

# Stabilization and Tracking for P/PI Combustion Control over a Communication Channel <sup>★</sup>

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**Abstract:** In the present work, motivated by the recent inclusion of optical variables in combustion processes, we consider the control of a reduced Hammerstein plant model over an additive white noise (AWN) channel located at the feedback path. For comparison, assuming uncertainty in the knowledge of the static nonlinearity of the Hammerstein plant model, we first propose in a one degree-of-freedom (DOF) scheme, the design of a proportional controller for robust stability. We then introduce a PI controller, in a 3 DOF scheme, to achieve not only robust stability, but also AWN channel Signal-to-Noise Ratio (SNR) minimization and setpoint first moment tracking.

*Keywords:* Communication control applications. Control system design. Discrete digital dynamic control. Feedback control. P and PI controllers. Networked Control Systems.

## 1. INTRODUCTION

For the last two decades the research area of Networked Control Systems (NCS) has concentrated the research interest of the control community, (J.Chen et al., 2011). Theoretical developments have proceeded apace based on information theory (Nair and Evans, 2004; Martins and Dahleh, 2008), optimization and linear control (Elia, 2004; Braslavsky et al., 2007; Rojas, 2012), multivariable and multiagent approaches (Jadbabaie et al., 2003; Middleton and Braslavsky, 2010; Knorn et al., 2016) and more recently event-triggered control (Heemels et al., 2012; da Silva Jr. et al., 2014; Campos-Delgado et al., 2015). All of the previous theoretical results and more, are now the basis for an informed control practice, revisiting established control processes, as well as facing new and demanding ones, toward achieving increased performance.

Such a revisited process is the combustion process, ubiquitous in most industrial setups Ballester and Garcia-Armingol (2010), which has recently benefited with the addition of optic based sensors Huang et al. (2010); Garcés et al. (2016a). Commonly, combustion processes have their flue gasses monitored through analyzers or chromatographs, which inevitably introduce transport time-delays in the plant models and a biased measurement (Kamimoto et al., 2016; Baukal, 2010). On the other hand, optical measurements of the flame can be performed in-situ, avoiding the transport time-delay and providing a

more accurate description of the combustion real time status (Ballester and Garcia-Armingol, 2010). Albeit, the obtained measurement is ideally transmitted through a communication channel, see Figure 1, avoiding unnecessary and prone to malfunction cabling due to the harsh environment and demanding conditions close to the furnace.

In Garcés et al. (2016b, 2015) it is argued that for such combustion processes, monitored through optic based sensors, an Hammerstein first order model is representative of the real process, under the hypothesis of a mild nonlinearity in the stationary response of the optical measurements. Since the linear part of the plant model is a first order system, this motivate the use of P/PI controller. On the other hand, the input static nonlinearity (characteristic of an Hammerstein model) is treated in Alonge et al. (2015); Gao et al. (2015); Guo et al. (2015); Sun et al. (2009) with a 2 degree-of-freedom (DOF) controller scheme.

In our first contribution, we offer explicitly the design requirement for robust stability and its trade-off with the AWN channel SNR requirement when using a P controller.

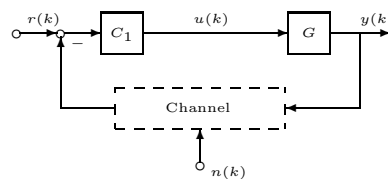


Fig. 1. Controlled combustion process scheme.

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Our second contribution extends the previous to a PI controller in a 2 DOF scheme that solves the trade-off between robust stability and AWN channel SNR reduction.

In our third and last contribution we consider a 3 DOF scheme to the setpoint tracking objective, and thus highlight the trade-off between robust performance and tracking.

The significance of this work is to present a controller design strategy for optical measurements in a combustion process, to fill the gap between instrumentation systems and a feedback operation to increase energy efficiency or reduce the pollutants emissions in a complex operation, subjected to more restricted profit margins and fuels with non-constant composition.

