PID Tuning Based on Forced Oscillation for Plants Without Ultimate Frequency

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Abstract: In this article, a method to tune proportional-integral-derivative controllers for the class of plants with relative degree one is proposed, since this class includes plants that are not amenable to application of the traditional Ziegler-Nichols (ZN)-like methods. The method presented here is based on a modified relay feedback experiment with inclusion in the loop of a transfer function of constant phase in a defined range of frequencies. Thus, with a single experiment and simple tuning formulas this method enlarges the class for which the ZN-like methods can be applied.

Keywords: Frequency domain controller design, process control, PID controllers, fractional-order systems, Ziegler-Nichols (ZN) methods.

1. INTRODUCTION

The PI(D) controllers are the most commonly used control structures (Åström and Hägglund, 1995). One of the reasons for this is the existence of the two Ziegler-Nichols (ZN)-like tuning methods initially proposed in the seminal work (Ziegler et al., 1942): the one based on the open-loop step response of the plant and the one based on a closed-loop experiment. The first method applies only to plants with a characteristic reaction curve, and the second method applies only to plants with a finite ultimate frequency, i.e., whose Nyquist diagram crosses the negative real axis. Thus, there are plants that are not amenable to the application either of the ZN-like tuning methods – plants that have no ultimate frequency and have no reaction curve that allows the application of this method.

A PID tuning method based on a modified relay feedback experiment has been proposed in Bazanella et al. (2017) for the class of plants with relative degree greater than one. The identification of a particular point of the plant's frequency response is performed in a single experiment through the relay feedback experiment with inclusion of a fractional-order integrator (FOI) in the loop. Then, the controller parameters are calculated from this identified information by simple tuning formulas, which maintains the same simplicity and rapidity of the ZN-like methods.

In this article, similar to the tuning method presented in Bazanella et al. (2017), it is proposed to tune PID controllers for plants with relative degree one. The modified relay feedback experiment is changed to be applicable to this class of plants and tuning formulas are proposed. Thus, this development enlarges even further the class for which the closed-loop ZN-like methods can be applied.

This article is organized as follows. In Section 2, some preliminary concepts and a brief review are presented. The tuning method is described in Section 3, where the PI(D) tuning formulas are developed based on the knowledge and positioning of a particular point of the plant's frequency response. The modified relay feedback experiment that allows to identify this point is presented in Section 4. A detailed analysis of the methodology is performed in Section 5 with two different plants that are not amenable to either of the traditional ZN methods. The concluding remarks are shown in Section 6.

2. PRELIMINARIES

2.1 Plants

In this article, the class of linear time invariant causal (LTIC) plants is considered. This class of plants is characterized by

$$Y(s) = G(s)U(s), \tag{1}$$

where $G(s)$ is the plant's transfer function, $U(s)$ and $Y(s)$ are respectively the Laplace transforms of the input and the plant's output (the controlled variable). The plant is controlled with unitary feedback by a LTIC controller, that is

$$E(s) = R(s) - Y(s), \quad U(s) = C(s)E(s), \tag{2}$$

where $R(s)$ is the reference, $E(s)$ is the tracking error, and $C(s)$ is the controller's transfer function.

2.2 PI(D) Controllers

The transfer function of a PI controller can be represented as:

$$C_{pi}(s) = K_p \left(1 + \frac{1}{T_i s}\right), \tag{3}$$

where T_i is the integral time and K_p is the proportional gain, which are the parameters to be designed. The tuning of a PID controller is performed considering ideal derivative action and the following transfer function:

$$C_{pid}(s) = K_p \left(1 + \frac{1}{T_i s}\right)(1 + T_d s). \tag{4}$$

Ideal derivative action cannot be exactly implemented in practice, so the transfer function of a implementable PID controller usually is

$$C_{pid}(s) = K_p \left(1 + \frac{1}{T_i s}\right)\left(1 + T_d \frac{s}{Ns+1}\right), \tag{5}$$

where N is a fixed parameter determined according to a procedure detailed in Section 3.

