Optimized Retuning of PID Controllers for TITO Processses

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Abstract: In this paper we propose a methodology to evaluate the performance of decentralized PID controllers for two-inputs-two-outputs processes and to retune the parameters. In particular, the model of the process is estimated based on a technique that exploits the final value theorem. Then, an evolutionary algorithm is applied in order to find the Pareto front by considering a multiobjective optimization problem. Finally, the performance obtained with the current tuning is compared with the optimal ones and, in case it is necessary, the PID parameters are retuned accordingly. A procedure to achieve the Nash optimal point is also proposed.

Keywords: TITO systems, PID controllers, process parameters estimation, performance assessment, retuning

1. INTRODUCTION

It is well-known that proportional-integral-derivative (PID) controllers are widely employed in the process industry but they are often poorly tuned because of the lack of time and/or skill of operators. Indeed, the parameters of the controller selected during the commissioning of the plant are often never modified despite possible changes in the process dynamics. This issue is especially critical if multiple-input-multiple-output (MIMO) processes are considered, because the presence of different objectives (for example, related to the different process variables) makes the tuning task more difficult. From another point of view, the industry 4.0 revolution yields the availability of a huge quantity of process data, which can be fruitfully exploited in order to automatically assess the performance of a controller and, in case it is evaluated as unsatisfactory, to provide new controller parameters in order to optimize the selected performance.

A large number of performance assessment techniques for PID controllers have been proposed in the literature, by taking into account either stochastic or deterministic performance (Hägglund, 2005; Jelali, 2006; Huang and Shah, 1999; Qin, 1998). In general, one of the most desirable feature that a performance assessment technique should possess is the employment of routine operational data as the need to perform special (possibly expensive) experiments can be detrimental for the process operations (Bauer et al., 2016). Further, it is very important that, once the performance has been recognized to be improvable, the controller can be retuned automatically without a specific intervention of the operator.

In this context, a technique based on the final value problem has been recently proposed in the literature for different processes and control structures in order to estimate the process model and, based on that, to assess the performance of the controller and to retune it (Veronesi and Visioli, 2009, 2012, 2010a,b, 2011, 2014; Normey-Rico et al., 2014). In fact, the achieved performance can be compared to that provided by a properly tuned controller which serves as a benchmark. One of the main advantages of this method is that its exploits the integral of suitable signals and it is therefore inherently robust to the measurement noise.

Nowadays, it is also recognized that the availability of high-performance software tools and of hardware components with high computational capabilities can be exploited to obtain an optimal tuning of PID controllers, where, for example, the trade-off between different control specifications can be explicitly considered (Garpinger and Hägglund, 2015; Sanchez et al., 2017; Boyd et al., 2000). In particular, evolutionary algorithms such as genetic algorithm and particle swarm optimization have been employed for this purpose, especially for the control of MIMO processes (Reynoso Meza et al., 2017; Turco Neto et al., 2017).

In this paper we present a method to evaluate the performance of decentralized PID controllers for two-inputs-two-outputs (TITO) processes and to optimally retune them by considering a Pareto front related to the integrated absolute errors of the two process variables. In particular, first the process model is estimated from set-point step response data (Pereira et al., 2017). Then, based on the model, Pareto fronts are built (one for each control specification) by means of an evolutionary algorithm and the performance of the PID controllers is evaluated by comparing it to the optimal ones. Finally, the PID controllers are retuned (in case the current performance is far from an optimal one) by selecting the desired trade-off between the different specifications. A technique to obtain also the tuning corresponding to the Nash point of the Pareto front is also proposed.

2. CONTROL SYSTEM

We consider a linear, time-invariant, continuous-time TITO system whose matrix transfer function is:

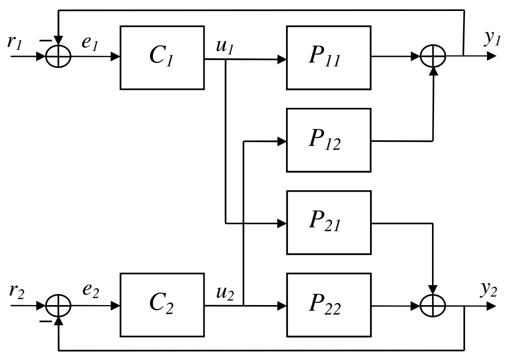

Fig. 1. The considered TITO control scheme.