This paper is organized as follows. Section 2 presents the standing assumptions behind this work, briefly reviewing the proposed plant model and the channel SNR definition. Section 3 derives the design requirements for a P controller to achieve robust stability. Section 4 extends the results in two directions by considering now a PI controller in a 2DOF scheme achieving robust stability and channel SNR reduction. Section 5 introduces a 3DOF scheme, together with the established PI controller, to highlight the trade-off between Robust stability, AWN channel SNR reduction and setpoint tracking. Conclusions are given in Section 6. All control results in this preliminary work are simulation based, we expect to achieve experimental confirmation at a later stage.

## 2. PRELIMINARIES

In this section we present the standing assumptions for this work.

### 2.1 Assumptions

**Plant model:** In Garces et al. (2016b, 2015) it has been established that a Hammerstein first order model can be a reasonable representation of an optically sensed combustion process

$$y(z) = \frac{K_G}{\alpha z - 1} f(u), \quad (1)$$

where  $K_G > 0$  and  $\alpha > 1$  are the plant model gain and time constant, whilst  $f(u)$  is the input static nonlinearity. The signal  $y(k)$  corresponds to the measured total radiation  $Rad_t(k)$  and the input signal  $u(k)$  to the percentage of fan speed  $\lambda(k)$  injecting air into the combustion chamber. In Figure 2 we show the scatter plot and average value of stationary total radiation  $Rad_t(\infty)$  measured in a ladle furnace preheating process (described in Samuelsson and Sohlberg (2010); Zabadal et al. (2004)), where the nonlinear static response is verified as function of the stationary value of fan speed  $\lambda(\infty)$ . A schematic of optically sensed ladle furnace preheating process is presented in Figure 3 where the main stages of measurements are summarized as the field flame spectra from the flame measured with the spectrometer and the spectral processing stage where the optical variables are finally calculated.

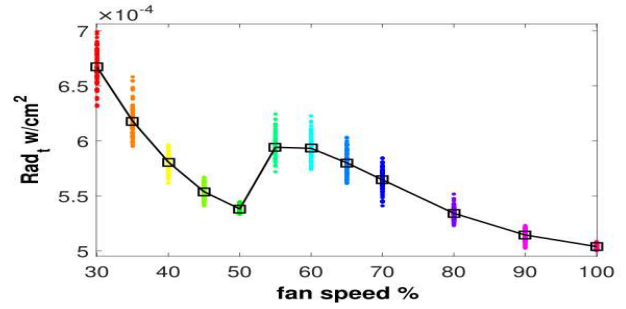


Fig. 2. Scatter plot of optically sensed ladle furnace preheating process.

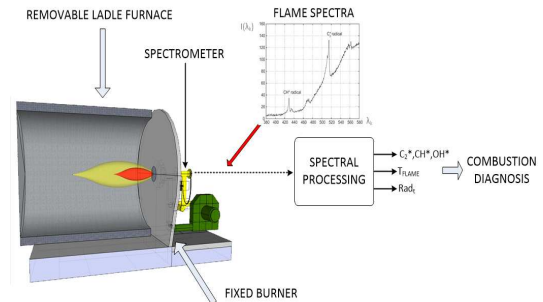


Fig. 3. Schematic of ladle furnace optically sensed.

- **Channel model:** The channel model is in general characterized by the admissible channel input power level  $\mathcal{P}$  and the channel additive noise process  $n(k)$ .
- **Channel additive noise process:** The channel additive noise scalar process  $n(k)$  is a zero-mean, i.i.d., white noise process with known variance  $\sigma^2$ .
- **Setpoint process:** the setpoint process is an i.i.d. Gaussian white process with mean  $\mu_r$  and variance  $\sigma_r^2$ , independent of the channel additive noise process  $n(k)$ .