2.3 ZN-like methods

There is a large amount of literature dedicated to PID tuning rules, and several methods have been proposed and successfully applied, for example, λ-tuning, Cohen-Coon, and MIGO – an overview is given in Åström and Hägglund (1995). These methods and their extensions, like the one proposed in (Pereira and Bazanella, 2015) to tune resonant controllers, consist in variations of the methods proposed in the seminal work Ziegler et al. (1942). A tuning method presented in (Ziegler et al., 1942) consists in causing an oscillation with a plant in closed-loop, measuring the oscillations' frequency and amplitude, then obtaining the controller parameters by means of simple formulas dependent on these parameters. In another seminal work, Åström and Hägglund (1984) have improved this method with the relay feedback experiment and the explicit consideration of gain and phase margins. Here, it is referred as the *classical forced oscillation (CFO)* method.

The CFO method is based on the knowledge of the *ultimate point* of the plant's frequency response. The ultimate point of a given transfer function is the point at which its Nyquist plot crosses the negative real axis – the point corresponding to the lowest frequency where its phase is $-\pi$. This point is characterized by the ultimate frequency ω_u and the ultimate gain K_u, which are defined as

$$\omega_u = \min_{\omega \geq 0} \omega : \angle G(j\omega) = -\pi, \quad K_u = \frac{1}{|G(j\omega_u)|}. \tag{6}$$

These definitions allow to summarize the CFO method as follows.

(1) Identify the ultimate point of the plant's frequency response, that is, determine ω_u and K_u.
(2) Design the parameters of the controller such that $C(j\omega_u)G(j\omega_u) = p$, or equivalently
$$C(j\omega_u) = -K_u p \tag{7}$$
where p is a prespecified location in the complex plane.

The first step of the CFO method is generally performed through the relay feedback experiment, which consists in a closed-loop experiment with the following nonlinear control action:

$$u(t) = d\,\mathrm{sign}(e(t)) + b, \tag{8}$$

where $\mathrm{sign}(\cdot)$ is the sign function [$\mathrm{sign}(x) = 1$ for positive x and $\mathrm{sign}(x) = -1$ for negative x], $d \in \mathbb{R}^+$ is a parameter to be chosen, and $b \in \mathbb{R}$ is the bias. The parameter d regulates the oscillation amplitude at the plant's output and b must be adjusted to obtain a symmetrical oscillation. Once a symmetric oscillation is obtained, its amplitude A_u and period T_u are measured and the ultimate characteristics are given by (Åström and Hägglund, 1984)

$$K_u = \frac{4d}{\pi A_u} \quad \text{and} \quad \omega_u = \frac{2\pi}{T_u}. \tag{9}$$

In the second step of this method, the controller's parameters K_p, T_i and T_d are designed by solving (7) for a defined location p. Under the assumption that the frequency response of the plant is sufficiently smooth, moving the ultimate point away from $-1 + j0$ in the complex plane with the controller implies that the whole open-loop frequency response is shifted away from it, thus leading to appropriate stability margins. Over time, different locations p have been proposed, each one resulting in different stability margins and transient performance. The tuning formulas presented in Ziegler et al. (1942) correspond to $p = -0.4 + j0.08$ for PI controllers and $p = -0.6 - j0.28$ for PID controllers.

This method can not be applied to plants that have no ultimate point. This is the case of all the minimum-phase open-loop stable first and second-order plants, and most plants with relative degree smaller than three, for example. Therefore, a PI(D) tuning method based on a modified relay feedback experiment for a larger class of plants was presented in Bazanella et al. (2017). The *extended forced oscillation (EFO)* method was developed based on the same theoretical approach as the CFO method, that is to place one particularly relevant point of the open-loop frequency response at a specified location in the complex plane, and it can be applicable to plants with relative degree larger than one. To identify a particular point of the plant's frequency response, the methodology requires only one experiment without designer intervention and the controllers parameters are designed trough simple formulas, keeping the same simplicity of the CFO method.