$$\mathbf{P}(s) = [P_{ij}(s)] \quad i, j = 1, 2 \tag{1}$$

where $(i, j = 1, 2)$

$$P_{ij}(s) = \frac{\mu_{ij} e^{-s\theta_{ij}}}{q_{ij}(s)} \tag{2}$$

and

$$q_{ij}(s) = \prod_{k=1}^{n_{ij}} (1 + s\tau_{ij,k}) = s^{n_{ij}} \prod_{k=1}^{n_{ij}} \tau_{ij,k} + ... + s \sum_{k=1}^{n_{ij}} \tau_{ij,k} + 1 \tag{3}$$

Define as

$$T^0_{ij} := \sum_{k=1}^{n_{ij}} \tau_{ij,k} + \theta_{ij} \tag{4}$$

the sum of the dead time and of the time constants of the single transfer function $P_{ij}(s)$. We consider a decentralized control law (where the input-output pairings have been previously selected), where the PID controller is in ideal form, namely,

$$\mathbf{C}(s) = \begin{bmatrix} C_1(s) & 0 \\ 0 & C_2(s) \end{bmatrix}. \tag{5}$$

where

$$C_j(s) = K_{pj} \left(1 + \frac{1}{T_{Ij}s} + T_{dj}s \right) \quad j = 1, 2 \tag{6}$$

and K_{pj}, T_{Ij}, T_{dj}, $j = 1, 2$ are, respectively, the proportional gain, the integral time constant and the derivative time constant of the PID controller that handles loop j. Note that, in order to make the controller proper, a low-pass filter has also to be implemented. Its cut-off frequency should be selected in order for the filter to be negligible for the PID relevant dynamics and to filter the high frequency noise at the same time. Hereafter we will neglect the presence of the filter in the autotuning procedure, but we will include it in the simulation results (see Section 6). For the analysis made in the following sections, it is convenient to write the PID controller transfer function as

$$C_j(s) = \frac{K_{pj} c_j(s)}{sT_{Ij}}, \quad j = 1, 2 \tag{7}$$

where

$$c_j(s) = T_{dj} T_{Ij} s^2 + T_{Ij} s + 1, \quad j = 1, 2. \tag{8}$$

The control scheme is shown in Fig. 1.

3. ESTIMATION OF THE PROCESS PARAMETERS

As indicated in (Pereira et al., 2017), the estimation of the process parameters can be performed by evaluating the response of the system to two separated step signals applied to each of the two set-points starting from steady-state conditions. This means that the transient response caused by a set-point step has to be terminated before another set-point step is applied and should not be perturbed by external disturbances.

Thus, by considering a step change of amplitude A_{s1} to the set-point of the first loop and then, at the end of the transient, a step change of amplitude A_{s2} to the set-point of the second loop, the four process gains and the values of the four sums of the lags and dead times T^0_{ij} $(i, j = 1, 2)$ can be computed as:

$$\begin{aligned}
\mu_{11} &= \frac{T_{i1} IE_{2,2}}{K_{p1}(IE_{1,1}IE_{2,2} - IE_{1,2}IE_{2,1})} A_{s1}, \\
\mu_{12} &= \frac{T_{i2} IE_{1,2}}{K_{p2}(IE_{1,2}IE_{2,1} - IE_{1,1}IE_{2,2})} A_{s1}, \\
\mu_{21} &= \frac{T_{i1} IE_{2,1}}{K_{p1}(IE_{1,2}IE_{2,1} - IE_{1,1}IE_{2,2})} A_{s2}, \\
\mu_{22} &= \frac{T_{i2} IE_{1,1}}{K_{p2}(IE_{1,1}IE_{2,2} - IE_{1,2}IE_{2,1})} A_{s2}.
\end{aligned} \tag{9}$$

$$\begin{aligned}
T^0_{11} &= \frac{\mu_{21} IV_2}{\mu_{11} A_{s2}} + \frac{IV_1}{A_{s1}}, \quad T^0_{12} = \frac{\mu_{22} IV_2}{\mu_{12} A_{s2}} + \frac{IV_1}{A_{s1}}, \\
T^0_{21} &= \frac{\mu_{11} IW_1}{\mu_{21} A_{s1}} + \frac{IW_2}{A_{s2}}, \quad T^0_{22} = \frac{\mu_{12} IW_1}{\mu_{22} A_{s1}} + \frac{IW_2}{A_{s2}}.
\end{aligned} \tag{10}$$

where $IE_{i,j}$ is the integral of the errors $e_1(t)$ and $e_2(t)$ after the step change in the set-point signals $r_1(t)$ and $r_2(t)$, respectively, IV_1 and IW_1 are the integrals of the following variables

$$\begin{aligned}
v(t) &:= \mu_{11} u_1(t) + \mu_{12} u_2(t) - y_1(t) \\
w(t) &:= \mu_{21} u_1(t) + \mu_{22} u_2(t) - y_2(t)
\end{aligned} \tag{11}$$

after the step change in the set-point signal $r_1(t)$, and IV_2 and IW_2 are the integrals of the same variables after the step change in the set-point signal $r_2(t)$.

