Design of PI Controller using Optimization Method in Fractional Order Control Systems

Tufan Dogruer*, Nusret Tan**

*Gaziosmanpasa University, Department of Electronic and Automation, 60250, Tokat, TURKEY (Tel: 356-256-1212; e-mail: tufan.dogruer@gop.edu.tr).
**Inonu University, Department of Electrical and Electronics, 44280, Malatya, TURKEY (e-mail: nusret.tan@inonu.edu.tr)

Abstract: In control systems, controller preference and design is an important issue for meeting the desired design criteria. In this paper, PI controller design was performed by using optimization method for fractional order systems. First, all the PI controller parameters that make the control system stable are calculated by using the stability boundary locus method. However, each controller parameter selected in the stability region may not be able to optimally control the system. Optimal controller parameters that provide the best control from the PI controller parameters that make the system stable by using the optimization method are obtained. In the optimization process, the optimal PI controller parameters are calculated by using the integral performance criterion based on the error. Simulation studies have been done for closed loop control system including a fractional order transfer function with time delay. It has been shown that the presented method can be successfully applied to fractional order control systems.

Keywords: PI controller, Optimization, Fractional order control system, Stability boundary locus

1. INTRODUCTION

Fractional calculus has become more popular in recent years, although it is a known topic for a long time. The contribution of computer technology that develops in this popularity of fractional calculus is major. Analysis and design using fractional calculus requires long and complicated mathematical operations. Therefore, few studies have been done in the first years. The first application of fractional calculus, first mentioned in 1695, was made by Abel in 1823. First systematic studies have been made in the nineteenth century by Lioville, Riemann and Holmgren (Monje et al., 2010). Fractional calculus, which is used in many scientific fields, started to be used in the field of control science with the study made by Tustin in 1958 (Tustin et al., 1958). In 1961 and 1963, Manabe has applied the fractional order integrator in the control systems (Manabe, 1961, 1963). In the last two decades, the fractional order calculations have been rediscovered by scientists and engineers, and applied gradually in many areas. In the modelling of real systems, fractional order systems show more successful results than integer order systems. Therefore, fractional order systems have become the focus of scientist and fractional order calculus is the basis of many scientific study in today's world.

Nowadays, PID (Proportional-Integral-Derivative) controllers are still the most frequently used controller structures in control loops due to many advantages. The derivative effect is not used many times due to the noise in the control process (Monje et al., 2010). Therefore, PI (Proportional-Integral) controllers are mostly preferred rather than PID controllers. It is reported that most of the PID controllers used in industrial application are PI controllers. PI controllers have two parameters that need to be calculated and they provide good results in most control systems.

Many methods have been developed for determining the parameters of PI controllers. The Ziegler-Nichols method, the Cohen-Coon method, and the Aström-Hagglund method are the most basic methods known (Åström and Hägglund, 2001; Tan et al., 2006b; Zhuang and Atherton, 1993; Ziegler and Nichols, 1942). In addition, the refined Ziegler-Nichols method, gain and phase margin based methods and methods that use integral performance criteria are used. These methods do not always provide good results. Different controller parameters may be available to make the control system response better. Optimization methods have been developed to obtain optimum controller parameters. The purpose of these methods is to obtain the controller parameters that provide the best response. All these tuning methods can give different responses in different control systems. Therefore, it is not true to say that a specific method is the best controller tuning method.

Calculation of controller parameters which make the systems stable is a very important issue and various methods have been developed for this. One of these methods is the stability boundary locus (SBL) method. SBL analysis is a graphical method used to obtain controller parameters that make a closed-loop system stable (Tan, 2005; Tan et al., 2003; Tan et al., 2006a). The aim of this study is to design a PI controller for fractional order control system with a fractional order transfer function and fractional order transfer function may include time delay as well. First, the PI controller parameters that make the systems to be controlled stable are determined by the SBL method. Then, the optimal PI controller parameters are determined by applying the optimization method in the stable region. In the optimization method, the PI controller parameters are determined when the error reaches the minimum value by using the integral performance criteria. In this way, the most available control of the fractional order systems is achieved with PI controller. Also in this study, integer approximation models of fractional order systems are used according to the Oustaloup and Matsuda methods (Krishna, 2011; Oustaloup et al., 2000).

