Reduced Order Model based Optimally Tuned Fractional Order PID controller for Pressurized Water Nuclear Reactor

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Abstract: — In this paper, the reduced model of the Pressurized Water Nuclear Reactor (PWR) is derived based on the point kinetics equations and thermal equilibrium relations. The power level of the nuclear reactor is controlled by adjusting the insertion reactivity of the rod. Several controllers such as Genetic Algorithm based PID controller (GAPID), Fractional Order PID controller (FOPID) and Genetic Algorithm based Fractional Order PID Controller (GAFOPID) are used to control the power level of the PWR type of nuclear reactor. The simulation results depict that the Genetic Algorithm based Fractional Order PID Controller (GAFOPID) shows the satisfactory response than other control techniques.

Keywords: reactor power, reactivity, controller, genetic algorithm, performance indices.

1. INTRODUCTION

Nowadays, nuclear reactor is a major component of a nuclear power plant and adopts integrated design. Over the years, one of the several nuclear engineering problems is to control the reactor power level. The reactor power control is important from the standpoint of safety concerns and for regular and appropriate operation of nuclear power plants, Guimarães, et al., (2011). This also includes the reliability and cleanliness of nuclear plants over the globe in industrial technology. Gupta, A et al., (2017).

The dynamic behavior of the Pressurized Water Reactor nuclear reactor is very complex and non-linear. Because of this non-linear structure, the reactor dynamics changes with time according to power levels, Aleksei et al., (2011). Hence it is necessary to use non-linear controllers for controlling the power level of nuclear reactors. Conventional reactor regulating system controls the average temperature of the reactor core according to its referenced temperature determined by the turbine load, Da Costa et al., (2011). This control has disadvantage of controlling under low power demand variations. It is hard to get the satisfying performance with the classic control strategy to control nuclear reactor power. Advanced intelligent control gives a bright future to nonlinear time dependent control system. In recent years, there has been a growing interest and research in the design of intelligent systems using soft computing methodologies such as Fuzzy Logic (FL), Artificial Neural Networks (ANN), Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Evolutionary Strategy (ES), Simulated Annealing (SA), and so on for nuclear engineering problems.

The paper is organized as follows. In Section 2, the mathematical model of the nuclear reactor is presented. In Section 3, the controller design is presented. Conclusion is given in Section 4.

2. MATHEMATICAL MODEL OF PWR TYPE OF NUCLEAR REACTOR

The nuclear reactor is an uncertain, nonlinear system and its parameter varies with time as a function of operating power level, fuel burn up and control rod worth.

2.1 Model

A reduced model of a PWR type of nuclear reactor is derived based on the point kinetics and on thermal hydraulic equilibrium relations. The kinetics equations are used to describe the neutron balance in the core and the thermal equilibrium relations are used to describe the energy exchange between the different loops, German, G et al., (2010). The plant geometry and the operating conditions play an important role in deciding the number of effective parameters. Thus, these parameters need to be calibrated for each plant and situation considered, Hetrick, L et al., (1971). The block diagram of the nuclear reactor model is shown in the Fig. 1

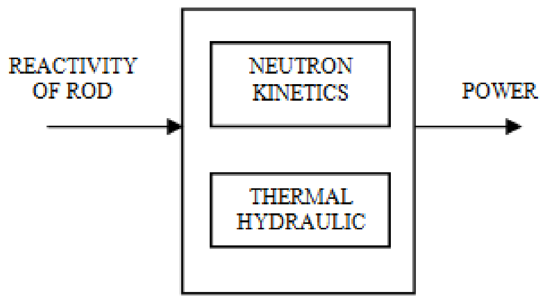

Fig. 1. Block diagram of Nuclear reactor model

The reduced model can be written as follows:

$$\frac{dP}{dt} = \beta\frac{P}{\Lambda}\left[\alpha_f\left(T_f - T_f^0\right) + \alpha_{av}(T_{av} - T_{av}^0) + \rho_{in} - 1\right] + \sum_{i=1}^{6}\lambda_i c_i \quad (1)$$

$$\frac{dC_i}{dt} = \beta_i\frac{P}{\Lambda} - \lambda_i C_i \quad (2)$$

Where i = 1 to 6.

