# PID Tuning Method for Integrating Processes Having Time Delay and Inverse Response

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Abstract: In this paper, a PID tuning method for integrating processes having time delay and inverse response is presented. The method is based on the stability boundary locus method and geometrical center (WGC) approach. The systematic procedure of the method is first to obtain the stability region in the PI controller parameters (proportional gain: $k_p$ and integral gain: $k_i$) plane according to derivative gain ($k_d$) using the stability boundary locus method and then to find the weighted geometrical center point of this region. The WGC controllers are obtained by using different values of $k_d$. Simulation examples have demonstrated that PID controller designed by using the proposed method gives good results.

*Keywords:* PID, the WGC method, Time delay, Integrating system

## 1. INTRODUCTION

The PID controller is the most commonly used controller structure in industrial applications. However, even for linear systems, fundamental stability problems are still an area of investigation in control theory. This is especially true if the system includes a time delay (Hohenbichler, 2009). Their common utilization in practice motivates researchers in developing better PI/PID adjustment methods (Chidambaram, 2000; Chidambaram and Sree, 2003; Lee, 2008; Sree et al., 2004; Shamsuzzoha and Lee, 2008; Silva et al., 2005; Taylor et al., 2006; Wang and Cluett, 1997). Some important studies can be seen in (Åström and Hagglund, 2001; Ho et al., 1995; Ho et al., 1998; Hohenbichler, 2009; Oliveira et al., 2009; Silva et al., 2005; Söylemez et al., 2003; Wang et al., 2016; Wang et al., 1999; Zhuang and Atherton, 1993). On the other hand, one of the complicated problems faced by engineers in industry is to design the PI/PID controller for integrating processes, whose dynamics also possess both time delay and inverse response characteristics. When the difficulty of controlling the integrating systems with time delay and inverse response and the simplicity of the PID control algorithm are considered together, the control of these processes requires a special attention (Lee, 2008; Normey and Camacho, 2007). Most frequently encountered example of such systems in practice is a boiler steam drum (Pai et al., 2010). Despite the common usage of boilers in the process industry, studies for developing control of such systems are relatively few in the literature. Åström and Bell have presented a non-linear dynamic model for natural circulation boilers (Åström and Bell, 2000). Kim and Chio proposed a model based on the principles of momentum and energy conservation (Kim and Choi, 2005). However, for the control of boilers, an approximate dynamic model for practical control purposes is needed more than a complicated and precise model. For the dynamics of such systems, Luyben proposed the inverse responded open-loop transfer function given in Equation 1 (Luyben, 2003).

$$G(s) = \frac{K(-\tau_a s+1)e^{-\theta s}}{s(\tau s+1)} \tag{1}$$

Accordingly, system parameters are determined by a model identification method based on open loop set point response that is obtained by the Matlab software. Luyben proposed a PI/PID controller tuning method in frequency domain by the Matlab software, as well (Luyben, 2003). But it is a significant drawback that this method needs the solution of synchronous non-linear algebraic equations. However, as a model-based approach the direct synthesis (DS) design method allows to directly determine the closed-loop behaviours desired from the system model (Chen and Seborg, 2002). Pai et al. expressed the control parameters based on obtaining the optimum adjustment parameter λ for the minimum IAE (integral absolute error) criterion and system model based DS-design using the golden ratio searching technique (Pai et al., 2010). Furthermore, it is indicated that the tuning method given in (Pai et al., 2010) provides better performance results for load/disturbance effects than Luyben's method. However, the method is based on minimum IAE criterion which is obtained by the golden-section searching method and hence an iterative design process. Accordingly, the method is not appropriate for practical design purpose.

Recently, a design method is given for the PI control of time-delay systems using the weighted geometrical center (WGC) (Onat, 2013) of stability region obtained from stability boundary locus method (Tan, 2005). The WGC method is based on the calculation of the weighted geometric center of the region of stabilizing controller parameters. The controller designed for WGC point generally provides good unit step response although it is not theoretically proved. In this study, the WGC method is extended to PID control of the inverse responded and time-delayed integrating processes. The design is fulfilled by using of derivative gain ($k_d$) as a sweep parameter. Simulation examples are given to investigate application of the method presented. The robustness of the proposed method is also shown by applying to an integrating transfer function with time delay and inverse response including parametric uncertainty.

This study is organized as follows. The second section explains PID tuning based on the stability boundary locus and the WGC methods for an integrating process with inverse response and time delay. The third section presents three simulation examples. Finally, discussion and results are given in the fourth section.