The paper is organized as follows. Section 2 provides information on the stability boundary locus method using PI controller. Section 3 deals with the design of controller, optimization method and implementation of the method. Finally, conclusions are drawn in Section 4.

2. STABILITY BOUNDARY LOCUS ANALYSIS USING PI CONTROLLER

In control systems, it is very important to obtain controller parameters that make the system stable. One of the methods used to obtain these parameters is the SBL method. The SBL method is a graphical method used to determine the controller parameters that make the control system stable.

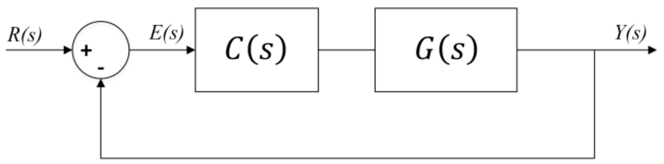

Fig. 1. A SISO control system

For the given SISO (single-input-single-output) control system in Figure 1, the transfer function of the system to be controlled is given in Equation 1. If there is a time delay in the system to be controlled, the exponential term is written as the product of the numerator of Equation 1. The transfer function of the PI controller can be expressed by Equation 2 (Tan et al., 2003; Tan et al., 2006a).

$$G(s)=\frac{N(s)}{D(s)} \tag{1}$$

$$C(s)=k_p+\frac{k_i}{s}=\frac{k_p.s+k_i}{s} \tag{2}$$

The closed loop characteristic polynomial of the system can be written as in Equation 3 (Tan et al., 2003; Tan et al., 2006a).

$$\Delta(s)=s.D(s)+(k_p s+k_i)N(s) \tag{3}$$

If we write the numerator and the denominator polynomials of Equation 1 into their even and odd parts and substituting $s=j\omega$, Equation 4 is obtained (Tan et al., 2003; Tan et al., 2006a).

$$G(s)=\frac{N_e(-\omega^2)+j\omega N_o(-\omega^2)}{D_e(-\omega^2)+j\omega D_o(-\omega^2)} \tag{4}$$

Then the characteristic polynomial can be written as Equation 5.

$$\begin{aligned}\Delta(j\omega)&=[k_i N_e(-\omega^2)-k_p\omega^2 N_o(-\omega^2)-\omega^2 D_o(-\omega^2)]\\&+j[k_p\omega N_e(-\omega^2)+k_i\omega N_o(-\omega^2)+\omega D_e(-\omega^2)]\end{aligned} \tag{5}$$

$$\Delta(j\omega)=R_\Delta+jI_\Delta=0 \tag{6}$$

If the real and imaginary parts of the characteristic polynomial are made to be equal to zero, Equation 7 and 8 are obtained (Tan et al., 2003; Tan et al., 2006a).

$$k_p(-\omega^2 N_o(-\omega^2))+k_i(N_e(-\omega^2))=\omega^2 D_o(-\omega^2) \tag{7}$$

$$k_p(\omega N_e(-\omega^2))+k_i(\omega N_o(-\omega^2))=-\omega D_e(-\omega^2) \tag{8}$$

$$\begin{aligned}&Q(\omega)=-\omega^2 N_o(-\omega^2),\quad R(\omega)=N_e(-\omega^2)\\&S(\omega)=\omega N_e(-\omega^2),\quad U(\omega)=\omega N_o(-\omega^2)\\&X(\omega)=\omega^2 D_o(-\omega^2),\quad Y(\omega)=-\omega D_e(-\omega^2)\end{aligned} \tag{9}$$

