Tuning for Fractional Order PID Controller based on Probabilistic Robustness

Zhenlong Wu*, Donghai Li*, Yali Xue*, Ting He*, Song Zheng.**

State Key Lab of Power System, Department of Energy and Power Engineering, Tsinghua University, Beijing, China. 100084 (Corresponding Author, **Donghai Li.; e-mail: lidongh@mail.tsinghua.edu.cn).*
***Department of Automation, Fuzhou University, Fuzhou, China, 350000.*

Abstract: To improve the control performance and robustness of fractional order PID (FOPID) controller for the uncertainty model, a tuning method for FOPID controller based on probabilistic robustness is proposed in this paper. Based on the Monte Carlo simulation, a probabilistic robustness index is formulated to represent the controller sensitivity to the uncertainty model. Stability boundaries of FOPID is depicted to provide the search space, in which the optimal group of parameters are selected based on the probabilistic robustness index. The procedure of the proposed method is designed to obtain the optimal controller parameters for the uncertainty model. Numerical examples are performed to verify the efficacy of the proposed method, and simulation results show that the proposed method has better performance, stronger robustness and ability of handling uncertainties.

Keywords: Factional order PID controller, Probabilistic robustness, Stability boundaries, Uncertainty model, Monte Carlo simulation.

1. INTRODUCTION

Fractional order systems (FOS) have attracted many attentions in the past decades, because the fractional calculus can descript the real-word phenomena more rigorously. FOS are often obtained by system identification approaches and show more dynamic information of a complicated system than integer-order systems, such as heat solid model (Petráš et al. 2002), gas turbine (Nataraj et al. 2010), perturbed pressurized heavy water reactor (Lamba et al. 2017) and lead-acid battery (Sabatier et al. 2010).

Fractional order proportional–integral–derivative (FOPID) controller is naturally suitable to control FOS, which is considered as a generalization of the classical PID controller. FOPID has more parameters to tune which means greater flexibility. Many tuning methods have been proposed to design FOPID. Multi-objective iterative optimization algorithms and other iterative optimization algorithms were used to optimize the parameters of FOPID (Zamani et al. 2017; Wu et al. 2016), one or more evaluation indexes for the normal model (NM) are defined and then used for parameter optimization. Internal model control (IMC) is also developed for FOPID (Bettayeb et al. 2014), and the design procedure of IMC-FOPID is similar to the classical PID. Other FOPID design procedures are proposed to meet the control specifications such as phase margin specification, gain crossover frequency, et al. (Badri et al. 2013; Luo et al. 2010). Other modified FOPID is also proposed to enhance the control performance such as fuzzy FOPID, adaptive FOPID, two-degree of freedom FOPID controllers (Sharma et al. 2014; Delavari et al. 2012). These modified FOPID controllers are all designed based on the NM and then checked whether the robustness constraint is satisfied.

Randomized algorithms for the uncertain systems control has been studied for decades and a lot of achievements have been obtained (Calafiore et al. 2014). As one of randomized algorithms, probabilistic robustness is a practical and powerful tool to design the controller and analyse the robustness considering the uncertainty model, so a tuning method based on probabilistic robustness for PID controller was developed which could satisfy the control requirements such as overshoot, setting time and showed an obvious advantage in the robustness (Wang et al. 2011). To the best of authors' known, the tuning method based on probabilistic robustness for FOPID has not been reported. So a new tuning method for FOPID controller based on probabilistic robustness is proposed in this paper, which combines the NM and the uncertainty model during the tuning process. The NM could offer stability boundaries of FOPID and the parameter optimization is developed based on the uncertainty model.

This paper comprises of 6 sections and the rest of the paper is organized as follows. Section 2 formulates the description of the uncertainty model and the realization of FOPID. The stability boundaries of FOPID parameters are analyzed in section 3. The tuning procedure based on probabilistic robustness is designed for FOPID in section 4. In section 5, two simulation examples illustrate the effectiveness of the proposed method. Finally, section 6 offers concluding remarks.