From Figure 1 we assume that the controller  $C_1$  is such that the output feedback system is stable in the sense that for any distribution of initial conditions, the distribution of all signals in the loop will converge to a stationary distribution. The power of the channel input, defined by  $\|s\|_{Pow}^2 \triangleq \lim_{k \rightarrow \infty} \mathcal{E} \{s^2(k)\}$ , is then required to satisfy a user defined power constraint  $\mathcal{P}$ . Under the stationarity assumption presented in (Åström, 1970, §4.4), the power of the channel input can then be computed as  $\|s\|_{Pow}^2 = \|T_{sr}(z)\|_2^2 (\sigma_r^2 + \mu_r^2) + \|T_{sn}(z)\|_2^2 \sigma^2$ , where the  $H_2$  norm is defined as

$$\|T_{\{\cdot\}}(z)\|_2^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} T_{\{\cdot\}}(e^{j\omega}) T^H(e^{j\omega}) d\omega$$

The power constraint at the channel input can be then redefined as an SNR constraint  $\frac{\mathcal{P}}{\sigma^2}$  which must be satisfied by the squared  $H_2$  norm of  $T_{sr}(z)$  and  $T_{sn}(z)$

$$\frac{\mathcal{P}}{\sigma^2} > \|T_{sr}(z)\|_2^2 \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + \|T_{sn}(z)\|_2^2. \quad (2)$$

From (2) we then have bound on the AWN channel SNR.

We treat the partial knowledge of  $f(u)$ , represented by  $\hat{f}(\cdot)$ , in a worst case scenario for which we define

$$\gamma = \max_v \left[ f \left( \hat{f}^{-1}(v) \right) \right]. \quad (3)$$

The inner feedback loop is then analyzed for this worst-case scenario, see for example Figure 5 for the case  $C_1 = K_p$ , with  $\gamma$  in place.

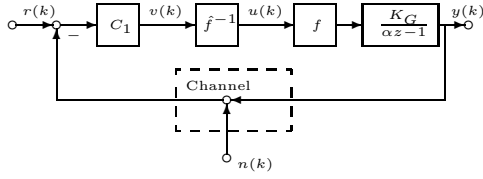


Fig. 4. 1 DOF configuration for a Hammerstein first order model over an AWN channel.

A sufficient condition for robust stability, assuming a nominal plant model  $G_o$  and a real plant model  $G = G_o(1 + G_\Delta)$  can be inferred for example from (Goodwin et al., 2001, Theorem 5.3), as  $\|T_o\|_\infty \|G_\Delta\|_\infty < 1$ . In the previous expression  $T_o = G_o C_1 / (1 + G_o C_1)$  is the nominal complementary sensitivity, that is considering the nominal plant model  $G_o$ . Applied to the case under study, we recognize  $G_\Delta = \gamma - 1$  and thus the condition

$$\|T_o\|_\infty |\gamma - 1| < 1 \quad (4)$$

is sufficient for robust stability. If we wish to achieve robust performance is again sufficient to increase the demand on the above condition as  $\|T_o\|_\infty |\gamma - 1| \ll 1$ .

### 3. 1 DOF ROBUST P CONTROL

In this section we consider the use of a proportional controller  $K_p$  in place of  $C_1$  in Figure 4. More so, we discuss the design of  $K_p$  for the worst case scenario of mismatch knowledge on the input static nonlinearity, see Figure 5. We start by considering the nominal complementary sensitivity function resulting from a proportional controller

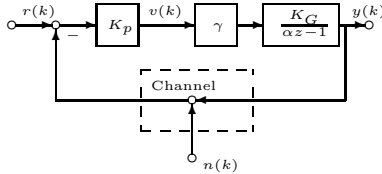


Fig. 5. 1 DOF configuration for a P robust control of Hammerstein first order model over an AWN channel.