## 2. THE STABILITY BOUNDARY LOCUS METHOD AND THE WGC CONCEPT

In order to explain the method, an integrating process with time delay and inverse response is used. The process is described by (2). The considered closed loop system with PID controller is shown in Fig. 1. Where, C(s) is the transfer function of the PID controller and given by (3).

$$G(s)=\frac{0.6(-0.3s+1)e^{-0.2s}}{s(s+1)} \tag{2}$$

$$C(s)=k_p+\frac{ki}{s}+k_d s \tag{3}$$

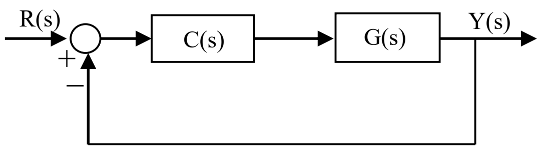


Fig. 1. Block diagram of a simple feedback control system

The closed loop characteristic polynomial $\Delta(s)$ of the system of Fig.1, i.e. the numerator of $1+C(s)G(s)$, can be written as

$$\Delta(s)=sD(s)+(k_d s^2+k_p s+k_i)N(s)e^{-\theta s}=0 \tag{4}$$

Decomposing the numerator and the denominator polynomials of $G(s)=N(s)/D(s)$ into their even and odd parts, and substituting $s=j\omega$, gives

$$G(j\omega)=\frac{N_e(-\omega^2)+j\omega N_o(-\omega^2)}{D_e(-\omega^2)+j\omega D_o(-\omega^2)} \tag{5}$$

For simplicity $(-\omega^2)$ will be dropped in the following equations. Equating the real and imaginary parts of $\Delta(j\omega)$ to zero, $k_p$ and $k_i$ parameters according to fixed value of $k_d$ are obtained by (6) and (7)

$$k_p=\frac{(\omega^2 N_o D_o+N_e D_e)\cos(\omega\theta)+\omega(N_o D_e-N_e D_o)\sin(\omega\theta)}{-(N_e^2+\omega^2 N_o^2)} \tag{6}$$

$$k_i=\frac{\omega^2(N_o D_e-N_e D_o)\cos(\omega\theta)-\omega(N_e D_e+\omega^2 N_o D_o)\sin(\omega\theta)-k_d\omega^2(N_e^2+\omega^2 N_o^2)}{-(N_e^2+\omega^2 N_o^2)} \tag{7}$$

For integrating transfer function with time delay and inverse response given in (1), $k_p$ and $k_i$ values for fixed $k_d$ are obtained as

$$k_p=\frac{\omega^2(\tau_a+\tau)\cos(\omega\theta)+\omega K(\tau_a\tau\omega^2-1)\sin(\omega\theta)}{K(1+\omega^2\tau_a^2)} \tag{8}$$

$$k_i=\frac{-\omega^2(\tau\tau_a\omega^2-1)\cos(\omega\theta)-\omega^3(\tau+\tau_a)\sin(\omega\theta)+k_d K\omega^2(1+\omega^2\tau_a^2)}{K(1+\omega^2\tau_a^2)} \tag{9}$$

The WGC based PID design method consists of five steps. All steps are applied for a numerical value of the derivative gain ($k_d$). Accordingly, the closed loop characteristic equation of the control system shown in Fig. 1 corresponded to PID controller parameters ($k_p$, $k_i$ and $k_d$) is obtained at first. In second step, the transformation of $s=j\omega$ in the closed loop characteristic equation is done. Where, $j$ is equal to $\sqrt{-1}$ and $\omega$ denotes frequency. In third step, the real and imaginary terms are separated from the characteristic equation and two equations are obtained by equating them to zero. In fourth step, the equation system is solved with respect to frequency for different values of $k_d$, and then the stability region on the $k_p$-$k_i$ plane is obtained. The stabilizing controller parameters regions for different values of $k_d$ are shown in Fig. 2. It can be seen from Fig. 2 that increase in the value of $k_d$ makes the stability region bigger. This means that the number of PID controllers which stabilizing the closed loop control system increases with the increase of derivative gain. Thus, designing a PID controller which satisfies desired performances will be possible within a large set of controller parameters. Here, the main problem is to find the appropriate point or points in the stability region for controller design. WGC based controller design is a suitable approach for finding the appropriate point since the application of the method is simple.