2. PROBLEM FORMULATION

2.1 Parameter Uncertainties for FOS

Much simplification for modelling and the nonlinearity of the real-word phenomena are inevitable exists for FOS, which is called uncertainties of FOS. Uncertainties of FOS could be considered as the parameter perturbation in a large space when dynamic characteristics are far from the nominal working condition. And this problem formulation could be depicted as following.

$$G_p(s)=\frac{b_1 s^{\gamma_1}+b_2 s^{\gamma_2}+\cdots+b_m s^{\gamma_m}}{a_1 s^{\eta_1}+a_2 s^{\eta_2}+\cdots+a_n s^{\eta_n}} \tag{1}$$

where $a_i(i=1,2,\cdots n)$, $\eta_i(i=1,2,\cdots n)$, $b_k(k=1,2,\cdots m)$ and $\gamma_k(k=1,2,\cdots m)$ are the coefficients and the order of denominator, the coefficients and the order of numerator, respectively. It should be noted that the highest order of denominator is no smaller than the highest order of numerator, and the order of denominator and numerator can be nonnegative integer or non-integer. Due to the existence of parameter uncertainties for FOS, define $\boldsymbol{q}=\{a_i,\eta_i,b_k,\gamma_k\}$ as the random vector of parameter uncertainties throughout the parameter space Q according to the probability density function of *pr*.

The aforementioned controller design methods for FOPID in section 1 are based on the NM and can have good control performance for the nominal working condition. However, the control performance would deteriorate when the model varies far from the nominal working condition even though FOS is designed with robustness constraints. So a tuning method considering the parameter uncertainties of FOPID is proposed to design FOPID and still has satisfactory control performance even the model varies far from the nominal working condition.

2.2 The Realization of FOPID

The classical PID controller is the most widely used controller currently, which is described in the form

$$G_{PID}(s)=K_p+\frac{K_i}{s}+K_d s \tag{2}$$

FOPID controller is a generalization of the classical PID controller and has a similar structure with the classical PID controller which can be described as

$$G_c(s)=k_p+\frac{k_i}{s^{\lambda}}+k_d s^{\mu} \tag{3}$$

where λ and μ can take any values within the range $(0,2)$.

Considering the hardware implementations difficulties of fractional order calculus operator s^{α} in MATLAB & Simulink, an integer order transfer function is proposed to approximate it by using the recursive distribution of poles and zeros in a particular frequency band by Oustaloup (Oustaloup et al. 2000). In this paper, an improved Oustaloup filter introduced by Xue is used (Xue et al. 2006), which is accurate enough for the order between 0 and 2. The approximate transfer function can be depicted by the following equation:

$$s^{\alpha}=s^{\alpha}_{[\omega_b,\omega_h]}=\left(\frac{p\omega_h}{b}\right)^{\alpha}\left(\frac{ps^2+b\omega_h s}{p(1-\alpha)s^2+b\omega_h s+p\alpha}\right)\prod_{k=1}^{M}\frac{s+\omega_k}{s+\omega_k'} \tag{4}$$

where $[\omega_b,\omega_h]$ is the fitting frequency band, b and p are adjustable parameters and the valve of them is 10 and 9 respectively. M is the order of approximation, zeroes ω_k and poles ω_k' are described as

$$\begin{cases}\omega_k'=\omega_b\left(\dfrac{\omega_h}{\omega_b}\right)^{(2k-1-\alpha)/M}\\ \omega_k=\omega_b\left(\dfrac{\omega_h}{\omega_b}\right)^{(2k-1+\alpha)/M}\end{cases} \tag{5}$$

3. THE STABILITY BOUNDARIES OF FOPID

To simplify the stability boundaries analysis, the NM of FOS is represented as

$$G_p(j\omega)=r(\omega)e^{j\vartheta(\omega)}=a(\omega)+jb(\omega) \tag{6}$$

And it should be noted that fractional order operator s^{α} is depicted by the mathematical identity:

$$j^{\lambda}=\cos\left(\frac{\pi}{2}\lambda\right)+j\sin\left(\frac{\pi}{2}\lambda\right) \tag{7}$$

$$(j\omega)^{\lambda}=\cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+j\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda} \tag{8}$$