Using Equation 9, Equations 7 and 8 can be written as

$$\begin{aligned}&k_p Q(\omega)+k_i R(\omega)=X(\omega)\\&k_p S(\omega)+k_i U(\omega)=Y(\omega)\end{aligned} \tag{10}$$

Solving Equation 10, k_p and k_i can be derived as

$$k_p=\frac{X(\omega)U(\omega)-Y(\omega)R(\omega)}{Q(\omega)U(\omega)-R(\omega)S(\omega)} \tag{11}$$

and

$$k_i=\frac{Y(\omega)Q(\omega)-X(\omega)S(\omega)}{Q(\omega)U(\omega)-R(\omega)S(\omega)} \tag{12}$$

If Equation 9 is substituted into Equations 11 and 12, the following equations are obtained.

$$k_p=\frac{\omega^2 D_o(-\omega^2)N_o(-\omega^2)+D_e(-\omega^2)N_e(-\omega^2)}{-(\omega^2 N_o^{\,2}(-\omega^2)+N_e^{\,2}(-\omega^2))} \tag{13}$$

and

$$k_i=\frac{\omega^2 D_e(-\omega^2)N_o(-\omega^2)-\omega^2 D_o(-\omega^2)N_e(-\omega^2)}{-(\omega^2 N_o^{\,2}(-\omega^2)+N_e^{\,2}(-\omega^2))} \tag{14}$$

By using Equations 13 and 14 the stability boundary locus can be plotted in the (kp, ki)-plane and the locus will divide (kp, ki)-plane into stable and unstable regions. The k_p and k_i parameters that can be selected in the stability region will make the system stable. Optimum controller parameters within the stability region can be obtained by optimization method.

3. CONTROLLER DESIGN USING OPTIMIZATION METHOD

3.1 PI Controllers

Control systems are used in many ways from our daily lives to numerous applications in the industry. The choice of the suitable controller type is crucial to achieve the desired design criteria. In most applications, simple structured controllers are preferred. PID controllers are often preferred by the industry for reasons such as simple structure and robust performance characteristics. PID controllers are more than 90% of the controller structures used in the industry (Monje et al., 2010). The derivative component of the PID controller is not used many times because it causes measurement noise in the control process. In these processes, PI controllers are preferred rather than PID controllers.

3.2 Optimization Method

The block diagram used to calculate PI controller parameters is given in Figure 2.

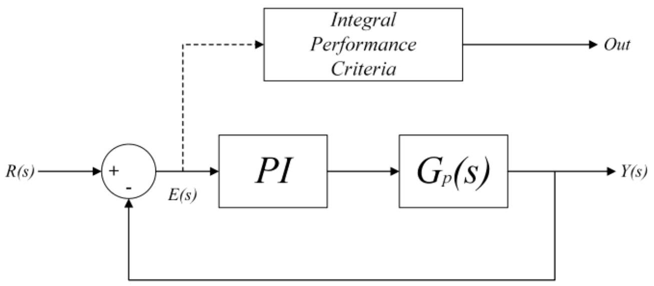

Fig. 2. Block diagram of Simulink model

E(s) denotes the error, is defined as the difference between the input signal and the output signal, and is expressed by Equation 15 (Atherton, 2009). Where *r(t)* is the input signal, and *y(t)* is the output signal.

$$e(t) = r(t) - y(t) \tag{15}$$

Based on the error in the control system, integral performance criteria have been developed to determine the suitable controller parameters. The IAE is expressed as the integral absolute error and is denoted by Equation 16. ISE is the integral squared error and is expressed by Equation 17. The ITAE criterion, shown by Equation 18, is the integral of the time-weighted absolute of the error. Finally, the ITSE criterion is the integral of time-weighted squared error and is calculated by Equation 19 (Atherton, 2009; Tavazoei, 2010). In equations, *e(t)* refers to the error that occur in the control system and t is time.