In this paper, the EFO method is further developed to be applicable to minimum-phase open-loop stable plants with relative degree one. In what follows, PI(D) tuning formulas are obtained. Then, the modified relay feedback experiment presented in Bazanella et al. (2017) is changed to be applicable to this class of plants.

3. EXTENDED FORCED OSCILLATION METHOD

The control design objective of the CFO method is to obtain appropriate stability margin, that is, gain and phase margins for the class of plants that has an ultimate point (Åström and Hägglund, 1984). If a plant has no ultimate point, then the gain margin will be infinite, provided that the controller does not contribute with too large phase lag. Thus, the phase margin becomes the only explicit control design objective for this class of plants (Bazanella et al., 2017). Based on the theoretical approach of the CFO method, the EFO's idea is to identify a specific point of the plant's frequency response, and then design a controller to achieve a desired phase margin at this particular frequency. This idea is analytically developed as follows.

Let M_ϕ be the desired phase margin and $\theta \triangleq M_\phi - 180°$. Identify the frequency ω_ν defined as $\angle G(j\omega_\nu) = \nu$, and the magnitude of the plant's frequency response at this specific frequency $M_\nu = |G(j\omega_\nu)|$. If the controller is tuned such that

$$C(j\omega_\nu)G(j\omega_\nu) = 1\angle\theta, \tag{10}$$

then the phase margin will be exactly the desired one at ω_ν, provided that the magnitude of the loop transfer function monotonically decreases for frequencies higher than this identified frequency. Thus, the controller must be tuned to satisfy

$$C(j\omega_\nu) = \frac{1}{M_\nu}\angle(\theta - \nu). \tag{11}$$

For those plants with relative degree larger than one and those plants with relative degree one whose phase crosses the $-120°$ line, Bazanella et al. (2017) have proposed to identify the point of the plant's frequency response whose phase is $\nu = -120°$. Then, the controller is designed to guarantee $M_\phi = 50°$ with the PI and $M_\phi = 60°$ with the PID structure.

In this paper, for those plants with relative degree one whose phase does not cross the $-120°$ line, the point of the plant's frequency response to be identified and the desired phase margin have been defined after several tests. PI and PID controllers have been designed and the resulting performance from the closed-loop reference step response has been evaluated for a wide array of plants [1]. Different values of ν and M_ϕ, which result in different stability margins and performance, can be chosen. The proposed values that have emerged as the best ones in tests are: identify the point of the plant's frequency response whose phase is $\nu = -60°$ and design the controller to guarantee $M_\phi = 50°$ with the PI and $M_\phi = 60°$ with the PID structure. In this case, (11) can be rewritten as

$$C_{pi}(j\omega_{60}) = \frac{1}{M_{60}}\angle - 70°, \tag{12}$$

for the PI, and

$$C_{pid}(j\omega_{60}) = \frac{1}{M_{60}}\angle - 60°. \tag{13}$$

for the PID controller.

In the following, the tuning formulas for PI and PID controllers are obtained to satisfy (12) and (13), respectively.

3.1 PI

The PI tuning formulas are obtained by substituting (3) with $s = j\omega_{60}$ into the tuning equation (12), which results in

$$C_{pi}(j\omega_{60}) = K_p\left(1 - \frac{j}{T_i\omega_{60}}\right) = \frac{1}{M_{60}}\angle - 70°. \tag{14}$$

Equating real and imaginary parts of the last equation yields the tuning formulas for a PI controller proposed in this article

$$K_p = \frac{\cos(70°)}{M_{60}},\ \ T_i = \frac{1}{\omega_{60}\tan(70°)} = \frac{T_{60}}{2\pi\tan(70°)}, \tag{15}$$

where $T_{60} = 2\pi/\omega_{60}$.

[1] The results of these tests are not shown for lack of space.