The closed loop transfer function of the whole system combining the NM and FOPID is obtained

$$G_{cl}(s)=\frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} \tag{9}$$

Hence, the characteristic equation of the whole system becomes

$$W(s;k_p,k_i,k_d,\lambda,\mu)=1+G_c(s)G_p(s)=0 \tag{10}$$

Substitute (3) and (6) into (10) and separate the real and imaginary parts, the functions of stability boundaries can be obtained

$$\begin{cases}\cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+\left(k_p\cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+k_i+k_d\cos\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}\right)\\ a(\omega)-\left(k_p\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+k_d\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}\right)b(\omega)=0 \quad (11\text{-}1)\\ \sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+\left(k_p\cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+k_i+k_d\cos\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}\right)\\ b(\omega)+\left(k_p\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+k_d\sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}\right)a(\omega)=0 \quad (11\text{-}2)\end{cases} \tag{11}$$

When ω, k_d, λ and μ are fixed, the stability boundaries of FOPID are obtained by solving (11) and repeating the calculation for a set of k_d-values, λ-values and μ-values gives the whole stability boundaries.

The expressions of k_p and k_i can be obtained by solving (11) as following

$$k_p = -\frac{1}{\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}}\left[\begin{array}{l} k_d \sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu} \\ +\dfrac{b(\omega)\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+a(\omega)\cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}}{a^2(\omega)+b^2(\omega)} \end{array}\right] \quad (12)$$

$$k_i = \left(-\sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}\text{-}\left(k_p \sin\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda}+k_d \sin\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}\right)a(\omega)\right) \quad (13)$$
$$/\, b(\omega) - k_p \cos\left(\frac{\pi}{2}\lambda\right)\omega^{\lambda} - k_d \cos\left(\frac{\pi}{2}(\lambda+\mu)\right)\omega^{\lambda+\mu}$$

Now the procedure of the stability boundaries calculation is summarized as follows:

i) The NM and the value of k_d, λ and μ are fixed.

ii) The stability boundaries of FOPID can be obtained by solving the expressions (12) and (13) with ω varies from $-\infty$ to $+\infty$.

iii) The whole stability boundaries are obtained by repeating the calculation for a set of k_d-values, λ-values and μ-values.

4. THE TUNING METHOD FOR FOPID BASED ON PROBABILISTIC ROBUSTNESS

Define parameters of FOPID controller as $d = \{k_p, k_i, k_d, \lambda, \mu\}$ which should be located in the stability boundaries in (12) and (13). To measure the design requirements on stability and performance quantitatively, a binary indicator function I_i is defined as following

$$I_i = \begin{cases} 0 & \text{design requirements are not satisfied} \\ 1 & \text{design requirements are satisfied} \end{cases} \quad (14)$$

When parameters *d* are fixed and the uncertainties of FOS are defined, the closed system performance can be evaluated by examining whether the system satisfies the design requirements or not. The probability *P* that FOPID controller satisfies design requirements can be described as the integral of the binary indicator function in the whole parameter space

$$P_i(d) = \int_Q I\left[G_p(\boldsymbol{q}), G_c(d)\right] p_r(\boldsymbol{q}) d\boldsymbol{q} \quad (15)$$

By using this probabilistic framework in (15), control indices can be examined. In this paper, the settling time t_s and the overshoot σ are chosen as control indices which are conflicted and can weigh the control performance well. Now the probabilistic robustness index is defined as

$$J(d) = fcn\left(P_1(d), P_2(d), \cdots\right) \quad (16)$$

where *fcn* is defined as the weights for each binary indicator.