$$\frac{dT_f}{dt} = -A_1(T_f - T_{av}) + A_1\frac{T_f^0 - T_{av}^0}{P^0}P \quad (3)$$

$$\frac{dT_{av}}{dt} = A_3(T_f - T_{av}) - A_3\frac{T_f^0 - T_{av}^0}{T_{av}^0 - T_{in}}(T_{av} - T_{in}) \quad (4)$$

The constants A_1 and A_3 are given by the following relations:

$$A_1 = \frac{UA}{M_f C_{pf}} \qquad A_3 = \frac{UA}{M_C C_{pc}} \quad (5)$$

where P denotes the power level of the reactor (MW),
β, β_i, λ_i, Λ are effective nuclear constants,
C_i is amplitude related with the concentration of the i^{th} group of neutron precursors (atom/cm^3)
T_f is the average temperature of the fuel in the vessel (K)
T_{av} is the average temperature of the coolant in the vessel (K)
A is the area of the core (cm^2),
U is an effective heat transfer parameter for the core (MW/°K),
M_f is the effective fuel mass (tons),
C_{pf} is the specific heat coefficient for the fuel (MJ/°K),
M_C is the effective coolant mass (tons),
C_{pc} is the specific heat coefficient for the coolant (MJ/°K),
T_{out} and T_{in} are the temperatures of the coolant at the outlet and at the inlet of the vessel (K),
T^0_f and T^0_{av} are the temperatures of the fuel and of the coolant at the initial steady state (K),
ρ_{in} is the insertion of reactivity that generates the transient (dollars),
α_f and α_{av} are the reactivity feedback coefficients associated to the temperature of the fuel and of the coolant, respectively ((Δk/k/°K).

The numerical values considered for the neutronic constants (Concentrations of the Six Neutron Precursors Groups and Relative Decay Rate) and the steady state data are given in Table 1 and Table 2 respectively.

Table 1. Neutronic Constants

ß values	λ values
$ß_1 = 2.47 \times 10^{-4}$	$\lambda_1 = 1.27 \times 10^{-2}$
$ß_2 = 1.38 \times 10^{-3}$	$\lambda_2 = 3.17 \times 10^{-2}$
$ß_3 = 1.22 \times 10^{-3}$	$\lambda_3 = 1.15 \times 10^{-1}$
$ß_4 = 2.64 \times 10^{-4}$	$\lambda_4 = 3.11 \times 10^{-1}$
$ß_5 = 8.32 \times 10^{-4}$	$\lambda_5 = 1.40$
$ß_6 = 1.69 \times 10^{-4}$	$\lambda_6 = 3.87$
$ß = 6.5 \times 10^{-3}$	$\lambda = 5 \times 10^{-4}$

Table 2.Values of the dependent variables at Steady State

Variables at steady state	Values
P^0 (MW)	738.637
T^0_f (K)	1104.4
T^0_{av} (K)	549.79
T_{in} (K)	533
α_f	-4.19×10^{-3}
α_{av}	-3.70×10^{-2}
A_1	2.77×10^{-1}
A_3	4.10×10^{-1}

In Table 3, the main parameters of the reactor are reported.

Table 3. Reactor parameters

Parameters	Values
Fuel rods/assembly	14 x 14
Fuel rod diameter	1.07×10^{-2} m
Fuel rod assembly size	0.199 m
Fuel thermal conductivity	3.461 W/m/K
Fuel specific heat	2.428×10^2 J/kg/K
Fuel pellet diameter	1.016×10^{-2} m
Fuel density	1.028 x 10 kg/m^3
Clad thermal conductivity	15.23 W/m/K
Clad specific heat	2.428×10^2 J/kg/K
Clad density	6.487×10^3 kg/m^3
Clad thickness	0.254×10^{-3} m
Fuel-to-clad (gap) heat transfer coefficient	5.674×10^3 W/m^2/K
Pressure	15.282 MPa
Average inlet mass flux	3.362 x 103 kg/m^2/s
Coolant inlet temperature	533.15 K
Initial thermal power	738.637 MW

3. CONTROLLER DESIGN FOR NUCLEAR REACTOR

Well-developed analytical techniques that require accurate modeling of the process under control have been used successfully as a means of control for years. However, not all of the processes can be modeled with sufficient accuracy, and in some cases noisy operating conditions or drift in process variables may render the controller developed in this fashion useless, Torabi, Keivan et al. (2011). In this work, various controllers are developed for a validated model of the PWR type of nuclear reactor.