Table 1. Tuning formulas for PI(D) controllers

Controller	K_p	T_i	T_d	N
PI	$\frac{0.34}{M_{60}}$	$0.058T_{60}$	-	-
PID	$\frac{0.34}{M_{60}}$	$0.058T_{60}$	$0.028\,T_{60}$	$1.6\times10^{-4}\,T_{60}$

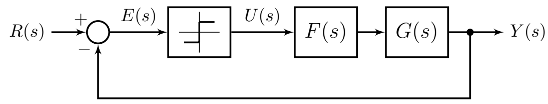


Fig. 1. Relay feedback experiment for identification of the ultimate point of $F(s)G(s)$.

3.2 PID

The PID tuning formulas are obtained from a PI tuned by (15), which inserts a phase lag of 70° at the identified frequency ω_{60}. To obtain a phase margin of 60°, the PD block $1 + T_d s$ must have a phase lead of 10° at this specific frequency, that is

$$\angle 1 + j\omega_{60}T_d = \arctan(\omega_{60}T_d) = 10°. \tag{16}$$

Then, the parameter T_d is given by

$$T_d = \frac{\tan(10°)}{\omega_{60}} = \frac{\tan(10°)}{2\pi}T_{60}. \tag{17}$$

Through this choice, the PD block changes the magnitude of the controller's transfer function at ω_{60} by a factor of

$$\sqrt{1 + (T_d\omega_{60})^2} = \sqrt{1 + \tan^2(10°)} = \frac{1}{\cos(10°)}. \tag{18}$$

To achieve unitary magnitude of the loop transfer function at ω_{60}, the controller's gain K_p given in (15) must be corrected by the inverse of the factor (18). Thus, for the PID, the proportional gain is

$$K_p = \frac{\cos(70°)\cos(10°)}{M_{60}}. \tag{19}$$

The ideal transfer function (4) is used for the controller design, whereas in the numerical examples, the implementable transfer function (5) is used with $N = 10^{-3}/\omega_{60}$. The proposed tuning formulas for the PI and PID controllers are presented in Table 1, where the trigonometric functions have been rounded up to two significant digits.

4. IDENTIFICATION OF THE ω_{60}-POINT

The relay feedback experiment described in Section 2.3 is a classical way to experimentally identify the ultimate point of a plant, but other points of a plant's frequency response can be identified with a slight modification of this experiment (Åström and Hägglund, 1995). When a known transfer function, say $F(s)$, is inserted in the loop in addition to the relay, as in Fig. 1, if a self-oscillation condition is obtained then it will have the ultimate frequency of the transfer function $F(s)G(s)$, that is, at $\omega_1 : \angle F(j\omega_1)G(j\omega_1) = -180°$. Thus, the plant's magnitude and phase at this frequency can be calculated as (Bazanella et al., 2017):

$$|G(j\omega_1)| = \frac{\pi A}{4d\,|F(j\omega_1)|},\ \angle G(j\omega_1) = -180° - \angle F(j\omega_1), \tag{20}$$

since $F(j\omega_1)$ is known.

Table 2. Coeficients of $F(s)$

	$m = 1/3$ for $\gamma = -30°$		$m = 2/3$ for $\gamma = -60°$	
k	a_k	b_k	a_k	b_k
0	0	0.3452	0	0.7152
1	111.1	1309	11.11	1446
2	8.49×10^4	5.4×10^5	1.097×10^4	4.387×10^5
3	1.15×10^7	4.302×10^7	1.918×10^6	2.678×10^7
4	3.232×10^8	7.22×10^8	6.963×10^7	3.473×10^8
5	1.942×10^9	2.598×10^9	5.403×10^8	9.672×10^8
6	2.509×10^9	2.013×10^9	9.016×10^8	5.799×10^8
7	6.986×10^8	3.36×10^8	3.24×10^8	7.487×10^7
8	4.195×10^7	1.211×10^7	2.506×10^7	2.08×10^6
9	5.462×10^5	9.508×10^4	4.164×10^5	1.238×10^4
10	1569	167.8	1466	15.45
11	1	0.06905	1	0.003576

To implement the proposed development in the EFO method, the point of the plant's frequency response whose phase is $-60°$ must be identified. If the identification is performed through the ultimate point of $F(s)G(s)$, a transfer function $F(s)$ whose phase is $-120°$ at the frequency ω_{60} must be chosen, but this specific frequency is not known in advance since it is one of the two values that the experiment aims at identifying [2].