$$T_o(z) = \frac{K_p K_G}{\alpha z - 1 + K_p K_G} \quad (5)$$

From the above we can obtain the range of values of  $K_p$  that ensures nominal stability, given  $\alpha$  and  $K_G$

$$\frac{1 - |\alpha|}{K_G} < K_p < \frac{1 + |\alpha|}{K_G} \quad (6)$$

Observe that, under the assumption proposed in Section 2 of  $\alpha, K_G > 0$ , the range of  $K_p$  spans negative and positive values. We proceed now by considering the channel SNR resulting for the nominal case. As explained in Section 2, the channel SNR is lower bounded by the expression

$$\begin{aligned} \|T_o\|_2^2 & \left( \frac{\sigma_r^2 + \mu^2}{\sigma^2} + 1 \right) \\ & = \frac{K_G^2 K_p^2}{\alpha^2 - (1 - K_G K_p)^2} \left( \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + 1 \right) \end{aligned} \quad (7)$$

As expected, as  $K_p$  approaches any of the extreme values for stability, the above lower bound tends to infinity. Also,

$K_p^2$	$K_G^2(2 - \gamma)\gamma$	$(\alpha + 1)^2$
$K_p^1$	$-2K_G(\alpha + 1)$	0
$K_p^0$	$(\alpha + 1)^2$	

Table 1. Routh Hurwitz algorithm on  $K_p$ .

it is clear that as  $K_p \rightarrow 0$ , then this SNR lower bound tends to zero, which is correct in so far the proposed linear part of the plant model is stable,  $\alpha > 1$ . On the other hand, if  $\alpha < 1$ , it is not difficult to verify that the minimum value of  $\|T_o\|_2^2$  is given by  $(1 - \alpha^2)/\alpha^2$  which is in agreement, for example, with (Rojas, 2012, Theorem 2). For robust stability, in light of Equation (4), we need to study the  $H_\infty$  norm of  $T_o$ . In general a closed-form expression for this type of norm is not readily available. However, in this occasion this is possible due to the rather simple expression for  $T_o(z)$  we are considering in this section. Nevertheless, there are two cases to study depending on the value of the product  $K_p K_G$ . Specifically, for

$$K_p K_G > 1 \quad (8)$$

the  $H_\infty$  norm of  $T_o(z)$  is then

$$\|T_o\|_\infty = \frac{|K_p K_G|}{\sqrt{\alpha^2 + (K_p K_G - 1)(K_p K_G - 1 - 2\alpha)}} \quad (9)$$

Since we want to impose by design the condition in Equation (4), this result in the following equation

$$\begin{aligned} 0 & < K_G^2(2 - \gamma)\gamma \cdot K_p^2 - 2K_G(\alpha + 1) \cdot K_p + (\alpha + 1)^2 \\ & = K_G^2(2 - \gamma)\gamma(K_p - K_{p1})(K_p - K_{p2}) \end{aligned} \quad (10)$$

A Routh-Hurwitz argument, see Table 1, identifies both  $K_{p1}$  and  $K_{p2}$  as positive roots. Assuming  $0 < K_{p1} < K_{p2}$  we expect  $K_{p1}$  to be an upper bound on  $K_p$  for robust stability.

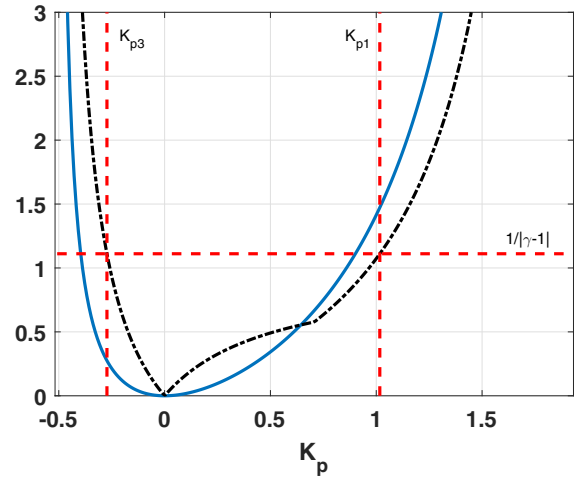


Fig. 6. AWN channel SNR, blue solid line, and  $H_\infty$  norm of the nominal complementary sensitivity  $T_o(z)$ , black dash-dotted line.