The desired performance criteria chosen for power level are,

- Minimum Overshoot (Relative)
- Quicker Settling Time (2% band).

3.1 PID Controller optimized by Genetic Algorithm (GAPID)

The structure of the PID controller is as follows:

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s \tag{6}$$

The best response can be achieved by proper tuning of K_P, K_I and K_D values of PID controller. Hence to obtain the best values of K_P, K_I and K_D, optimization algorithms are used. Genetic algorithm is one of the nontraditional optimization methods. GA tries to imitate natural genetics and natural selection. Survival of the fittest is the main philosophy behind the Genetic Algorithm. The genetic algorithm solves optimization problems by mimicking the principles of biological evolution, repeatedly modifying a population of individual points using rules modeled on gene combinations in biological reproduction.

The parameters of the Genetic Algorithm used for tuning PID controller are shown in the table 4.

Table 4. Parameters of the Genetic Algorithm

Parameters	Values
Number of population	100
Number of generation	50
Number of parameters to be optimized	3
Crossover probability	0.99
Mutation probability	0.01

The optimal values of K_P, K_I and K_D obtained by Genetic Algorithm are 1.0235, 0.9321 and 0.612 respectively. The response of the PWR type of nuclear reactor controlled by GAPID is shown in the following Fig. 2.

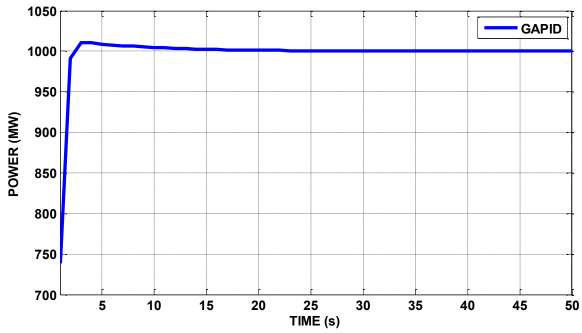

Fig. 2. Servo response of the nuclear reactor using GAPID controller

The settling time of this controller is around 20 seconds and it contains overshoot of about 15 MW.

3.2 Fractional Order PID Controller

PID controllers belong to the dominating form of feedback industrial controllers and there is a continuous effort to improve their quality and robustness. In recent years, there is an increasing number of studies related to the application of fractional controllers in many areas of science and engineering. Fractional Order PID (FO-PID) controllers could benefit the industry significantly with a wide spread impact when FO-PID parameter tuning techniques have been well developed. This fact is due to a better understanding of the Fractional Calculus (FC), Marzio, Marseguerra et al. (2004).

The FC concepts are adapted to frequency-based methods. The introduction of fractional order calculus idea to conventional controller design extends the opportunity of added performance improvement. Research activities are now focused to develop new tuning rules for fractional controllers for real systems. Some of these techniques are based on an extension of the classical PID control theory. Clearly, depending on the values of the orders λ and μ, we get an infinite number of choices for controller's type (defined through the (λ, μ)–plane). Conventional systems are derived from differential equations of integer order whereas fractional order systems are derived from fractional order differential equations (Zarabadipour, H et al.). Since PID control is popular in many industry sections, $PI^{\lambda}D^{\mu}$ controller should provide additional potentials to achieve better performance. The structure of the FOPID controller is shown in Fig. 3.