In the last step, WGC point of the stability region of the considered control system is calculated. WGC point is computed by using the points on the stability boundary locus. WGC points for different values of $k_d$ are shown in Fig. 3. This locus enclosing the stability region consists of $n$ points of which the coordinates are named as $(k_{p1}, k_{i1}), (k_{p2}, k_{i2}), \dots, (k_{pn}, k_{in})$ and $(k_{p1}, 0), (k_{p2}, 0), \dots, (k_{pn}, 0)$. Accordingly, $k_p$ and $k_i$ coordinate values of the WGC point are computed by using (10) and (11), respectively (see (Onat, 2013) for the details). Thus, PID controller parameters based on the WGC are obtained as given in Table 1. For the PID controllers given in Table 1, the closed loop responses for unit step change are illustrated in Figure 4.

$$k_p = \frac{1}{n}\sum_{m=1}^{n} k_{pm} \tag{10}$$

$$k_i = \frac{1}{2n}\sum_{m=1}^{n} k_{im} \tag{11}$$

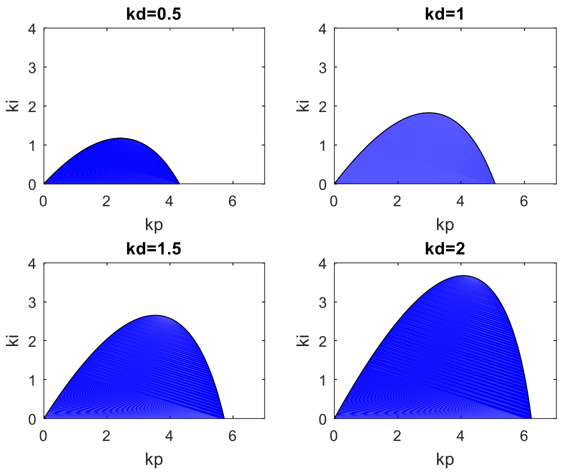


Fig. 2.Stability regions (blue) for different values of $k_d$

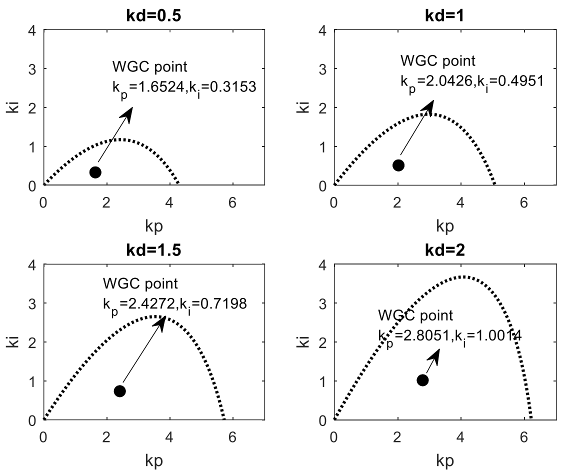


Fig. 3.The WGC points for different values of $k_d$

**Table 1. PID controller parameters based on WGC.**

| $k_p$ | $k_i$ | $k_d$ |
|---|---|---|
| **1.6524** | 0.3153 | **0.5** |
| **2.0426** | 0.4951 | **1.0** |
| **2.4272** | 0.7198 | **1.5** |
| **2.8051** | 1.0014 | **2.0** |

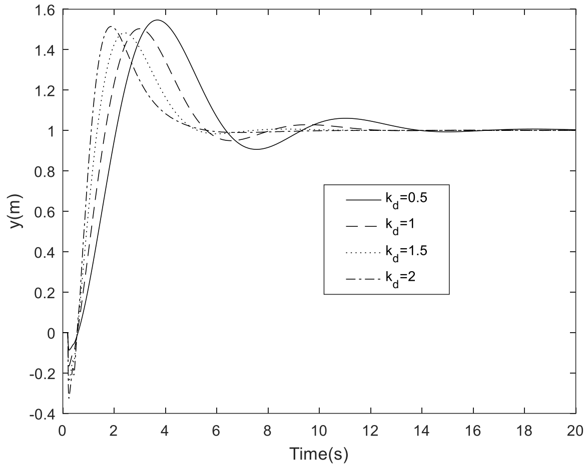


Fig.4. Unit step responses of PID controllers for closed loop system based on WGC points.