The second case, to complete the study on the  $H_\infty$  norm of  $T_o(z)$  is imposed by

$$K_p K_G \leq 1 \quad (11)$$

which results in an expression for  $\|T_o\|_\infty$  of

$$\|T_o\|_\infty = \frac{|K_p K_G|}{\sqrt{\alpha^2 + (K_p K_G - 1)(K_p K_G - 1 + 2\alpha)}} \quad (12)$$

Again, the above norm expression, together with Equation (4) lead us to the following inequality

$$0 < K_G^2(2 - \gamma)\gamma \cdot K_p^2 - 2K_G(-\alpha + 1) \cdot K_p + (-\alpha + 1)^2 = K_G^2(2 - \gamma)\gamma(K_p - K_{p3})(K_p - K_{p4}) \quad (13)$$

A similar Routh- Hurwitz, not presented, identifies  $K_{p3}$  and  $K_{p4}$  as negative roots. Assuming  $K_{p4} < K_{p3} < 0$  we now expect  $K_{p3}$  to be a lower bound on  $K_p$  for robust stability. That is, in conclusion, we have for robust stability, the requirement on  $K_p$  of

$$K_{p3} < K_p < K_{p1} \quad (14)$$

To better understand the obtained expressions for the SNR reduction and robust stability we present next an example.

*Example 1.* In this example we consider  $K_G = \sqrt{2}$ ,  $\alpha = \sqrt{3}$ ,  $\sigma_r^2 + \mu_r^2 = \sigma^2 = 1$  and  $\gamma = 0.1$ . The resulting expressions for this example are presented in Figure 6. The AWN channel SNR is shown in a blue solid line, whilst the  $H_\infty$  norm of  $T_o(z)$  is shown in a black dash-dotted line. For stability the range of  $K_p$  is limited to  $-0.5176 < K_p < 1.9319$ , whilst for robust stability this range is further reduced to  $K_{p3} = -0.2724 < K_p < 1.0168 = K_{p1}$ .

The previous example highlights the limitations of using a proportional controller to solve the problems at hand, that is robust stability, SNR reduction and setpoint tracking. The proportional controller best solution for the first two is to do nothing (since the plant model is stable), and it is well known that a proportional controller is not suitable for setpoint tracking with zero steady state error.

#### 4. 2 DOF ROBUST PI CONTROL

The last comment in the previous section motivates the proposal of using a PI controller, instead of a P controller. We also take the opportunity to analyze the introduction of a second degree of freedom in the controller solution, represented by  $C_2$  in Figure 7. As in Freudenberg et al. (2010) we consider  $C_2$  as a simple gain value. More so, we define  $C_1(z)$  as

$$C_1(z) = \left( K_p + K_i \frac{z}{z-1} \right) \cdot \frac{1}{C_2} \quad (15)$$

that is a modified PI controller. The nominal closed-loop relationship of interest in this case is represented by

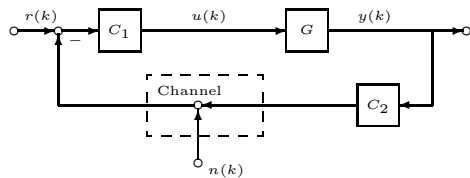


Fig. 7. General 2 DOF scheme over an AWN channel.

$$\frac{P}{\sigma^2} > \left\| \frac{C_2 G_o C_1}{1 + C_2 G_o C_1} \right\|_2^2 \left( \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + 1 \right) \quad (16)$$

Since  $C_1(z)$  contains the inverse of  $C_2$  we have that the lower bound on the AWN channel SNR can be designed exclusively through the PI part of  $C_1(z)$ , that is through the user selection of  $K_p$  and  $K_i$ . From Rojas and Garces (2017), adapting the notation, we can directly quote the appropriate result for completeness, which specifies that

$$\frac{P}{\sigma^2} > \frac{2b_1 b_o a_1 - (b_o^2 + b_1^2)(a_o + 1)}{(a_o - 1)((a_o + 1)^2 - a_1^2)} \left( \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + 1 \right), \quad (17)$$

$$\text{with } b_1 = \frac{K_G(K_p + K_i)}{\alpha}, b_o = -\frac{K_G K_p}{\alpha}, a_1 = -1 - \frac{1}{\alpha} + \frac{K_G(K_p + K_i)}{\alpha}, a_o = \frac{(1 - K_G K_p)}{\alpha}.$$