$$IAE = \int_0^\infty |e(\mathrm{t})|.dt \tag{16}$$

$$ISE = \int_0^\infty e^2(\mathrm{t}).dt \tag{17}$$

$$ITAE = \int_0^\infty t.|e(\mathrm{t})|.dt \tag{18}$$

$$ITSE = \int_0^\infty t.e^2(\mathrm{t}).dt \tag{19}$$

The optimization process can be defined as selecting the most appropriate one from the current situations. Simulink models have been developed for optimizations based on the integral performance criteria. The optimization process begins by entering initial values in controller parameters. When the smallest error value is reached, the optimization stops and the most suitable controller parameters are obtained. In the model given in Figure 2, fractional order plant is transformed to integer order transfer function by using integer approximation methods.

3.3 Implementation of the Method

Example 1 Consider the fractional order transfer function with time delay in the control system given in Figure 1 as follows.

$$G_{p1}(s) = \frac{1}{s^{1.5}+1}e^{-s} \tag{20}$$

The unit step response of the closed loop system for the transfer function given in Equation 20 is given in Figure 3. If the figure is examined, it is seen that the output signal does not follow the input signal and the steady state error is too great.

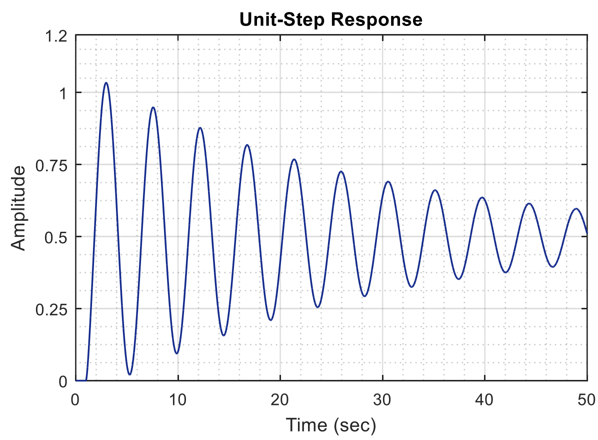

Fig. 3. Unit step response of the closed loop system with *C(s)*= 1

The equations for the k_p and k_i parameters obtained by the SBL method for the transfer function in Equation 20 are given in Equations 21 and 22.

$$k_p = 0.707\omega^{1.5} sin(\omega) + cos(\omega)(0.707\omega^{1.5} - 1) \tag{21}$$

and

$$k_i = 0.707\omega^{2.5} cos(\omega) + sin(\omega)(\omega - 0.707\omega^{2.5}) \tag{22}$$

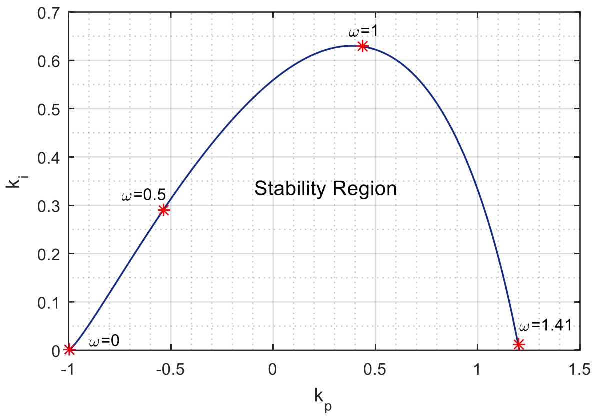

Fig. 4. Stability boundary locus and stability region for $G_{p1}(s)$

The stability boundary locus and stability regions obtained for the stable and unstable values of k_p and k_i are given in Figure 4 ($\omega \in [0,1.42]$).

A Simulink model was constructed using the Oustaloup 5th order integer approximation model for fractional order system with time delay. In the Simulink model, various integral performance criterions were used to minimize the error. The model based on IAE is shown in Figure 5.