Finally, the apparent dead time $\tilde{\theta}_{ij}$ of each transfer function $\tilde{P}_{ij}(s)$ can be evaluated by using a noise band approach as explained in (Pereira et al., 2017).

Summarizing, the identification procedure initially consists of evaluating $IE_{1,1}$, $IE_{2,1}$, $IE_{1,2}$, $IE_{2,2}$, IV_1, IW_1, IV_2, IW_2, and then by determining μ_{ij} and T^0_{ij} $(i, j = 1, 2)$ by applying (9) and (10). Note that the values of the parameters of the PID controllers employed (those that need to be retuned) are obviously known. Once μ_{ij} and T^0_{ij} $(i, j = 1, 2)$ have been determined, each transfer function $P_{ij}(s)$ can be approximated as a FOPDT transfer function, namely:

$$\tilde{P}_{ij}(s) = \frac{\mu_{ij} e^{-s\tilde{\theta}_{ij}}}{\tilde{\tau}_{ij} s + 1}, \tag{12}$$

where $\tilde{\tau}_{ij} = T^0_{ij} - \tilde{\theta}_{ij}$.

Finally, it has to be highlighted that closed-loop data are employed to estimate the process model, which means that PID controllers have to be in place. The tuning of the controllers is not an issue for the model estimation as only steady-state values of the signals are considered, provided the stability of the control system is ensured.

4. PERFORMANCE ASSESSMENT

Once the model of the process have been obtained, an evolutionary algorithm (for example, a genetic algorithm (Mitchell, 1998) or a particle swarm optimization algorithm (Kennedy and Eberhart, 2001)) is used in order to solve a multiobjective optimization problem (MOP) that takes into account performance of the two process variables. For this purpose, we can consider the integrated absolute error (IAE), defined as

$$IAE_i = \int |e_i(t)| dt, \quad i = 1, 2 \tag{13}$$

and therefore we can define the following objective functions ($0.1 \leq \alpha \leq 0.9$):

$$J_1 = \alpha IAE_1^{r1} + (1-\alpha) IAE_2^{r2} \tag{14}$$

$$J_2 = \alpha IAE_1^{r1} + (1-\alpha) IAE_2^{r1} \tag{15}$$

$$J_3 = \alpha IAE_1^{r2} + (1-\alpha) IAE_2^{r2} \tag{16}$$

where IAE_1^{r1} is the integrated absolute error related to the first process variable when a step signal is applied only to the first set-point, IAE_1^{r2} is the integrated absolute error related to the first process variable when a step signal is applied only to the second set-point, IAE_2^{r2} is the integrated absolute error related to the second process variable when a step signal is applied only to the second set-point and, finally, IAE_2^{r1} is the integrated absolute error related to the second process variable when a step signal is applied only to the first set-point.

By considering different values of α ranging from 0.1 to 0.9 (note that the case $\alpha = 0$ and $\alpha = 1$ are avoided as this would imply that the performance related to one of the process variables can be completely neglected and therefore the problem reduces to consider a single-input-single-output case), each MOP can be solved by finding the PID parameters that minimize the considered objective function. Formally, we can consider the following MOPs:

$$\min_{K_{p1}, T_{I1}, T_{d1}, K_{p2}, T_{I2}, T_{d2}} J_i, \quad i = 1, \ldots, 3 \tag{17}$$

and, for each MOP, a Pareto front can be determined.

Then, the IAE values obtained with the current PID controllers can be compared with those achieved by the optimal PID controllers determined by the evolutionary algorithm. In other words, the controller performance can be assessed by evaluating the distance of the achieved integrated absolute errors from the determined Pareto fronts.