As proposed in Bazanella et al. (2017), if the phase of $F(s)$ was the same for all frequencies, i.e. $\angle F(j\omega) = \gamma \ \forall\omega$, then only one experiment would be necessary, keeping the same characteristics of the CFO method. A system that has a transfer function with a flat phase frequency response that is not an entire multiple of $-90°$ is an FOI and it is represented by

$$F(s) = \frac{1}{s^m}, \tag{21}$$

where it can be verified that

$$\angle F(j\omega) = -\angle\left(\frac{j}{\omega}\right)^m = -\angle\left(\frac{e^{\frac{j\pi}{2}}}{\omega}\right)^m = -\frac{\pi}{2}m. \tag{22}$$

Defining $m = -\gamma/90°$ in (22) yields $\angle F(j\omega) = \gamma \ \forall\omega$ and, for example, choosing $\gamma = -30°$ results in $m = 1/3$ and for $\gamma = -60°$ is obtained $m = 2/3$.

In general, fractional-order systems are approximately implemented by integer order systems. To obtain transfer functions that approximate the magnitude and phase characteristics of a desired FOI, the MATLAB package FOMCON (Tepljakov et al., 2011), (Tepljakov, 2013) was used. The transfer function shown in (23) with the two sets of parameters presented in Table 2 describes two FOIs approximations with magnitude characteristics of $-m \times 20$ dB/decade and constant phase value of $-m \times 90°$ for $m = 1/3$ and $2/3$, considering the range of frequencies from 10^{-3} to 10^3 rad/s.

$$F(s) = \frac{\sum_{k=0}^{11} b_k s^k}{\sum_{k=0}^{11} a_k s^k} \tag{23}$$

Fig. 2 presents the Bode diagrams of these two FOIs approximations with magnitude and phase curves having: -6.66 dB/decade and $-30°$; -13.33 dB/decade and $-60°$. Furthermore, as desired, a transfer function with constant

[2] For the class of plants with relative degree larger than one the point of the plant's frequency response whose phase is $-120°$ is identified with a transfer function $F(s)$ whose phase is $-60°$ at the frequency ω_{120}.

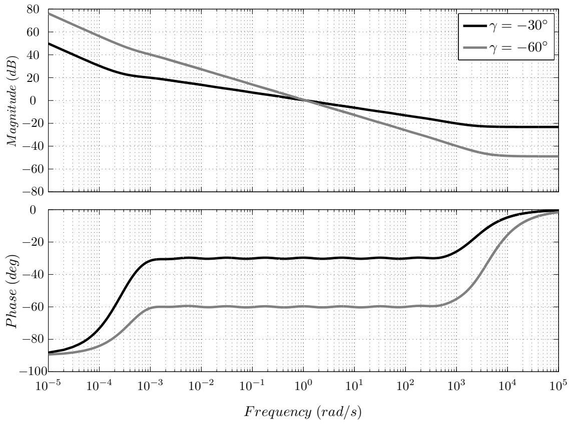


Fig. 2. Frequency response of the FOI approximations.