Considering the integral in (15) is difficult to calculate analytically in most cases, a method to obtain the estimate of probability *P* and the probabilistic robustness index *J* based on Monte Carlo simulation is depicted as following

$$\hat{P}(\boldsymbol{d}) = \frac{1}{N}\sum_{k=1}^{N} I\left[G_p(\boldsymbol{q}), G_c(d)\right] \quad (17)$$

$$\hat{J}(d) = fcn\left(\hat{P}_1(d), \hat{P}_2(d), \cdots\right) \quad (18)$$

The estimate $\hat{P}$ and $\hat{J}$ can approach respectively to the real probability *P* and *J* when $N \to \infty$. However, the value of *N* cannot be infinity in practice and a finite *N* results in estimation errors. A minimum *N* can be calculated based on Massart Inequality which guarantees a certain confidence level to a risk parameter (Chen et al. 2003)

$$N > \frac{2\left(1-\varepsilon+\frac{\theta\varepsilon}{3}\right)\left(1-\frac{\theta}{3}\right)\ln\frac{2}{\delta}}{\theta^2\varepsilon} \quad (19)$$

where ε denotes the given risk parameter, the confidence level is defined as $1-\delta$ and $\theta \in (0,1)$. Such a sample size can ensure $P_r\{|P_x - K/N| < \theta\varepsilon\} > 1-\delta$ with $P_x = 1-\varepsilon$. *K/N* is the estimated probability, *K* is the value of the design requirements satisfaction in *N* samples and the confidence interval is $[K/N-\theta\varepsilon,\ K/N+\theta\varepsilon]$.

The goal of tuning FOPID controller based on probabilistic robustness is to find the optimal controller parameters d^* that obtains the maximum value of $J(d^*)$ with the parameter uncertainties throughout the parameter space. Considering the difficulty of the non-convex for the parameters optimization, genetic algorithms (GA) are adopted to optimize the parameters of FOPID controller which has good global convergence ability and a high computational efficiency (Haupt et al. 2004).

It should be noted that the minimum value of *N* for Monte Carlo simulation is still too large for GA when the parameters of Massart Inequality are set. So a small value of *N* is set for the parameters optimization then a big enough value of *N* determined by (19) is used to test the probability of design requirements.

5. NUMERICAL EXAMPLES

In this section, FOPID controllers tuned by the proposed method are applied to two FOS with actual physical meaning. The definition of the probabilistic robustness index is varied based on design requirements, which can be weight coefficient, linear function and nonlinear function, etc. Considering the conflicts between the settling time and overshoot, a linear function with weight coefficient is defined in this paper

$$J(d) = 0.8P_{ts} + 0.2P_{\sigma} \quad (20)$$

where P_{ts} and P_{σ} denote the binary indicator functions of the closed system satisfies the design requirements of the setting time t_s and overshoot σ, respectively. It should be noted that the setting time t_s is decided by the desired dynamic characteristics of the model and the control performance, and the selection of the setting time is a comprehensive result.

Here are some parameter settings: The number of Monte Carlo simulation for each individual, the number of initial population and the maximum number of evolutionary iterations are set 500, 100 and 30, respectively. The parameters of Massart Inequality are set ε=0.01, δ=0.01 and θ=0.2, the value of N is selected as 24495.

5.1 Pressurized Heavy Water Reactor Model

A pressurized heavy water reactor (PHWR) model is identified by Das (Das et al. 2011). The parameter space Q is set by covering ± 50% around the nominated value of the coefficients and ± 20% around the order of PHWR model. This model with parameter uncertainties are depicted as

$$G_{p1}(s)=\frac{N_1(s)}{D_1(s)}=\frac{b_1}{s^{\eta_1}+a_2 s^{\eta_2}+a_3} \qquad (21)$$

where $b_1 \in [761.44735, 2284.3420]$, $a_2 \in [4.0972, 12.2916]$, $a_3 \in [3.8842, 11.6526]$, $\eta_1 \in [1.4680, 2.7262]$, $\eta_2 \in [0.7025, 1.3047]$. The parameters of the NM of the PHWR model are b_1=1522.89468, a_2=8.1944, a_3=7.7684, η_1= 2.0971 and η_2=1.0036.