Note that, in order to determine the search space of the optimization problem, we can use the current tuning as a starting point. In fact, the bounds of the search space can be obtained, for example, by multiplying and dividing the current values by ten (in case the derivative action is not used, the maximum value can be selected by considering a ratio of the integral time constant). The possible occurrence of instability is not a problem as the optimization algorithm uses the (previously obtained) model of the plant and the unstable cases can be handled by significantly penalizing the cost function. In this context, the user can obviously also choose to avoid to use the derivative action very simply by constraining the derivative time constants to be zero. It is worth underlying at this point that the computational burden of the evolutionary algorithm can be quite high as it needs a large number of simulations to obtain the global optimum. However, this is not a critical issue as the optimization is completely performed off-line.

5. RETUNING

If, according to the performance assessment procedure based on the Pareto fronts, it results that the performance achieved by the current controllers can be improved, the PID controllers can be retuned by considering one of the set of parameters determined by evolutionary algorithm for a given objective function and for a given value of α, depending on the control specifications for a given application. Actually, a default value of α can be assumed in order for the overall method to be fully automatic and to avoid any intervention from the operator.

An alternative option is to consider a point in the Pareto front that provides a suitable trade-off between the performance indexes IAE_1 and IAE_2 achieved for the two process variables y_1 and y_2. In this context, it is worth considering the following bargaining points (see Figure 2):

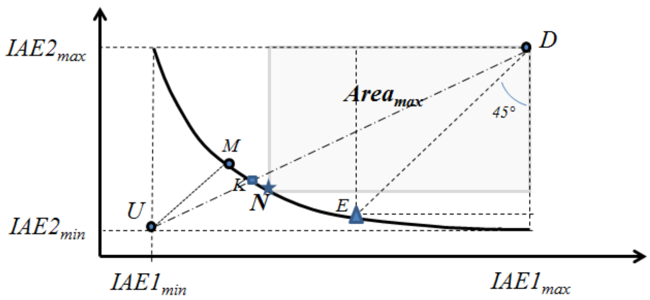

Fig. 2. Locations of the bargaining solutions with respect to the Pareto front.

- the disagreement point D, where both IAE_1 and IAE_2 are the worst ones;
- the utopia point U, where both IAE_1 and IAE_2 are the best ones;
- the egalitarian point E where $IAE_1(D) - IAE_1(E) = IAE_2(D) - IAE_2(E)$;
- the point M of the Pareto front at the minimum distance from the utopia point;
- The point K at the intersection between the Pareto front and the straight line connecting the utopia and the disagreement points;
- the Nash point N, for which the area having D and N as vertexes has a maximum value.

In this paper we choose to find the tuning of the PID parameters in order to achieve the Nash point, as in (Sanchez et al., 2017). The Nash point can be found by determining a convenient expression of the Pareto front by a suitable fitting function, so that the determination of the rectangular area having D and N as vertexes can be easily performed numerically by considering many points of the Pareto front.

Then, when the Nash point N is determined, the new set of PID parameters can be found by linear interpolation between the two available closer points, denoted as P and Q respectively, found through the evolutionary optimization.

Each PID parameter κ_j can then be computed as

$$\kappa_j = \frac{QN}{PN + QN} \kappa_P + \frac{PN}{PN + QN} \kappa_Q \tag{18}$$

where PN and QN is the distance from N to P and Q, respectively, and κ_P and κ_Q are the values of the parameter in P and Q, respectively.

In case the resulting performance is not yet close enough to the Nash point, a local search algorithm can be run to minimize this distance.

6. ILLUSTRATIVE EXAMPLES

Consider the well-known Wood and Berry distillation column model presented in (Wood and Berry, 1973):

$$\mathbf{P}(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{1+16.7s} & \dfrac{-18.9e^{-3s}}{1+21s} \\ \dfrac{6.6e^{-7s}}{1+10.9s} & \dfrac{-19.4e^{-3s}}{1+14.4s} \end{bmatrix}. \tag{19}$$

As an initial controller parameters, those proposed by applying the "biggest log modulus tuning" (BLT) technique proposed in (Luyben, 1986) are selected: $K_{p1} = 0.375$, $T_{i1} = 8.29$, $T_{d1} = 0$,

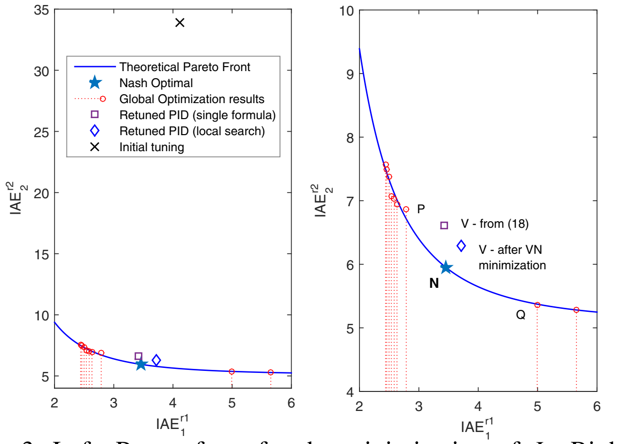

Fig. 3. Left: Pareto front for the minimization of J_1. Right: zoom of the results.