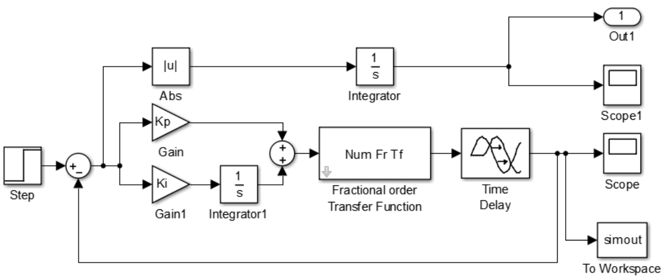

Fig. 5. Simulink model based on IAE performance index

The controller parameters are obtained by running the optimization Simulink block diagram. The PI controller parameters are given in Table 1 for the design performed using integral performance criteria.

Table 1. PI controller parameters for $G_{p1}(s)$

	IAE	ISE	ITAE	ITSE
k_p	0.246	0.402	0.47	0.258
k_i	0.303	0.321	0.25	0.311

The points where the obtained PI controller parameters are placed in the stability region are shown in Figure 6. It is seen that PI controller parameters are placed very close to each other for different integral performance criteria. In Figure 6, a geometric shape like a triangle since the points for ITSE and IAE are very close to each other is obtained by combining these points with lines.

The PI controllers are obtained by substituting the parameters in Tables 1 in Equation 2. The unit-step responses in Figure 7 are obtained by applying the PI controllers given in Table 1.

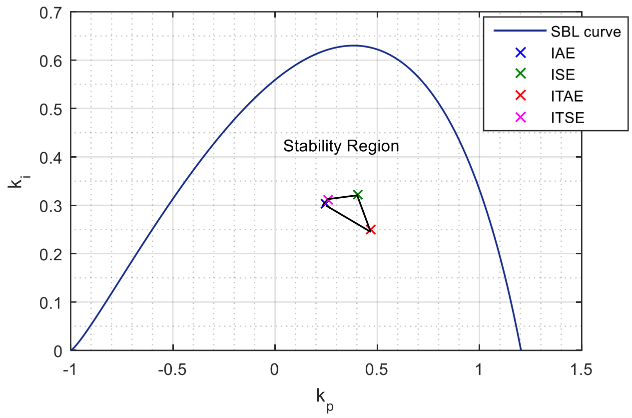

Fig. 6. Stability region with optimal PI controller parameters

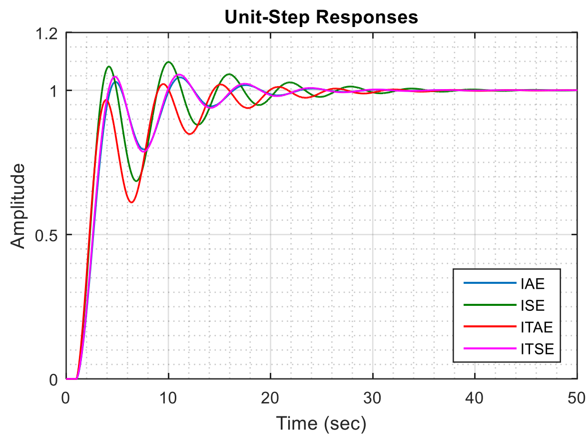

Fig. 7. Unit step responses of the systems with PI controller for different integral performance criteria

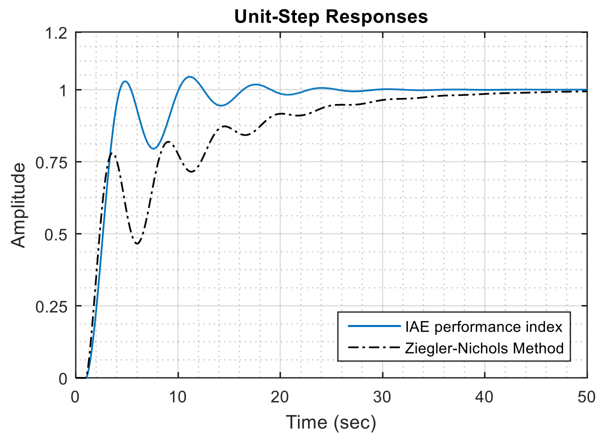

Fig. 8. Comparison of Ziegler-Nichols and Optimization method

Figure 8 shows the unit step responses obtained by applying the PI controller parameters calculated by the Ziegler Nichols method and the optimization method. It is seen that the controller obtained by the optimization method controls the system better.