The design requirements are set $t_s < 7s$, $\sigma < 5\%$ and the parameters of FOPID controller can be tuned by the proposed method. The proposed FOPID controller (PRFOPID) is obtained

$$G_{c1}(s)=0.1997+\frac{0.1671}{s^{0.5841}}+0.1170 s^{1.6519} \qquad (22)$$

The response of PRFOPID is compared to interval fractional-order proportional integral derivative (INFOPID) controller (Lamba et al. 2017), fractional-order internal model proportional integral derivative (FOIMCPID) controller (Sagar et al. 2016) and fractional-order proportional integral (FOPI) controller (Bhase et al. 2014). Note that these parameters of those controllers are tuned well by the researchers in relevant references and they are listed in Table 1.

Table 1. Parameters of different controllers for the PHWR model

Controllers	Tuning parameters
INFOPID	k_p=0.011, k_i=0.03, k_d=0.05, λ=0.9, μ=0.95
FOIMCPID	k_p=0.0108, k_i=0.0102, k_d=0.0013, λ=1.0036, μ=1.0935
FOPI	k_p=0.0016323, k_i=0.001506, λ=1.004

The step response for the NM of PHWR is shown in Fig. 1, and PRFOPID and INFOPID have the fastest response speed than other controllers while PRFOPID has a smaller overshoot than INFOPID for the NM of PHWR.

Monte Carlo simulation is carried out for N times with the uncertainty model in the whole parameter space, the indices of the integral of time multiply absolute error (ITAE), the integral of squared error (ISE), the integral of absolute error (IAE), t_s and σ are all recorded in the whole 20 s. The results of t_s and σ for each perturbed PHWR model with different controllers are shown in Fig. 2 and the ranges of ITAE, ISE and IAE are listed in Table 2.

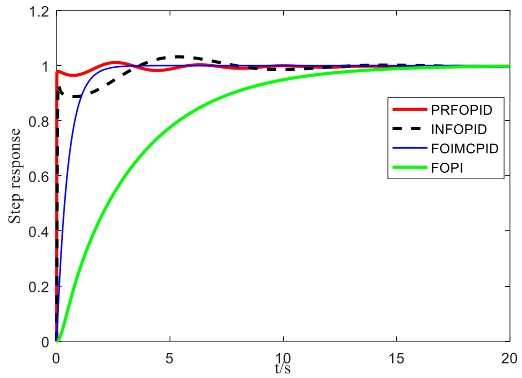

Fig. 1. The step response for the NM of PHWR with different controllers.

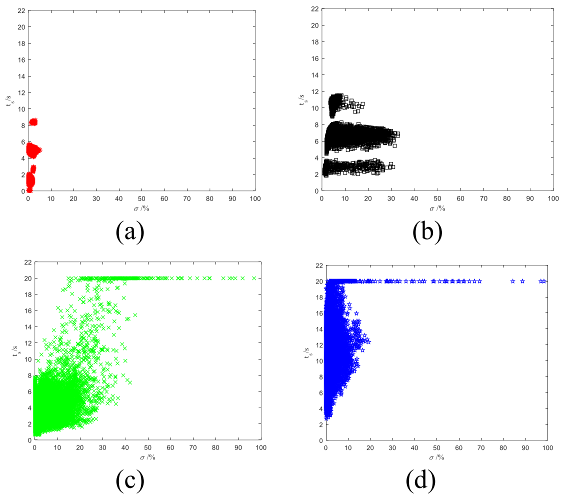

Fig. 2. The records for each perturbed PHWR model with different controllers (*(a): PRFOPID, (b): INFOPID, (c): FOIMCPID, (d): FOPI*).