For robust stability, on the other hand, we study the closed-loop relationship between  $y(k)$  and  $r(k), n(k)$  which is given by

$$y(k) = t_{ry}(k) * r(k) - t_{ry}(k) * n(k) \quad (18)$$

where the zeta transform of  $t_{ry}$  is given by

$$T_{ry}(z) = \frac{G_o C_1}{1 + C_2 G_o C_1} \quad (19)$$

The key observation is then that, given the proposed 2 DOF scheme and definition of  $C_1(z)$ , then  $T_{ry}(z)$  is inversely proportional to  $C_2$ . As a consequence, as  $C_2 \rightarrow \infty$ , then  $\|T_{ry}\|_\infty \rightarrow 0$  and thus we can satisfy the robust stability requirement imposed by  $\gamma$  through  $C_2$  independently of the selection of  $K_p$  and  $K_i$  selected for SNR reduction. Finally, as we achieve steady state through a stabilizing choice of  $C_1(z)$  and  $C_2$  plus taking expectation, we obtain

$$y_{ss} = \lim_{k \rightarrow \infty} E\{y(k)\} = \lim_{k \rightarrow \infty} t_{ry} * \mu_r = \mu_r \quad (20)$$

since  $E\{n(k)\} = 0$  and the presence of the integral action in  $C_1(z)$ . Thus, as long as  $k_i \neq 0$ , we can also achieve the proposed tracking objective.

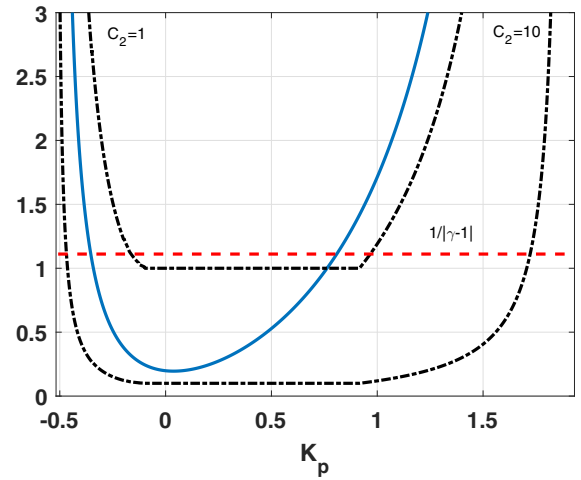


Fig. 8. AWN channel SNR, blue solid line, and  $H_\infty$  norm of the nominal complementary sensitivity  $T_o(z)$ , black dash-dotted line, for  $C_2 = 1$  and  $C_2 = 10$ .

*Example 2.* In this example, for consistency, we also consider  $K_G = \sqrt{2}$ ,  $\alpha = \sqrt{3}$ ,  $\sigma_r^2 = \mu_r^2 = \sigma^2 = 1$  and  $\gamma = 0.1$ . The integral gain  $K_i$  is selected to be 0.1. The resulting expressions for this example are presented in Figure 8. The AWN channel SNR is shown in a blue solid line, whilst the  $H_\infty$  norm of  $T_o(z)$  is shown in a black dash-dotted line, for both  $C_2 = 1$  and  $C_2 = 10$ . The range of  $K_p$ , as in the previous example and for comparison, is limited to  $-0.5176 < K_p < 1.9319$ . We observe that the choice of PI controller has resulted in an overall increased AWN channel SNR lower bound. We also observe, that as  $C_2$  increases (in this case from 1 to 10) the range of values for  $K_p$  compatible with the robust stability condition (below the red dashed line) increases. This accounts for more user



