\ 100\ 100]$. The control result by the RLS is shown in Fig. 3. The figure shows that the parameter estimation

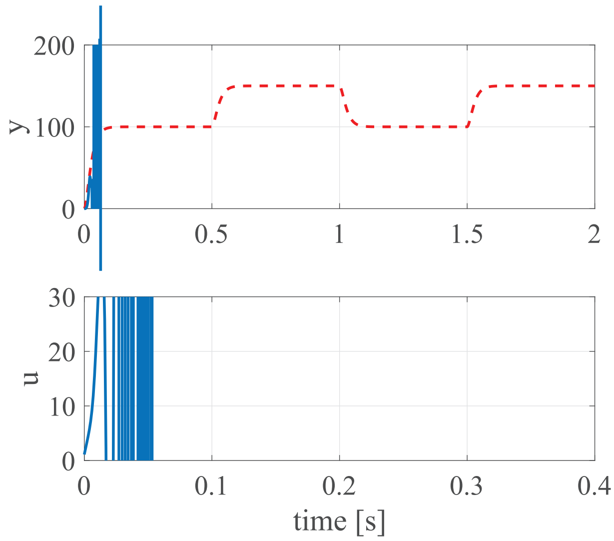

Fig. 3. Control result obtained by RLSM.

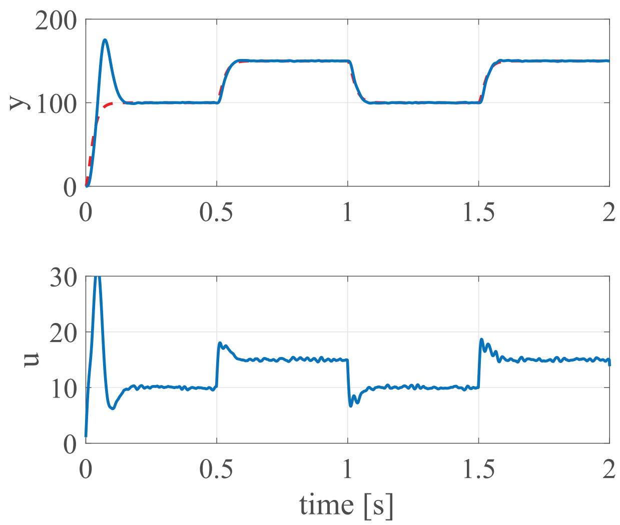

Fig. 4. Control result obtained by proposed method.

is not executed appropriately.

Finally, the control result by the proposed method is shown in Fig. 4 and Fig. 5. All initial values were set as in RLS and the setting parameters in the Kalman filter were set to $\sigma_v^2 = 1 \times 10^{-5}$ and $\sigma_w^2 = 1 \times 10^{-5}$. The figures show that the improved control result compared with the fixed PID controller is obtained by setting the estimated variances appropriately.

5. CONCLUSIONS

In this research, the Kalman filter based self-tuning PID controller was proposed. In the method, the extended output and its error function were firstly derived from the PID control law. Secondly, the PID parameter estimation problem was resolved as a Kalman filtering problem. This paper showed that the error function can be used as the innovation in this estimation problem. The effectiveness of the proposed method was evaluated by numerical results. These results showed that the proposed method works well when the variances of the noise are set properly. In other words, the method has expanded application area of the proposed PID controller than RLS based one. Parameter convergence condition and ensuring stability will be proved in a future work.

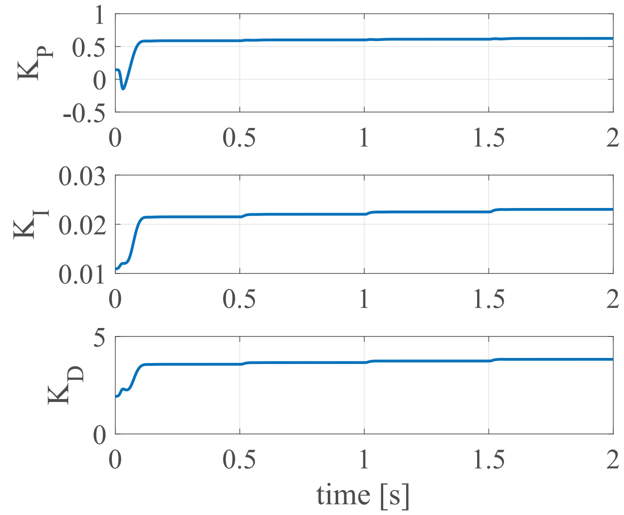

Fig. 5. Trajectories of PID gains corresponding to Fig. 4.

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