Table 2. The ranges of ITAE, ISE and IAE for each perturbed PHWR model with different controllers

	ITAE	ISE	IAE
PRFOPID	[0.2613, 2.1711]	[0.0053, 0.0204]	[0.0498, 0.3656]
INFOPID	[0.4814, 3.3611]	[0.0109, 0.2370]	[0.1472, 1.0441]
FOIMCPID	[0.0729, 592.7868]	[0.0899, 173.6517]	[0.2223, 41.7039]
FOPI	[1.2069, 431.3260]	[0.6808, 100.9813]	[1.1609, 29.5754]

Smaller indices suggest better performance while denser distribution denotes stronger robustness. Besides, the smaller ranges of ITAE, ISE and IAE suggest stronger ability of handling uncertainties. It is obvious that PRFOPID has the superiority in all performance, robustness and ability of handling uncertainties. It should be noted that FOIMCPID and FOPI may result in non-convergence for some perturbed

PHWR models because the settling time is no smaller than the whole 20 s.

K/N which is the estimated value of the probability is also calculated, and the value of *K/N* is 0.9972, 0.6699, 0.9096 and 0.2305, respectively. PRFOPID has the largest estimated value of the probability for perturbed PHWR models, which means PRFOPID has most probability to obtain the design requirements for the uncertainties PHWR model throughout the whole parameter space Q.

5.2 Heating Furnace Model

A heating furnace (HF) model is identified based on a real experimental heating furnace, the parameter space Q is set by taking ± 40% lower and upper bound uncertainties for the coefficients and ± 10% lower and upper bound uncertainties for the order of HF model. This model with parameter uncertainties are depicted as

$$G_{p2}(s)=\frac{N_2(s)}{D_2(s)}=\frac{1}{a_1 s^{\eta_1}+a_2 s^{\eta_2}+a_3} \tag{23}$$

where $a_1 \in [8996.58, 20992.02]$, $a_2 \in [\,3605.712, 8413.328]$, $a_3 \in [1.0140, 2.3660]$, $\eta_1 \in [1.1790, 1.4410]$, $\eta_2 \in [0.8730, 1.0670]$. The parameters of the NM of the HF model are a_1=14994.3, a_2=6009.52, a_3=1.69, η_1=1.31 and η_2= 0.97.

The design requirements are set $t_s < 140s$, $\sigma < 5\%$ and the parameters of FOPID controller can be tuned by the proposed method. The proposed controller (FOPID_1) is obtained

$$G_{c2}(s)=958.4717+\frac{50.7771}{s^{0.3821}}+312.5614 s^{0.1681} \tag{24}$$

The response of FOPID_1 is compared to FOPID_2 controller (Zhao et al. 2005), FOPID_3 controller (Bouafoura et al. 2010) and FOPID_4 controller (Merrikh et al. 2010). Note that these parameters of those controllers are tuned well by the researchers in relevant references and they are listed in Table 3.

Table 3. Parameters of different controllers for the HF model

Controllers	Tuning parameters
FOPID_2	k_p=736.8054, k_i=-0.5885, k_d=-818.4204, λ=0.6, μ=0.35
FOPID_3	k_p=714.9739, k_i=107.0099, k_d=287.7011, λ=0.6, μ=0.35
FOPID_4	k_p=1000, k_i=100, k_d=100, λ=0.5, μ=0.31

The step response for the NM of HF is shown in Fig. 3, FOPID_1, FOPID_3 and FOPID_4 have the same response speed while FOPID_1 proposed in this paper have the smallest overshoot than other controllers for the NM of HF.

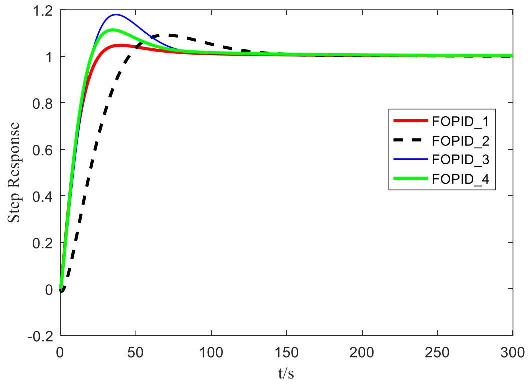

Fig. 3. The step response for the NM of HF with different controllers.