$K_{p2} = -0.075$, $T_{i2} = 23.6$, $T_{d2} = 0$.
With these values of the parameters, the indexes resulting by applying two (separated) steps to the two set-points are $IAE_1^{r1} = 4.1158$ and $IAE_2^{r2} = 33.8937$.
The evaluation of the two responses, according to the method reviewed in Section 3, yields

$$\begin{gathered} T_{11}^0 = 17.72, \quad T_{12}^0 = 24.16, \quad T_{21}^0 = 17.93, \quad T_{22}^0 = 17.83 \\ \tilde{\theta}_{11} = 1.08, \quad \tilde{\theta}_{12} = 3.32, \quad \tilde{\theta}_{21} = 7.1, \quad \tilde{\theta}_{22} = 3.2 \\ \mu_{11} = 12.80, \quad \mu_{12} = -18.92, \quad \mu_{21} = 6.61, \quad \mu_{22} = -19.46 \end{gathered} \tag{20}$$

and therefore:

$$\tilde{\tau}_{11} = 16.64, \quad \tilde{\tau}_{12} = 20.84, \quad \tilde{\tau}_{21} = 10.83, \quad \tilde{\tau}_{22} = 14.63 \tag{21}$$

The results obtained by applying a genetic algorithm in order to minimize J_1 for the estimated process are shown in Table 1 and the Pareto front is shown in Figure 3. It can be seen that the current performance is far away from the Pareto front and therefore, for the selected task, it is worth retuning the two PID controllers. As mentioned in Section 5, this can be done by simply selecting a suitable column in Table 1. If the Nash solution is pursued, the optimal points on the Pareto front can be fitted by the function $IAE_2^{r2} = \frac{b_1}{x^{b_3}} + b_2$, where $x = IAE_1^{r1}$ and $b_1 = 32.4221$, $b_2 = 5.0713$ and $b_3 = 2.9088$. Thus, the Nash optimal is determined as $N = (3.4590, 5.9486)$.
After the Nash optimal performance has been determined, the corresponding PID parameters can be estimated on the base of the distances from the two closest known points in the Pareto front. In this example they are the points P and Q obtained for $\alpha = 0.2$ and $\alpha = 0.3$, respectively.
As $PN = 1.6414$ and $QN = 1.1356$, by applying formula (18) the new set of PID parameters results in $K_{p1} = 0.7291$, $T_{i1} = 12.0881$, $T_{d1} = 0.7711$, $K_{p2} = -0.1713$, $T_{i2} = 7.6699$, $T_{d2} = 1.8777$, yielding $(IAE_1^{r1}, IAE_2^{r2}) = (3.4313, 6.6301)$, represented by the square in Figure 3. Denoting this point as V, it is possible to minimize the VN distance through a local search algorithm (based on descendent gradient), in order to obtain a tuning that yields a performance closer to the Nash point. Doing so, the distance from V to N can be improved from $VN = 0.6821$ to $VN = 0.4305$. The corresponding tuning is $K_{p1} = 0.5672$, $T_{i1} = 10.8147$, $T_{d1} = 1.1230$, $K_{p2} = -0.1715$, $T_{i2} = 8.5088$, $T_{d2} = 1.9446$, resulting in $IAE_1^{r1} = 3.6339$, $IAE_2^{r2} = 6.4607$, see the diamond in Figure 3.
The set-point step responses obtained before and after the retuning are shown in Figure 4, where the increment of the performance is evident.