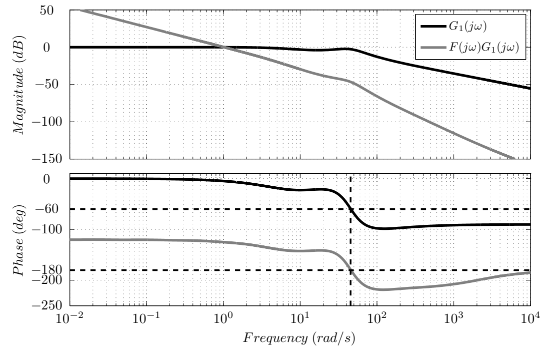


Fig. 3. Frequency response of $G_1(s)$ and $G_1(s)F(s)$ with $\gamma = -120°$.

phase of $-120°$ can be obtained from the FOI approximation with phase of $-30°$ and an integrator, that is, $F'(s) = F(s)/s$.

To properly identify the desired point of the plant's frequency response, the transfer function $F(s)$ given in (23) must have phase γ in the self-oscillation frequency obtained in the relay experiment with the approximated FOI in the loop. Therefore, $F(s)$ must have the largest possible range of frequencies.

5. CASE STUDIES

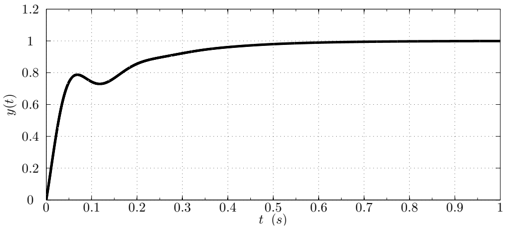
In order to validate the proposed tuning method, two different plants will be taken into account. For each of them, a detailed description of all steps of the controller design will be presented.

5.1 Plant 1

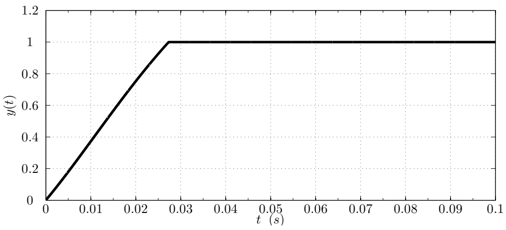
The first plant has the following transfer function:

$$G_1(s) = \frac{17s^3 + 1840s^2 + 5.2 \times 10^4 s + 4.5 \times 10^5}{s^4 + 80s^3 + 3850s^2 + 9 \times 10^4 s + 4.5 \times 10^5}. \tag{24}$$

The frequency response of the plant described by the transfer function (24) is presented in Fig. 3. Initially, the classical ZN methods are applied in order to tune the PI(D) controller's parameters. The output of the open-loop step response procedure is shown in Fig. 4(a), where it is seen that the reaction curve is not a well-defined S-curve, thus tuning based on this experiment can not be



(a) Reaction curve – unitary step



(b) Relay feedback experiment: $d = 2$ and $b = 0$

Fig. 4. Output of $G_1(s)$ for the ZN experiments.

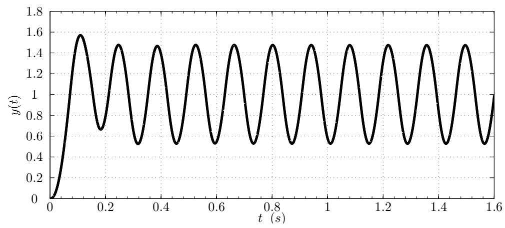


Fig. 5. Output of $G_1(s)$ for the relay feedback experiment with a FOI of $-120°$ in the loop.

Table 3. Parameters for the relay feedback experiment with FOI of $-120°$ and $G_1(s)$

d	b	A	$\lvert F(j\omega_{60})\rvert$	M_{60}	$T_{60}[s]$
80	0	0.474	0.00642	0.724	0.139

performed. Then, the CFO method is applied through a closed-loop relay feedback experiment whose plant's output is shown in Fig. 4(b) considering as reference input a step with amplitude one. The CFO method also can not be applied, since the plant has no ultimate point and, consequently, a self-oscillation condition is not obtained.