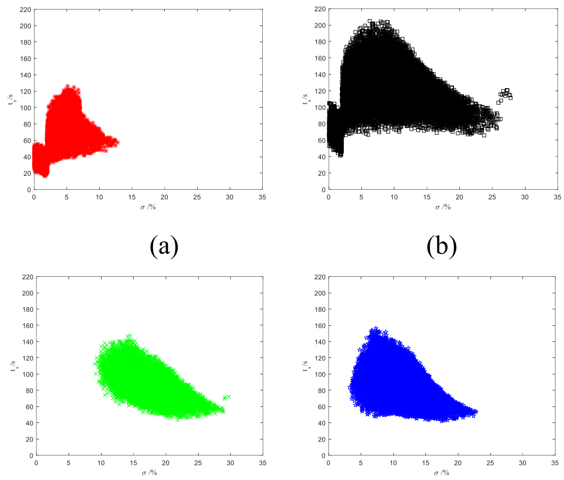

Fig. 4. The records for each perturbed HF model with different controllers (*(a): FOPID_1, (b): FOPID_2, (c): FOPID_3, (d): FOPID_4*).

Table 4. The ranges of ITAE, ISE and IAE for each perturbed HF model with different controllers

	ITAE	ISE	IAE
FOPID_1	[46.2865, 728.0836]	[3.4073, 9.1300]	[6.4727, 18.4817]
FOPID_2	[340.7160, 1795.113]	[10.1364, 23.0261]	[17.1536, 38.1190]
FOPID_3	[210.0180, 1193.163]	[4.2752, 10.1753]	[10.3243, 25.2119]
FOPID_4	[203.9462, 999.8548]	[3.8120, 9.0064]	[8.6272, 21.4678]

Monte Carlo simulation is carried out for *N* times with the uncertainty model in the whole parameter space, the indices of ITAE, ISE, IAE, t_s and σ are all recorded in the whole 300 s. The results of t_s and σ for each perturbed PHWR model with different controllers are shown in Fig. 4 and the ranges of ITAE, ISE and IAE are listed in Table 4. FOPID_1 has the smallest indices, densest distribution and the smallest ranges of ITAE, ISE and IAE than other FOPID controllers, so FOPID_1 has the superiority in all performance, robustness and ability of handling uncertainties.

The estimated value of the probability *K/N* is 0.9702, 0.7022, 0.7993 and 0.8162, respectively. FOPID_1 has the largest estimated value of the probability for perturbed PHWR models, which means FOPID_1 proposed in this paper has most probability to obtain the design requirements for the uncertainties HF model throughout the whole parameter space Q.

6. CONCLUSIONS

In this paper, a tuning method for FOPID controller based on probabilistic robustness is proposed to enhance the performance, robustness and ability of handling uncertainties of FOPID. The tuning procedure is designed for the parameter optimization and stability boundaries of FOPID is depicted to provide the search parameter space, which is the generating space of the initial population for GA. Numerical examples for PHWR model and HF model are performed to verify the efficacy of the proposed method and show the promising application value for the FOPID tuning.

7. ACKNOWLEDGEMENT

This work is supported by National Key Research and Development Program of China under Grant 2016YFB0901405, and the Open Fund of State Key Laboratory of Clean and Efficient Coal-fired Power Generation and Pollution Control.

REFERENCES

Badri V, Tavazoei M S. (2013). On tuning fractional order [proportional–derivative] controllers for a class of fractional order systems. *Automatica*, 49(7): 2297-2301.

Bettayeb, M., and R. Mansouri. (2014). Fractional IMC-PID-filter controllers design for non-integer order systems. *Journal of Process Control,* 24.4: 261-271.

Bhase S S, Patre B M. (2014). Robust FOPI controller design for power control of PHWR under step-back condition. *Nuclear Engineering and Design,* 274: 20-29.

Bouafoura M K, Braiek N B. (2010). $PI^{\lambda}D^{\mu}$ controller design for integer and fractional plants using piecewise orthogonal functions. *Communications in Nonlinear Science and Numerical Simulation*, 15(5): 1267-1278.