## 3. SIMULATION EXAMPLES

### *3.1 Example 1*

Consider the integrating system with time delay and inverse response given in (12). Accordingly, the PID controller parameters are obtained as $k_p$=1.8489, $k_i$=0.1791 for $k_d$=0.5, $k_p$=2.1749, $k_i$=0.2518 for $k_d$=1, $k_p$=2.5124, $k_i$=0.3371 for $k_d$=1.5, and $k_p$=2.8583, $k_i$=0.4375 for $k_d$=2 by using the WGC method. Stability regions and the WGC points for different values of $k_d$ can be seen in Fig. 5. Comparison of unit step responses of the closed loop system based on the WGC points for Example 1 is shown in Fig. 6. Here, it should be noted that parallel form of PID, $C(s) = k_p + \frac{ki}{s} + \frac{Nk_d s}{k_d s + N}$ (where $N$=100), is used to obtain simulation results.

$$G(s) = \frac{0.4(-0.5s+1)e^{-0.1s}}{s(2s+1)} \tag{12}$$

Comparison of unit step responses of Example 1 for different values of PID can be seen in Fig. 7. The simulation results given in Fig. 7 show that the unit step response obtained for the controller designed according to WGC point performs better than the controllers designed using different points in the stability region.

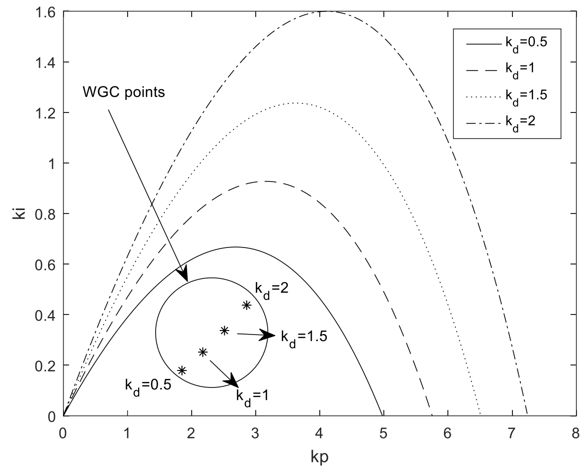


Fig. 5. Stability regions and the WGC points for different values of $k_d$

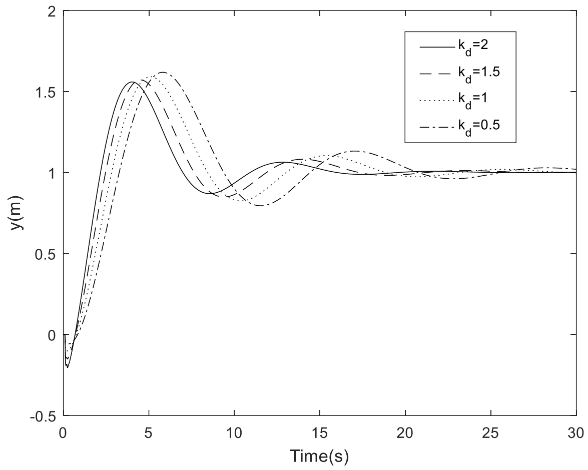


Fig. 6. Comparison of unit step responses of closed loop system based on WGC points for Example 1

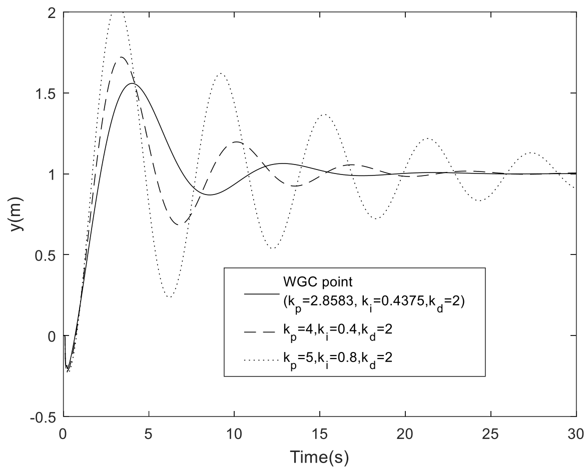


Fig. 7. Comparison of unit step responses of Example 1 for different values of PID

*3.2 Example 2*

Generally, industrial processes have very high order dynamics. In order to show the effectiveness of the WGC method, consider a fourth order integrating process with time delay and inverse response studied by (Pai et al., 2010)

$$G(s) = \frac{0.5(-0.5s+1)e^{-0.7s}}{s(0.4s+1)(0.1s+1)(0.5s+1)} \tag{13}$$

Approximated process model is obtained as in Eq. (14) by (Pai et al., 2010)

$$G(s) = \frac{0.5183(-0.4699s+1)e^{-0.81s}}{s(1.1609s+1)} \tag{14}$$

If WGC tuning procedure is applied for this system, PID controller parameters are obtained as $k_p$=0.9445, $k_i$=0.1429 for $k_d$=1. In addition, the controller parameters for the designs of Luyben and Pai et al are given by $k_p$=0.87, $k_i$=0.0362, $k_d$=0.9744 and $k_p$=1.27, $k_i$=0.2197, $k_d$=1.1811, respectively. The closed loop responses for a set-point change with magnitude of 0.2 are illustrated in Fig. 8. Fig. 8 clearly shows that the PID controller designed with WGC method performs better than Pai method in terms of percent overshoot and also gives a better result when compared with Luyben method in terms of settling time.