freedom at the moment of deciding the AWN channel SNR. Observe also that, due to the PI structure, even for  $K_p = 0$  we still have an SNR lower bound different from zero, since in this occasion, the integral part of  $C_1(z)$  maintains the loop closed. Finally, Figure 9 shows the dynamic response for a sub-set of  $SNR$  values  $\{0.2, 0.52, 1.68, 6.12\}$  imposed by  $K_p$  values of  $\{-0.01, 0.49, 0.99, 1.49\}$ , where the zero error for tracking is verified, but the dynamic response has an increased overshoot when the  $SNR$  increases. In particular, for the last value of  $K_p$  we observe a rather unacceptable overshoot of 30 % percent together with a poorly compensated oscillatory response.

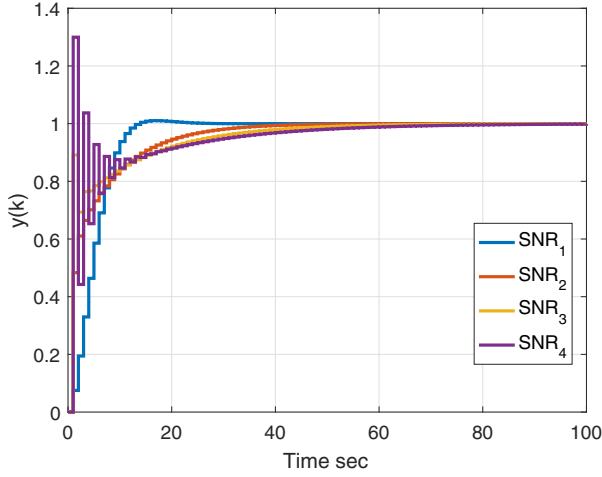


Fig. 9. Example of dynamic response for tracking and  $SNR_1 = 0.1999$ ,  $SNR_2 = 0.5176$ ,  $SNR_3 = 1.6822$ ,  $SNR_4 = 6.1168$

Another well known feature introduced by a communication channel is transmission delay. This feature can be studied directly with the present setup, see Figure 10, where we consider a one unit time delay.

The closed-loop relationship that defines the AWN channel SNR lower bound is now

$$\frac{P}{\sigma^2} > \left\| \frac{C_2 G_o C_1}{z + C_2 G_o C_1} \right\|_2^2 \left( \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + 1 \right) \quad (21)$$

For a specific PI structure for the linear part of  $C_1$  this in turns reports

$$\frac{P}{\sigma^2} > \frac{-(b_1 - b_2)^2 \beta_1 + 2b_1 b_2 (\alpha_1 - \beta_1)}{\alpha_2 \beta_1 - \alpha_1 \beta_2} \left( \frac{\sigma_r^2 + \mu_r^2}{\sigma^2} + 1 \right), \quad (22)$$

with  $b_2 = \frac{K_G(K_p + K_i)}{\alpha}$ ,  $b_1 = -\frac{K_G K_p}{\alpha}$ ,  $a_2 = -\frac{\alpha_1 + 1}{\alpha}$ ,  $a_1 = \frac{1 + K_G(K_p + K_i)}{\alpha}$ ,  $a_o = -\frac{K_G K_p}{\alpha}$ ,  $\alpha_1 = a_o a_1 - a_2$ ,  $\alpha_2 = -2a_o a_1 a_2 + a_o^2 + a_1^2 + a_2^2 - 1$ ,  $\beta_1 = a_o^2 + a_o a_2 - a_1 - 1$  and  $\beta_2 = 2(a_1 - a_o a_2)(a_o + a_2)$ . We omit the proof, but as intuition would have it, for the same values of  $K_p$ ,  $K_i$ ,  $K_G$  and  $\alpha$  an extra time delay increases the AWN channel SNR lower bound. Of course a transmission delay is not limited to only one unit time delay, but again, the more the transmission delay the bigger will be the AWN channel SNR lower bound and thus the overall requirement for a successful transmission over the AWN channel. The robust part of the design, when including a one time delay in the channel, follow the same approach as without it.