The time parameters and percent overshoot values for unit step responses given in Figure 7 are given in Table 2.

Table 2. Performance characteristics for $G_{p1}(s)$

	IAE	ISE	ITAE	ITSE
Rise t.	2.20	1.75	1.90	2.12
Settling t.	15.75	25.31	24.22	20.84
Peak t.	11.15	10.02	9.50	11.06
Overshoot	4.45	9.82	2.15	5.42

Example 2 Consider the fractional order transfer function in the control system given in Figure 1 as follows.

$$G_{p2}(s) = \frac{1}{s^{0.1}(s+1)^3} \tag{23}$$

The unit step response of the closed loop system for the transfer function given in Equation 23 is given in Figure 9.

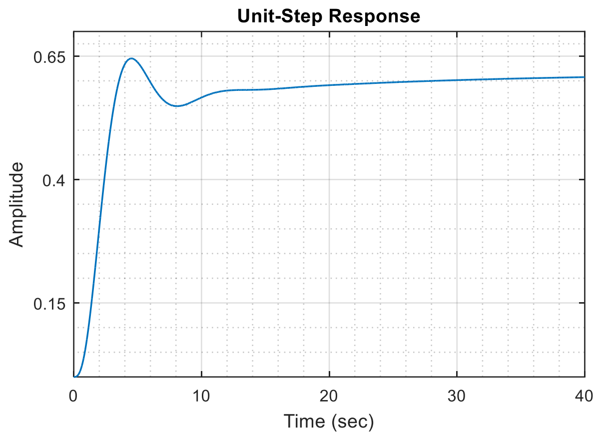

Fig. 9. Unit step response of the closed loop system with $C(s)$= 1

For the transfer function in Equation 23, the k_p and k_i obtained according to the SBL method are given in Equations 24 and 25, respectively.

$$k_p = (2.961\omega^{2.1} - 0.987\omega^{0.1} - 0.156\omega^{3.1} + 0.468\omega^{1.1}) \tag{24}$$

and

$$k_i = (-0.987\omega^{4.1} + 2.961\omega^{2.1} - 0.468\omega^{3.1} + 0.156\omega^{1.1}) \tag{25}$$

The stability boundary locus and stability regions obtained for the stable and unstable values of k_p and k_i are given in Figure 10 ($\omega \in [0, 1.55]$).

Figure 11 shows the points where the obtained PI controller parameters are placed in the stability region. It is seen that the obtained PI controller parameters are very close to each other. The unit step responses in Figure 12 are obtained by applying the PI controllers given in Table 3. If the Figure 12 is to be examined, it is seen that the smallest percent overshoot value is realized with the ITAE criterion. The unit step response with the shortest settling time is provided by the IAE criterion. The unit step responses with fastest rise time and peak time are provided by the ISE criterion.

Table 3. PI controller parameters for $G_{p2}(s)$

	IAE	ISE	ITAE	ITSE
k_p	1.509	2.325	1.351	1.556
k_i	0.322	0.336	0.308	0.355

The PI controller parameters are given in Table 3 for the design performed using integral performance criteria.