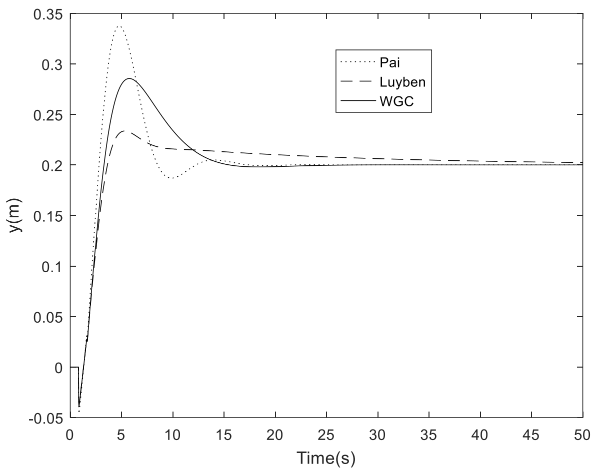


Fig. 8.Comparison of unit step responses of closed loop system for Example 2

*3.3. Example 3*

Consider the following integrating transfer function with time delay and inverse response which includes uncertain parameters.

$$G(s) = \frac{-[0.1,0.3]s+0.2}{s([0.8,1.2]s+1)}e^{-[0.1,0.3]s} = \frac{-[0.1,0.3]s+0.2}{[0.8,1.2]s^2+s}e^{-[0.1,0.3]s} \tag{15}$$

The aim is to design robust PID controller using the stability boundary locus approach and the WGC method. The stability regions (for fixed $k_d$=1) for the following 8 transfer functions and WGC points are shown in Fig. 9. The stability region for

$G_1(s)$ is the largest and the stability region for $G_8(s)$ is the smallest regions including stabilizing parameters. It is interesting that the stability region determined from $G_8(s)$ is the common stability region. This means that the controller designed from this common region will stabilize all the uncertain system. However, for example the PID controller designed for WGC point of $G_1(s)$ will not stabilize all the system such as the systems of $G_5(s)$ and $G_6(s)$. Therefore, it is not a robust controller. On the other hand, a PID controller selected from common region will be a robust controller since it will stabilize the given uncertain system. Unit step responses of 8 transfer functions with the PID controllers designed for the corresponding WGC points are shown in Fig. 10 where the percent overshoot and settling time values are approximately equal to each other.

$$G_1(s) = \frac{-0.1s+0.2}{0.8s^2+s}e^{-0.1s} \tag{16}$$

$$G_2(s) = \frac{-0.1s+0.2}{0.8s^2+s}e^{-0.3s} \tag{17}$$

$$G_3(s) = \frac{-0.1s+0.2}{1.2s^2+s}e^{-0.1s} \tag{18}$$

$$G_4(s) = \frac{-0.1s+0.2}{1.2s^2+s}e^{-0.3s} \tag{19}$$

$$G_5(s) = \frac{-0.3s+0.2}{0.8s^2+s}e^{-0.1s} \tag{20}$$

$$G_6(s) = \frac{-0.3s+0.2}{0.8s^2+s}e^{-0.3s} \tag{21}$$

$$G_7(s) = \frac{-0.3s+0.2}{1.2s^2+s}e^{-0.1s} \tag{22}$$

$$G_8(s) = \frac{-0.3s+0.2}{1.2s^2+s}e^{-0.3s} \tag{23}$$

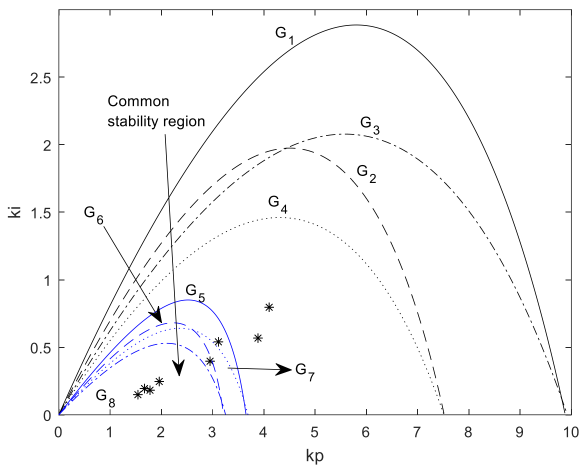


Fig. 9. Stability regions (for fixed $k_d$=1) and WGC points of 8 transfer function for Example 3