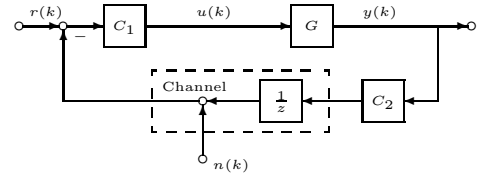


Fig. 10. General 2 DOF scheme over an AWN channel with a one unit time delay.

### 5. 3 DOF FOR ROBUST PERFORMANCE

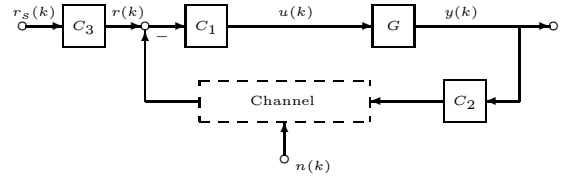


Fig. 11. General 3 DOF scheme.

In this section we consider the use of a 3 DOF scheme, see Figure 11. This approach is in line with what already proposed in Alonge et al. (2015); Gao et al. (2015); Guo et al. (2015); Sun et al. (2009), albeit for a 2 DOF scheme. The main objective is to use  $C_3(z)$  to impose a desired closed-loop relationship between  $y(k)$  and the pre-filtered setpoint  $r_s(k)$ . It is straightforward then, given  $T_{obj}(z)$  as such a desired closed-loop objective, that a suitable choice for  $C_3(z)$  is

$$C_3(z) = \frac{1}{C_2} \cdot T_{obj}(z) T_{ry}^{-1}(z) \quad (23)$$

For the above choice, and the scheme in Figure 11, the AWN channel SNR is now lower bounded by

$$\frac{P}{\sigma^2} > \|T_{obj}\|_2^2 \frac{\sigma_{r_s}^2 + \mu_{r_s}^2}{\sigma^2} + \|T_{sn}\|_2^2 \quad (24)$$

where  $\sigma_{r_s}$  and  $\mu_{r_s}$  are the variance and mean, respectively, of  $r_s(k)$ . The AWN channel SNR lower bound is then the result of the design of  $T_{sn}(s)(z)$  and the choice of  $T_{obj}(z)$ . For robust stability, on the other hand, we have

$$y(k) = \frac{1}{C_2} \cdot t_{obj}(k) * r(k) - t_{ry}(k) * n(k) \quad (25)$$

The key observation now is that, as long as  $C_3(z)$  is inversely proportional to  $C_2$ , we can still satisfy the robust stability requirement imposed by  $\gamma$  through  $C_2$ , independently of the selection of  $K_p$  and  $K_i$ , assuming a PI choice for  $C_1(z)$ . As in the previous section, taking expectation we obtain the tracking of  $r_s$  by  $y(k)$

$$y(k) = E\left\{ \frac{1}{C_2} \cdot t_{obj}(k) * r(k) - t_{ry}(k) * n(k) \right\} = \frac{\mu_{r_s}}{C_2} \cdot t_{obj}(k) \quad (26)$$

Therefore, with the user choice of  $t_{obj}(k)$  for all  $k$ , subject to  $\lim_{k \rightarrow \infty} t_{obj}(k) = C_2$ , we can impose as desired, a user defined closed-loop relationship between  $y(k)$  and the pre-filtered setpoint  $r_s(k)$ .

### 6. CONCLUSIONS

In the present work, motivated by the recent inclusion of optical variables in combustion processes, we have proposed different schemes for the control of a reduced Hammerstein plant model over an additive white noise

(AWN) channel located at the feedback path using P and PI controllers. We first proposed a 1 DOF scheme with a P controller to illustrate the design challenges involved. We then introduced a PI controller, in a 2 DOF and 3 DOF scheme, to achieve not only robust stability, but also AWN channel SNR reduction and setpoint first moment tracking. Future directions should consider other type of communication channel models, beyond the AWN channel, and also the inclusion of dynamical uncertainty in the linear part of the plant model.

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