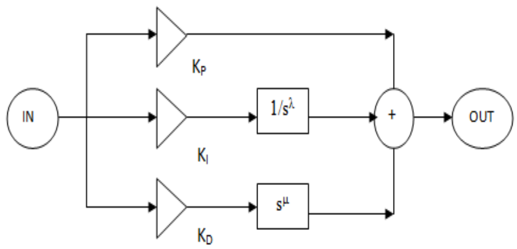

Fig. 3. Fractional Order PID control structure

$$G_c(s) = K_p + \frac{K_I}{s^{\lambda}} + K_D s^{\mu} \tag{7}$$

The overall closed loop transfer function can be written as,

$$\frac{y}{r} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)} \tag{8}$$

Fractional Order Closed Loop Characteristic Polynomial (FOCP) is given by,

$$1 + G_C(s)G_P(s) = 0 \tag{9}$$

On substituting the values of $G_C(s)$ and $G_P(s)$, the final value of K_P,K_I are derived as below,

$$K_P = \frac{\begin{bmatrix} (\sin(\lambda)*(\frac{\pi}{2}))*(\omega^2(d_2 n_0 - d_1 n_1) - d_0 n_0) \\ \\ +(\cos(\lambda)*(\frac{\pi}{2}))*(\omega(d_0 n_1 - d_1 n_0) - \omega^3(d_2 n_1)) \end{bmatrix}}{\{\omega^2(n_1)^2 + (n_0)^2\}*(\sin(\lambda)*(\frac{\pi}{2}))} \tag{10}$$

$$K_I = \frac{[(\omega^{\lambda})*[\omega^3(n_1 d_2) + \omega(n_0 d_1 - n_1 d_0)]}{\{\omega^2(n_1)^2 + (n_0)^2\}*(\sin(\lambda)*(\frac{\pi}{2}))} \tag{11}$$

ω changes from 0 to ω maximum, whose value is determined by substituting $K_I = 0$ .Using equations, K_P and K_I values are calculated for each value of λ (varying from 0.1 to 0.9, in steps of 0.1), by substituting ω from 0 to ω maximum. A stability curve in the K_P-K_I plane is constructed for each λ. All regions bounded in between stability curve and the stability line is represented as Global Stability Region. From the Global Stability Region, the average values of K_P and K_I corresponding to the each value of λ is obtained Among these values , the best fit of K_P average and K_I average and corresponding λ are identified by means of optimization techniques.

The 9th order transfer function model is obtained from the mathematical model of the nuclear reactor as shown below,

$$G_P(s) = \frac{\begin{matrix} 9602\,s^8 + 1.918e005\,s^7 + 8.968005\,s^6 + 1.321e006\,s^5 + \\ 6.776e005\,s^4 + 1.445e005\,s^3 + 1.24e004\,s^2 + 356.8s + 2.81 \end{matrix}}{\begin{matrix} s^9 + 32.97s^8 + 355.7s^7 + 1442\,s^6 + 2390\,s^5 + \\ 1767\,s^4 + 505.3s^3 + 49.56s^2 + 1.457\,s + 0.01142 \end{matrix}} \tag{12}$$

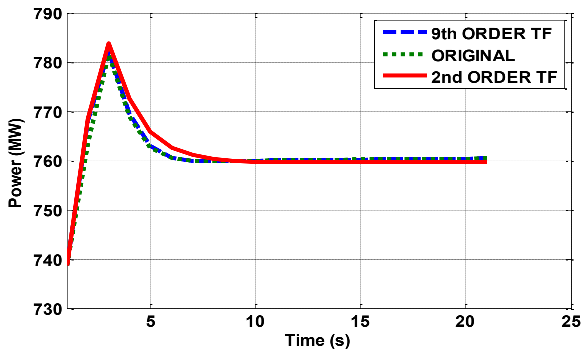

Fig. 4. Validation of the Transfer function

The 9th order transfer function model is reduced into second order transfer function by using Hankel singular value-based model reduction (Anuj et.al 2017).

The reduced second order transfer function is shown below,

$$G_P(s) = \frac{9500\,s + 2012}{s^2 + 12.7232\,s + 8.8662} \tag{13}$$

From Fig. 4, it is evident that the reduced second order transfer function is validated against the original model and 9th order model. The frequency response of the transfer function is also validated against the 9th order transfer function.