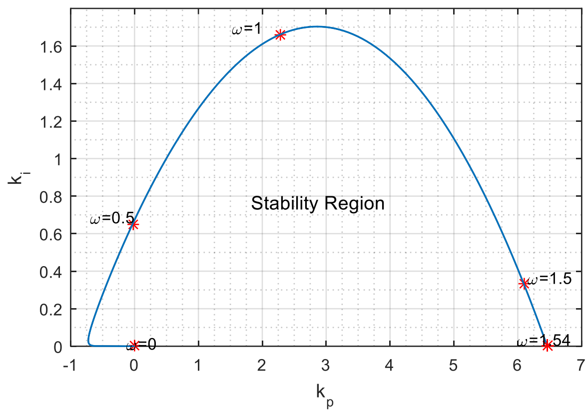

Fig. 10. Stability boundary locus and stability region for $G_{p2}(s)$

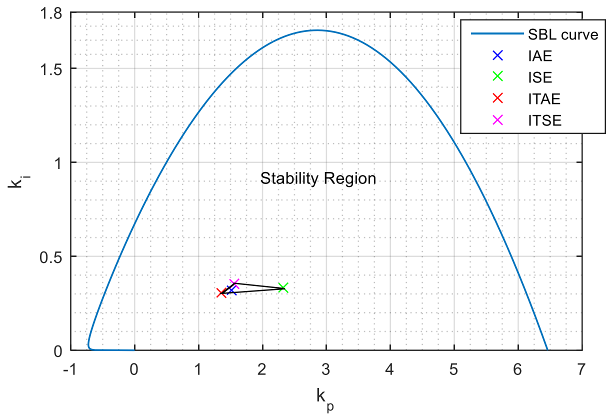

Fig. 11. Stability region with designed PI controllers

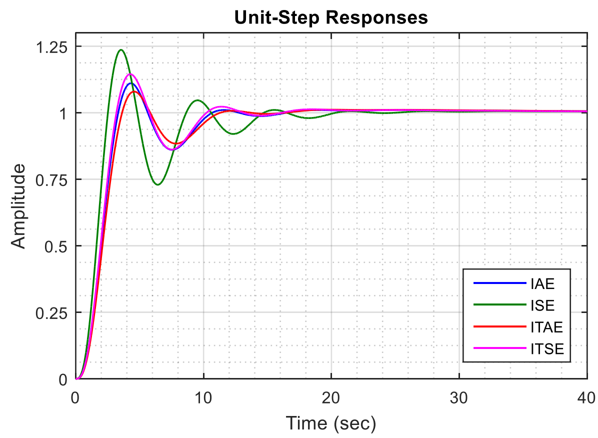

Fig. 12. Unit step responses of the systems with PI controller for different integral performance criteria

Table 4. Performance characteristics for *$G_{p2}(s)$*

	IAE	ISE	ITAE	ITSE
Rise t.	2.03	1.51	2.21	1.94
Settling t.	10.23	19.07	10.70	9.90
Peak t.	4.31	3.54	4.56	4.27
Overshoot	10.46	23.03	7.33	13.93

The time parameters and percent overshoot values for unit step responses given in Figure 12 are given in Table 4.

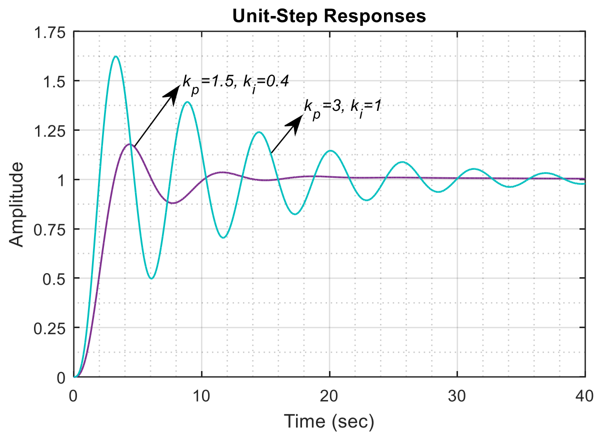

Fig. 13. Unit step responses of the systems with PI controller for two different points in stability region

The unit step responses are shown in Figure 13 for two different points selected in Figure 11 in the stability region. The system is stable for the both controllers designed using the parameters of selected points. In Figure 13, it is clear that the PI parameters close to the specified points for the integral performance criteria provide better control than the other.