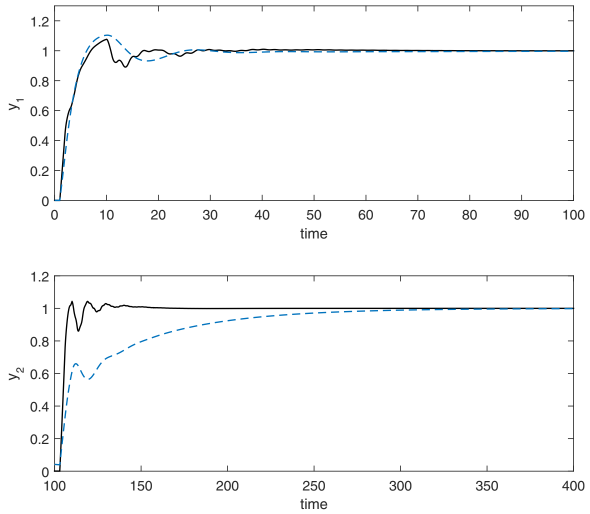

Fig. 4. Set-point step responses before (dashed line) and after (solid line) the retuning by minimizing J_1.

As a second illustrative example we consider again process (19) but in this case we address the control task where the minimization of J_2 is of concern. The two PID controllers are initially (well) tuned by applying the rules proposed in (Lee et al., 2004):

$$\begin{gathered} K_{p1} = 0.61 \quad T_{i1} = 8.42 \quad T_{d1} = 0.26 \\ K_{p2} = -0.12 \quad T_{i2} = 7.68 \quad T_{d2} = 0.73 \end{gathered} \tag{22}$$

By evaluating the two set-point step responses, the process parameters are estimated as

$$\begin{gathered} T_{11}^0 = 17.69, \quad T_{12}^0 = 23.98, \quad T_{21}^0 = 17.90, \quad T_{22}^0 = 17.40 \\ \tilde{\theta}_{11} = 1.04, \quad \tilde{\theta}_{12} = 3.20, \quad \tilde{\theta}_{21} = 7.05, \quad \tilde{\theta}_{22} = 3.1 \\ \mu_{11} = 12.80, \quad \mu_{12} = -18.89, \quad \mu_{21} = 6.60, \quad \mu_{22} = -19.40 \end{gathered} \tag{23}$$

and therefore:

$$\tilde{\tau}_{11} = 16.64, \quad \tilde{\tau}_{12} = 20.84, \quad \tilde{\tau}_{21} = 10.83, \quad \tilde{\tau}_{22} = 14.63 \tag{24}$$

Then, we have $IAE_1^{r1} = 3.799$ and $IAE_2^{r1} = 5.598$.
The application of a genetic algorithm in order to minimize J_2 yields the results shown in Table 2 and the Pareto front shown in Figure 5. In this case the performance is not very far away from the Pareto front but it can be improved. The Nash optimal point is found to be $N = (4.8800\ 2.5665)$. With the initial tuning the distance from the Nash point is equal to 5.5977.
Then, we have $PN = 1.1897$ and $QN = 2.7790$ so that the new set of PID parameters resulting from (18) is $K_{p1} = 0.4717$, $T_{i1} = 9.1693$, $T_{d1} = 0.5355$, $K_{p2} = -0.2542$, $T_{i2} = 6.0996$, $T_{d2} = 1.9321$, yielding $IAE_1^{r1} = 5.18893$ and $IAE_2^{r1} = 4.1828$.
The point V is still not very close to the Nash point. By minimizing VN through a local search algorithm, the following PID parameters are obtained: $K_{p1} = 0.3589$, $T_{i1} = 7.7921$, $T_{d1} = 1.2619$, $K_{p2} = -0.2241$, $T_{i2} = 4.9495$, $T_{d2} = 2.2680$; giving $IAE_1^{r1} = 4.9090$, $IAE_2^{r1} = 2.6085$ and $VN = 0.0511$, which means that the Nash point has been virtually achieved.
The set-point step responses obtained before and after the retuning are shown in Figure 6.

Finally, when J_3 is considered, by starting again from the tuning (22) we have $IAE_1^{r2} = 7.8175$ and $IAE_2^{r2} = 2.1358$. Results

Table 1. Results of the application of the evolutionary algorithm to minimize J_1.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
IAE_1	5.6526	4.9926	2.7879	2.6314	2.5849	2.5361	2.4982	2.4600	2.4406
IAE_2	5.2823	5.3634	6.8647	6.9437	7.0279	7.0734	7.3760	7.4975	7.5659
K_{p1}	0.2630	0.2545	1.0574	1.1127	1.1153	1.1734	1.2408	1.2400	1.2419
T_{i1}	8.4866	6.4997	15.9545	15.2145	14.2495	13.7192	13.2372	13.0012	12.5541
T_{d1}	0.9173	0.9534	0.6449	0.5750	0.5761	0.5030	0.4613	0.4701	0.4874
K_{p2}	-0.2000	-0.2012	-0.1506	-0.1452	-0.1388	-0.1394	-0.1124	-0.1150	-0.1176
T_{i2}	8.4814	8.5735	7.0448	7.0749	7.3445	7.2647	6.1091	7.0014	7.5987
T_{d2}	2.2884	2.2525	1.6184	1.5822	1.4126	1.4066	1.0402	0.9501	0.9388