Thus, the EFO method is used because it is not possible to obtain the PI(D) controller's parameters through either classical ZN tuning methods. A self-oscillatory behavior of the system is obtained in a closed loop experiment with a relay and a FOI of $-120°$, as presented in Fig. 1. The ultimate point of $F(s)G_1(s)$ has the same frequency that the point of the plant's frequency response whose phase is $-60°$, as shown in Fig. 3, and this specific point is identified. The plant's output considering as reference input a step with amplitude one is presented in Fig. 5 and the parameters obtained in this experiment are summarized in Table 3.

The sets of the controllers' parameters calculated from Table 1 and the performance measures are summarized in Table 4. The closed-loop responses to a unit step with both PI and PID controllers are shown in Fig. 6. The settling time (t_s) is about the same for both controllers, the maximum overshoot (M_o) is approximately 50% smaller with the PID controller, due to the larger phase margin.

Table 4. Tuning and performance for $G_1(s)$

PI				PID					
K_p	T_i	$t_s[s]$	M_o	K_p	T_i	T_d	N	$t_s[s]$	M_o
0.469	0.00805	0.31	14	0.469	0.00805	0.00388	2.22×10^{-5}	0.29	7.1

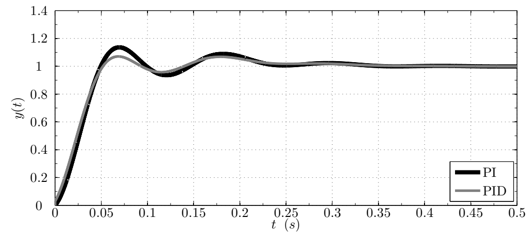


Fig. 6. Output of $G_1(s)$ in closed loop with the corresponding PI and PID controllers.

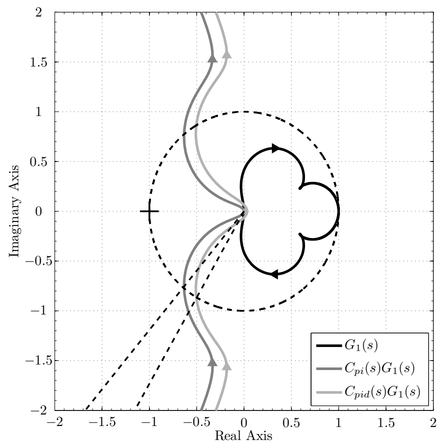


Fig. 7. Nyquist diagrams of $G_1(s)$, $C_{pi}(s)G_1(s)$, and $C_{pid}(s)G_1(s)$. Dashed lines are at $-120°$, $-130°$ and at unitary magnitude.

The Nyquist diagrams of the plant's transfer function $G_1(s)$ and of the open-loop transfer function with the PI(D) controllers, $C_{pi}(s)G_1(s)$ and $C_{pid}(s)G_1(s)$, are presented in Fig. 7. It can be seen that these diagrams do not encircle the point $-1 + j0$, and the controller's magnitude $1/M_{60}$ and phase at the plant's frequency ω_{60}: $-70°$ and $-60°$, for the PI and PID controllers, respectively, guarantees phase margin of 50° and 60°.

5.2 Plant 2

The second plant considered for analysis of the tuning method has an unknown transfer function. Initially, the relay feedback experiment is performed to tune the PI(D) controller's parameters. In this experiment a self-oscillation condition is not satisfied, thus this plant has no ultimate point and the CFO method can not be applied.

Then, the relay feedback experiment with a $-60°$ FOI in the loop is performed, and once again a self-oscillation condition is not obtained, so the phase of the plant's frequency response does not reach either $-180°$ or $-120°$. A last relay feedback experiment is performed with a $-120°$ FOI in the loop and a self-oscillatory behavior is achieved, as presented in Fig. 8. Hence, it is identified the point of the plant's frequency response whose phase is $\nu = -60°$. Indeed, the transfer function of this plant is given by

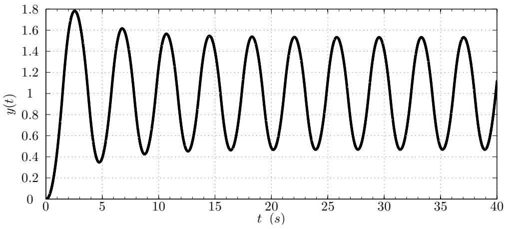


Fig. 8. Output of $G_2(s)$ for the relay feedback experiment with a FOI of $-120°$ in the loop.