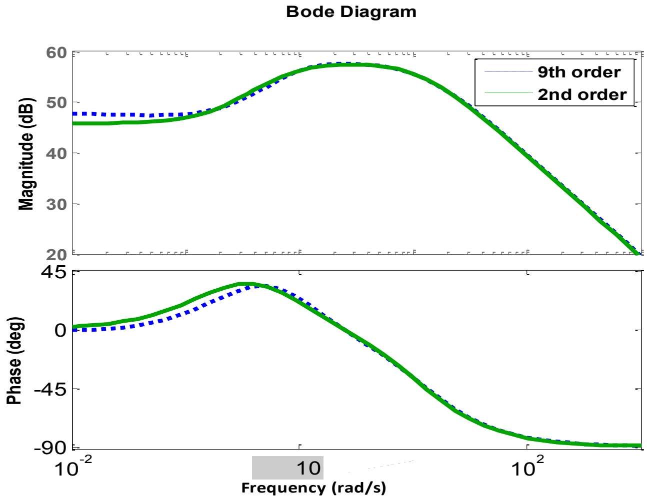

Fig. 5. Frequency of the original and reduced model

When $K_I = 0$, ω maximum is found to be 2.48. We will fix the value of K_D and find the stability region in the (K_P, K_I) plane. The value of K_D is fixed as 0.3912 and μ as 0.3968 from the stability plane, Subhransu Padhee et al. (2011). The average value of K_P and K_I for each value of λ is calculated and ISE, IAE of each tuning values are found out. From the values of ISE and IAE, the value of λ which gives minimum IAE and ISE is chosen as final K_P and K_I value. The performance indices comparison of various values of λ is shown in Table 5.

Table 5. Performance indices comparison of various values of λ

λ	K_P average	K_I average	ISE	IAE
0.1	0.3683	0.4235	1546	1522
0.2	0.25365	0.36321	1513	1510
0.3	0.3749	0.25656	1401	1505
0.4	0.30985	0.33902	1305	1436
0.5	**0.8653**	**0.9523**	**1115**	**1124**
0.6	0.1819	0.3886	1208	1356
0.7	0.1894	0.35196	1303	1346
0.8	0.184564	0.3298	1421	1436
0.9	0.190	0.31868	1525	1532

The response of the Fractional Order PID controller for controlling the nuclear reactor power is shown in Fig. 6.

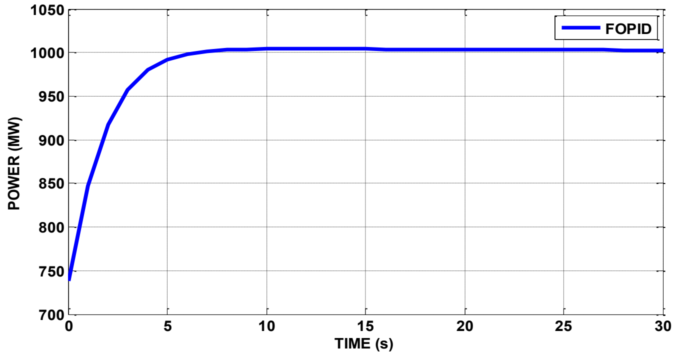

Fig 6. Servo response of the nuclear reactor using FOPID controller

The settling time of FOPID controller is about 10 sec.

3.3 Genetic Algorithm Based Fractional Order PID Controller

The workability of genetic algorithms (GAs) is based on Darwinian's theory of survival of the fittest. Genetic algorithms (GAs) may contain a chromosome, a gene, set of population, fitness, fitness function, breeding, mutation and selection. Genetic algorithms (GAs) begin with a set of solutions represented by chromosomes, called population. Solutions from one population are taken and used to form a new population, which is motivated by the possibility that the new population will be better than the old one. Further, solutions are selected according to their fitness to form new solutions, that is, offsprings.

In this work, Genetic Algorithm is used to optimize the FOPID parameters such as K_P, K_I, K_D, λ, and μ. The parameters of the Genetic Algorithm used for optimizing the FOPID controller are shown in Table 6.

Table 6. Parameters of the Genetic Algorithm

Parameters	Values
Number of population	50
Number of generation	50
Number of parameters to be optimized	5
Crossover probability	0.99
Mutation probability	0.01

The optimal values obtained from GA are K_P = 0.4299, K_I= 0.8878, λ= 0.7691, K_D=0.3912 and μ= 0.3968.