Table 2. Results of the application of the evolutionary algorithm to minimize J_2.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
IAE_1	10.492	6.8459	5.9964	2.8246	2.7496	2.5212	2.4835	2.4125	2.3647
IAE_2	1.2275	1.8489	2.1553	4.4368	4.4476	4.7178	4.794	5.088	5.3991
K_{p1}	0.1390	0.1577	0.2076	1.0887	1.1190	1.1271	1.1599	1.2283	1.2159
T_{i1}	7.8533	5.0086	5.8842	16.8429	16.9524	14.2210	13.9952	13.1143	12.2261
T_{d1}	1.9712	0.4683	0.5259	0.5578	0.5706	0.5113	0.4798	0.4918	0.4941
K_{p2}	-0.2876	-0.3183	-0.3123	-0.1185	-0.1398	-0.1413	-0.1352	-0.1239	-0.1117
T_{i2}	3.7779	5.8218	7.1765	3.5841	4.6199	8.6516	8.5712	9.5403	9.6724
T_{d2}	1.9473	1.5513	1.6172	2.6675	1.8176	1.5585	1.5527	1.0616	1.0178

Table 3. Results of the application of the evolutionary algorithm to minimize J_3.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
IAE_1	5.8233	3.7588	0.7954	0.7311	0.3537	0.3199	0.3008	0.2902	0.2481
IAE_2	5.4139	5.4491	7.263	7.2983	7.5628	7.6357	7.7028	7.7146	7.8216
K_{p1}	0.2452	0.2486	1.9246	1.9616	1.8348	1.8161	1.7434	1.6327	1.7153
T_{i1}	9.3670	5.4357	8.9008	7.7128	1.2858	1.4167	1.1760	1.1937	1.3099
T_{d1}	1.8343	1.0628	0.3379	0.3329	0.4644	0.4517	0.5086	0.5892	0.4811
K_{p2}	-0.1982	-0.2114	-0.1360	-0.1335	-0.1084	-0.1047	-0.1017	-0.0976	-0.0901
T_{i2}	8.8048	8.6809	5.5265	5.5162	4.7514	4.6910	4.6055	4.5356	4.5226
T_{d2}	2.0852	2.2343	1.4025	1.3755	0.7170	0.6201	0.2631	0.4696	0.3730

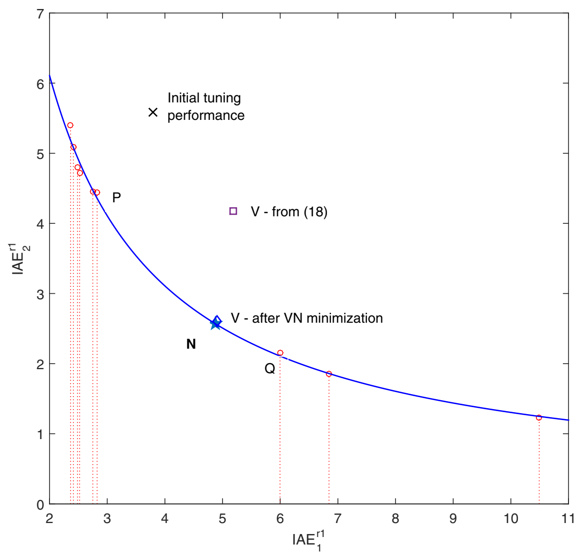

Fig. 5. Pareto front for the minimization of J_2.

obtained by the genetic algorithm are in Table 3 and in Figure 7. The Nash point is determined as $N = (1.9650, 6.2389)$. With the initial tuning, the distance from the Nash point is equal to 1.5878. Then, we have $PN = 1.9600$ and $QN = 1.5546$ so the set of PID parameters resulting from the (18) is $K_{p1} = 1.1833$, $T_{i1} = 7.3681$, $T_{d1} = 0.6585$, $K_{p2} = -0.1694$, $T_{i2} = 6.9218$, $T_{d2} = 1.7704$, giving $IAE_1^{r2} = 1.2623$ and $IAE_2^{r2} = 7.3385$, corresponding to $VN = 1.3049$. Even if both IAE_1^{r2} and IAE_2^{r2} are lower than the ones obtained by the initial tuning, this point