Table 5. Parameters for the relay feedback experiment with FOI of $-120°$ and $G_2(s)$

d	b	A	$\lvert F(j\omega_{60})\rvert$	M_{60}	$T_{60}[s]$
1.6	0	0.532	0.522	0.501	3.75

Table 6. Tuning and performance for $G_2(s)$

PI				PID					
K_p	T_i	$t_s[s]$	M_o	K_p	T_i	T_d	N	$t_s[s]$	M_o
0.679	0.218	4.5	20	0.679	0.218	0.105	6×10^{-4}	3.1	14

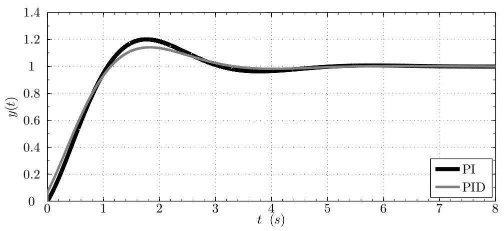


Fig. 9. Output of $G_2(s)$ in closed loop with the corresponding PI and PID controllers.

$$G_2(s) = \frac{1}{s+1}$$

and the desired point of the plant's frequency response is identified. The parameters obtained in this experiment are summarized in Table 3.

The sets of the controllers' parameters and the performance measures are shown in Table 6. In Fig. 9 the plant's output signals to a unit step with both the PI and PID controllers in closed loop are presented. In this case, the settling time and the maximum overshoot are approximately 30% smaller with the PID controller.

The Nyquist diagram of $G_2(s)$ and of the open-loop transfer function with the PI(D) controllers, $C_{pi}(s)G_2(s)$ and $C_{pid}(s)G_2(s)$, are shown in Fig. 10. For both controllers the diagrams of the control loop do not encircle the point $-1 + j0$ since of the frequency response is smooth enough around the negative real axis and the phase margin of 50° and 60°, for the PI and PID, respectively, is guaranteed.

6. CONCLUSION

In this paper, a development in the EFO method has been proposed to tune PID controllers for plants with relative degree one, which are not amenable to the application of the CFO method. Like the EFO method, its development is based on the identification of a particular point of the plant's frequency response through the relay feedback experiment with the inclusion of a FOI in the loop. PID tuning formulas were obtained considering a wide array of plants, then a detailed description of the controller design was presented in two case studies. The obtained closed-loop performance is similar to the performance that is achieved with the ZN-like methods. The proposed development enlarges the class of plants for which the EFO method can be applicable, thus making it possible the tuning of PID controllers for a larger class of plants that do not admit the application of either classical ZN-like methods. A possible extension of this work is to develop a single methodology to tune PID controllers for both plants that have and plants that have no ultimate point.

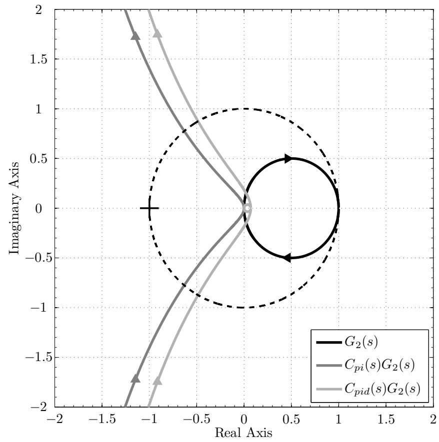


Fig. 10. Nyquist diagrams of $G_2(s)$, $C_{pi}(s)G_2(s)$, and $C_{pid}(s)G_2(s)$. Dashed lines are at $-120°$, $-130°$ and at unitary magnitude.

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