The response of the Genetic Algorithm based Fractional Order PID controller (GAFOPID) is shown in Fig. 7.

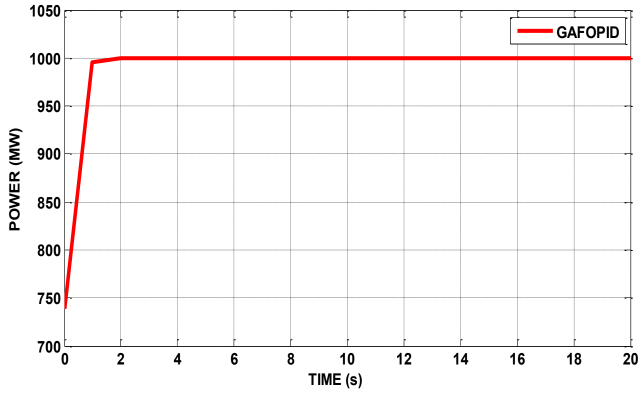

Fig. 7. Response of the GAFOPID controller

The settling time of GAFOPID controller is less than 2s which is very less when compared to other controllers. The servo operation comparisons of various controllers are shown Fig. 8.

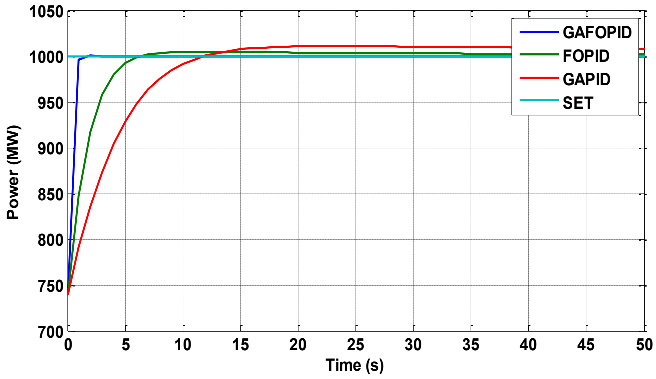

Fig. 8. Comparison of Servo Responses

The fuel temperature of the reactor acts as a disturbance variable. At 170s, the fuel temperature is increased by 100 ^{o}K. The regulatory response of the various controllers is shown in the Fig. 9. The GAFOPID controller rejects the disturbance effectively than other controllers.

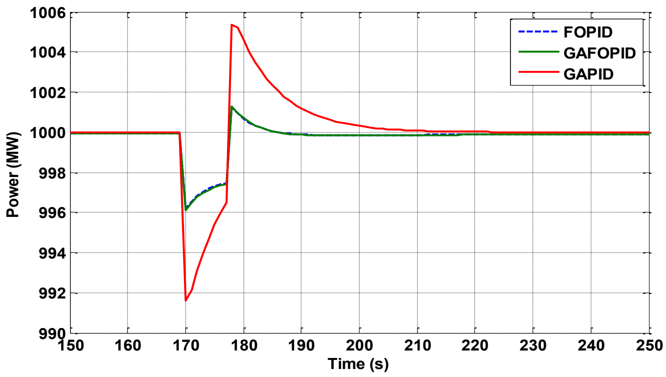

Fig. 9. Comparison of Regulatory Responses

The performance indices of various controllers are tabulated in Table 7.

Table 7. Comparison of Performance Indices

Types of Controllers	ISE	IAE	Settling time (s)
GAFOPID	1104	1110	2.2
FOPID	1115	1124	10.5
GAPID	1758	1765.8	20

4. CONCLUSIONS

The reduced model of the PWR type of nuclear reactor is developed by using the neutron balance equations and the heat exchange equations. The nuclear reactor is controlled by using Genetic Algorithm based PID controller, Fractional Order PID controller and Fractional Order PID controller optimized by Genetic Algorithm. From the results, it is clear that the Fractional Order PID controller optimized by Genetic Algorithm shows the satisfactory performance when it is compared to other controller strategies in both servo and regulatory level control.

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