4. CONCLUSIONS

In this paper, a method based on the stability boundary locus approach has been presented to design PI controllers for a closed loop control system including a fractional order transfer function. Optimization of parameters of PI controller which minimize the error signal in the closed loop system has been done over stability region. Two examples have been provided for demonstration of the application of the presented method. The first example that is simulated is the PI controller design for a fractional order system with time delay. Although there are many controller parameters that make the system stable in the simulated system, it has been seen that the controller parameters determined by the optimization have performed better control. In addition, the unit step responses of the system are compared for PI controller parameters obtained by Ziegler Nichols and the optimization method. It is clear that the optimization method is more successful. The best settling time for the first example was achieved with the IAE criterion, while the smallest percent overshoot value was achieved with the ITAE criterion. In the second example, a fractional order system was controlled with a PI controller. The optimization method was applied after the computation of stability region in the (k_p, k_i)-plane determined by the SBL method. Control of the system with the PI controller has been successfully accomplished.

ACKNOWLEDGMENT

This work is supported by the Scientific and Research Council of Turkey (TÜBİTAK) under Grant no. EEEAG 115E388.

REFERENCES

Åström, K. J., and Hägglund, T. (2001). The future of PID control. *Control engineering practice* **9**(11), 1163-1175.
Atherton, D. (2009). Control engineering, Bookboon.
Krishna, B. T. (2011). Studies on fractional order differentiators and integrators: A survey. *Signal Processing* **91**(3), 386-426.
Manabe, S. (1961). The noninteger integral and its application to control systems. *English Translation Journal Japan* **6**(83-87.
Manabe, S. (1963). The system design by the use of a model consisting of a saturation and noninteger integral. *English Translation Journal Japan*, 47-150.
Monje, C. A., Chen, Y., Vinagre, B. M., Xue, D., and Feliu-Batlle, V. (2010). Fractional-order systems and controls: fundamentals and applications, Springer Science & Business Media.
Oustaloup, A., Levron, F., Mathieu, B., and Nanot, F. M. (2000). Frequency-band complex noninteger differentiator: characterization and synthesis. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* **47**(1), 25-39.
Tan, N. (2005). Computation of stabilizing PI and PID controllers for processes with time delay. *ISA transactions* **44**(2), 213-223.
Tan, N., Kaya, I., and Atherton, D. P. (2003). Computation of stabilizing PI and PID controllers. *In* "Proceedings of 2003 IEEE Conference on Control Applications, 2003. CCA 2003.", Vol. 2, pp. 876-881 vol.2.
Tan, N., Kaya, I., Yeroglu, C., and Atherton, D. P. (2006a). Computation of stabilizing PI and PID controllers using the stability boundary locus. *Energy Conversion and Management* **47**(18), 3045-3058.
Tan, W., Liu, J., Chen, T., and Marquez, H. J. (2006b). Comparison of some well-known PID tuning formulas. *Computers & Chemical Engineering* **30**(9), 1416-1423.
Tavazoei, M. S. (2010). Notes on integral performance indices in fractional-order control systems. *Journal of Process Control* **20**(3), 285-291.
Tustin, A., Allanson, J., Layton, J., and Jakeways, R. (1958). The design of systems for automatic control of the position of massive objects. *Proceedings of the IEE-Part C: Monographs* **105**(1S), 1-57.
Zhuang, M., and Atherton, D. (1993). Automatic tuning of optimum PID controllers. *In* "IEE Proceedings D (Control Theory and Applications)", Vol. 140, pp. 216-224. IET.
Ziegler, J. G., and Nichols, N. B. (1942). Optimum settings for automatic controllers. *trans. ASME* **64**(11).