Calafiore G C, Dabbene F, Tempo R. (2011). Research on probabilistic methods for control system design. *Automatica*, 47(7): 1279-1293.

Chen X, Zhou K, Aravena J L. (2003). Fast universal algorithms for robustness analysis. *Decision and Control, 2003. Proceedings. 42nd IEEE Conference on. IEEE,* 2: 1926-1931.

Das S, Das S, Gupta A. (2011). Fractional Order Modeling of a PHWR under Step-Back Condition and Control of Its Global Power with a Robust $PI^{\lambda}D^{\mu}$ Controller. *IEEE Transactions on Nuclear Science,* 58(5): 2431-2441.

Delavari H, Ghaderi R, Ranjbar N A, et al. (2012). Adaptive fractional PID controller for robot manipulator. *arXiv preprint arXiv:1206.2027.*

Haupt R L, Haupt S E. (2004). *Practical genetic algorithms.* John Wiley & Sons, New Jersey.

Lamba R, Singla S K, Sondhi S. (2017). Fractional order PID controller for power control in perturbed pressurized heavy water reactor. *Nuclear Engineering and Design*, 323: 84-94.

Luo Y, Chen Y Q, Wang C Y, et al. (2010). Tuning fractional order proportional integral controllers for fractional order systems. *Journal of Process Control*, 20(7): 823-831.

Merrikh-Bayat F, Karimi-Ghartemani M. (2010). Method for designing $PI^{\lambda}D^{\mu}$ stabilisers for minimum-phase fractional-order systems. *IET control theory & applications,* 4(1): 61-70.

Nataraj P S V, Kalla R. (2010). Computation of spectral sets for uncertain linear fractional-order systems. *Communications in Nonlinear Science and Numerical Simulation*, 15(4): 946-955.

Oustaloup A, Levron F, Mathieu B, et al. (2000). Frequency-band complex noninteger differentiator: characterization and synthesis. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications,* 47(1): 25-39.

Petráš I, Vinagre B M, Dorčák Ľ, et al. (2002). Fractional digital control of a heat solid: Experimental results *Proceedings of International Carpathian Control Conference ICCC*. 2: 365-370.

Sabatier J, Cugnet M, Laruelle S, et al. (2010). A fractional order model for lead-acid battery crankability estimation. *Communications in Nonlinear Science and Numerical Simulation*, 15(5): 1308-1317.

Sagar S, Kaur S, Sondhi S. (2016). Fractional Order IMC-PID Controller Design for the Pressurized Heavy Water Reactor. *International Conference on Electrical, Electronics, Engineering Trends, Communication, Optimization and Sciences (EEECOS)*, 72-77.

Sharma R, Rana K P S, Kumar V. (2014). Performance analysis of fractional order fuzzy PID controllers applied to a robotic manipulator. *Expert systems with applications,* 41(9): 4274-4289.

Wang C F, Li D H, (2011). Decentralized PID controllers based on probabilistic robustness. *Journal of dynamic systems, measurement, and control,* 133: 061015

Wu Z, Li D, Wang L. (2016). Control of the superheated steam temperature: A comparison study between PID and fractional order PID controller. C*ontrol Conference (CCC), 2016 35th Chinese*, *IEEE,* 10521-10526.

Xue D, Zhao C, Chen Y Q. (2006). A modified approximation method of fractional order system. *Mechatronics and Automation, Proceedings of the 2006 IEEE International Conference*, 1043-1048.

Zamani A A, Tavakoli S, Etedali S. (2017). Fractional order PID control design for semi-active control of smart base-isolated structures: A multi-objective cuckoo search approach. *ISA transactions*, 67: 222-232.

Zhao C, Xue D, Chen Y Q. (2005). A fractional order PID tuning algorithm for a class of fractional order plants. *Mechatronics and automation, 2005 IEEE international conferenc*e. *IEEE*, 1: 216-221.