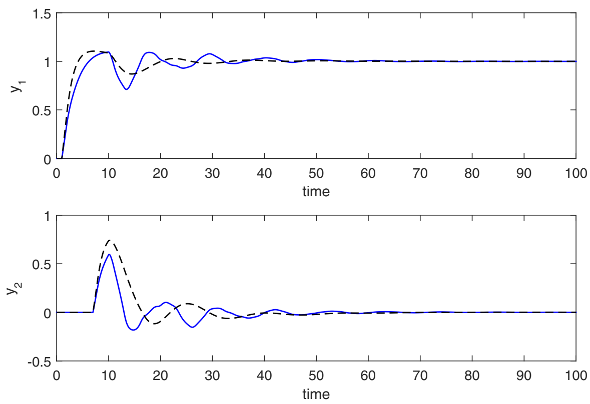

Fig. 6. Set-point step responses before (dashed line) and after (solid line) the retuning by minimizing J_2.

is still not very close to the Nash point. By minimizing VN through a local search algorithm the following PID parameters are obtained: $K_{p1} = 0.3978$, $T_{i1} = 4.2630$, $T_{d1} = 0.9607$, $K_{p2} = -0.1941$, $T_{i2} = 9.1714$, $T_{d2} = 2.2505$, yielding $IAE_1^{r2} = 1.9650$, $IAE_2^{r2} = 6.2389$ and $VN = 1.9015 \cdot 10^{-7}$, which means that a Nash optimally tuning has been achieved. The step responses before and after the retuning are shown in Figure 8.

7. CONCLUSIONS

An optimization-based performance assessment technique for decentralized PID controllers of TITO processes has been proposed in this paper. After a process model has been estimated by using the data of two separated set-point step responses, an

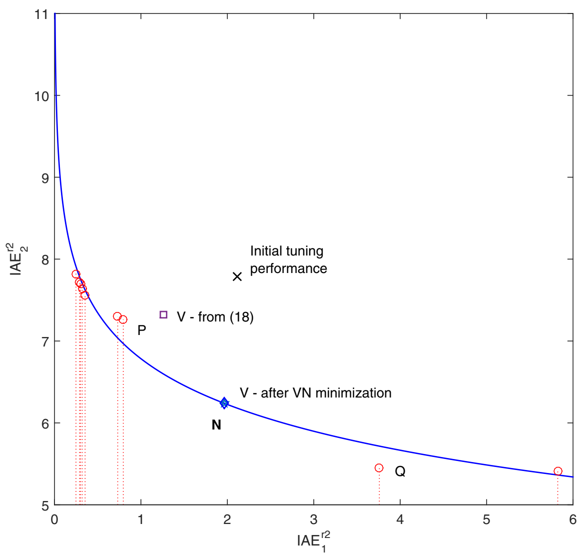

Fig. 7. Pareto front for the minimization of J_3.

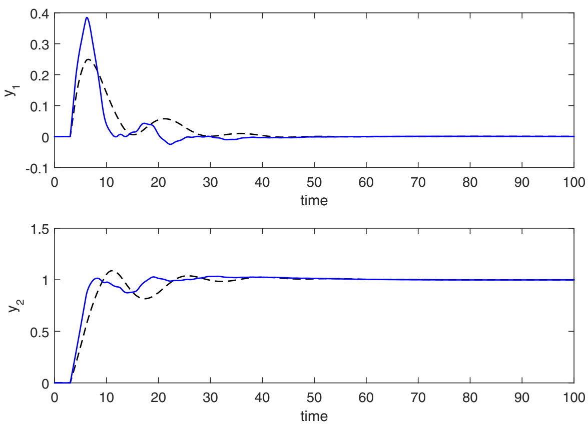

Fig. 8. Set-point step responses before (dashed line) and after (solid line) the retuning by minimizing J_3.

evolutionary algorithm is applied to build a Pareto front which can be used to evaluate the achieved performance, according to a selected task. The PID controllers can then be retuned by exploiting the results of the optimization or by considering the Nash point of the Pareto front, which can be reached by a simple local optimization procedure. The overall methodology is suitable for industrial frameworks where computational resources can be employed to process data available from the control systems.

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