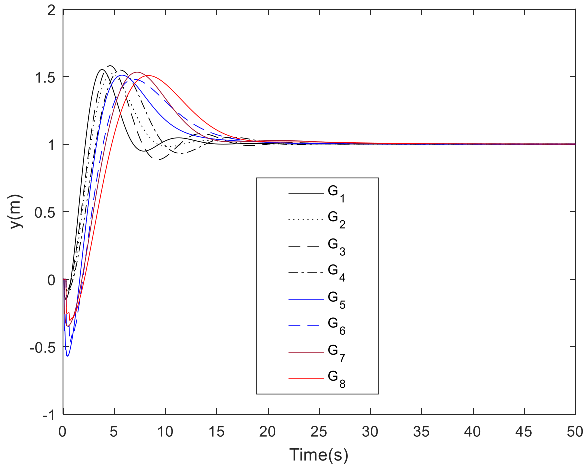


Fig. 10. Step responses of 8 transfer function based on WGC points.

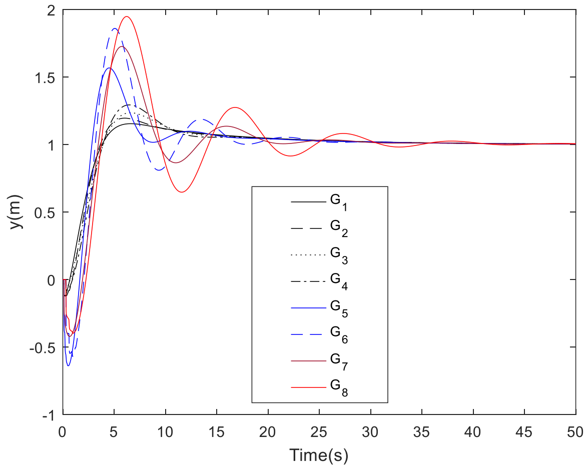


Fig. 11. Step responses of 8 transfer functions for $k_p$=2.3387, $k_i$=0.1618 and $k_d$=1 from the common stability region.

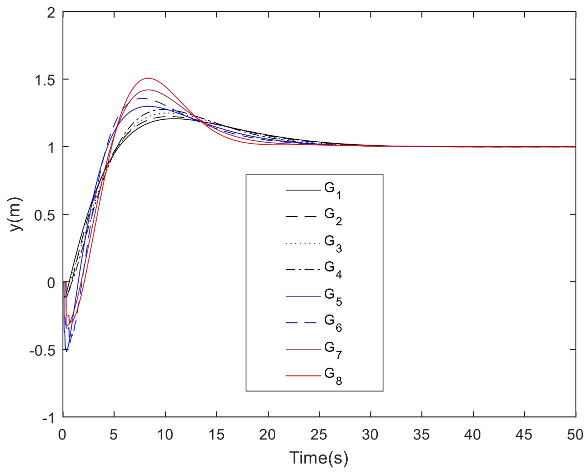


Fig. 12. Step responses of 8 transfer functions according to the WGC point- $k_p$=1.5425, $k_i$=0.1490 and $k_d$=1- from the common stability region.

The unit step responses of 8 transfer functions for $k_p$=2.3387, $k_i$=0.1618, and $k_d$=1 from the common stability region can be seen in Fig. 11. Besides, the unit step responses for the PID controller designed according to the WGC point ($k_p$=1.5425, $k_i$=0.1490, and $k_d$=1) which is in the common stability region are shown in Fig. 12 where it can be seen that the designed controller is robust against the uncertain parameters.

## 4. CONCLUSIONS

In this paper, the WGC method has been used to design PID controllers for integrating systems having time delay and inverse response. The main advantages of the proposed method are that the controller parameters are calculated numerically without using any graphical method or iterative process and the estimated controller parameters from WGC gives appropriate unit step responses. Numerical examples have been provided to show the benefit of the results. Also, it has been shown that the controller designed from WGC point is robust in case of parameter uncertainties. For the designers in the industry, the simplicity of the tuning method is an important advantage.

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