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A Numerical Study for Plant-Independent Evaluation of Fractional-order PID Controller Performance [1]

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Abstract: A stunning outcome of fractional calculus for control practice are fractional-order PID (FOPID) controllers. Based on their experimental and numerical results, several studies have reported improvements in control performance of closed loop control systems by FOPID controllers compared to classical PID controllers. However, the industry at large is still cautious about adopting FOPID controllers because of the lack of concrete data about the related cost-benefit trade-off. Main concerns arise at the point that there have not been a quantitative evaluation scheme that clearly demonstrates for which concrete cases FOPID controllers can provide considerable improvements in control. Therefore, there is a need for more thorough theoretical and quantitative demonstrations. To that end, this study presents a plant function independent evaluation methodology to reveal inherent advantages of FOPID control. Impacts of two additional controller coefficients, namely fractional orders of differentiator and integrator, are analyzed in the frequency domain and their contributions to open loop gain maximization, phase margin and Reference to Disturbance Rate (RDR) performance are investigated.

Keywords: Fractional-order PID Control, PID controller, robust performance, stability, disturbance rejection.

1. INTRODUCTION

Robust control performance is a major concern for real world applications. Robustness against unpredictable plant parameter perturbations and environmental disturbances strongly depends on the characteristics of controllers offering robustness in stability and disturbance rejection. To this end, the concept of FOPID control was introduced in Podlubny et al. (1997). This extension in dynamic responses of integrator and derivative components suggests improved control laws by increasing span of controller response. By the tuning the orders of the integrator and differentiator, it may be possible to obtain a controller which is more robust against parametric system perturbation and uncertain disturbance models. Therefore, the common expectation is that FOPID controllers should become a substitute of classical PID controllers in practical applications (Chen et al. (2009)).

Classical PID control has a history of almost a century. Today, the PID controller is a well-known, fundamental and widely utilized controller structure that has become industrial standard. Nevertheless, based on the experimental and numerical results published in last two decades, many researchers have reported superior control performance of FOPID controllers compared to the performance of classical PID control (Chen et al. (2009); Padula and Visioli (2011); Tepljakov et al. (2014); Sundaravadivu et al. (2011); Tepljakov et al. (2016); Gole et al. (2012)). However, higher computational complexity in FOPID realization and corresponding rise in substitution costs put forth the questions when the improvements, provided by a FOPID controller, are affordable and the concerns about usefulness of these assets in improvement of control performance. From the industrial point of view, FOPID should offer a considerable cost-benefit advantage for specific control requirements to justify substituting existing PID controllers with FOPID controllers. There is clearly a need for a thorough investigation and theoretical clarification of the origins of performance improvements and how to benefit from them in applications. These issues are the

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main motivation of this study and in this paper we focus on the theoretical assessment of contribution of fractional-order dynamics to robust control performance.

Some analytical derivations proving contribution of FO control dynamics to robust performance were presented through fundamental plant assumptions in Vinagre et al. (2003). The demonstration was based on a template, so-called *iso-damping*, providing flatness of phase response of open loop fractional-order transfer function around the crossover frequency. In another study, fragility analysis of PID and FOPID has been presented to show sensitivity of controller performance to controller coefficients and some design and implementation concerns were explored for this controller (Alfaro (2007); Padula and Visioli (2016)). However, due to assumption of a specific plant model, dynamics originating from plant function may raise the concern of a loss of generality. To see exact contribution of the controller, plant independence in analyses have a potential to yield more common properties related with controller performance.

The current study applies a plant function independent assessment methodology to show common contributions of FOPID controllers to system stability and disturbance rejection performance. To eliminate plant function dependent dynamics from our analyses, we assumed an ideal plant function with $G(s) = 1$. Thus, effects caused by plant parameters such as amplitude, phase and delay, are eliminated. After isolation of closed loop control from plant models, the impact of FOPID controller integration and differentiator orders can be analyzed in frequency domain. In the paper, the contribution of tuning of these coefficients to robustness indicators such as open loop gain maximization, phase margin and RDR performance is discussed.

2. THEORETICAL BACKGROUND FOR FRACTIONAL ORDER SYSTEMS

Fractional-order systems are represented by time domain system models in the form of fractional-order differential equations (Chen et al. (2009))

$$\begin{aligned} &a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + a_0 D^{\alpha_0} y(t) = \\ &\quad b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + \cdots + b_0 D^{\beta_0} u(t) \end{aligned} \tag{1}$$

that can be expressed in s-domain in the form of FO transfer functions of the form

$$T(s) = \frac{Y(s)}{U(s)} = \sum_{i=0}^{m} b_j s^{\beta_j} \Big/ \sum_{i=0}^{n} a_i s^{\alpha_i}, \tag{2}$$

where denominator polynomial coefficients a_i and numerator polynomial coefficients b_j are real numbers. The $\alpha_i \in \mathbb{R}^+$ and $\beta_j \in \mathbb{R}^+$ are non-integer orders, and setting of $\alpha_0 = 0$ and $\beta_0 = 0$ constitutes constant terms of the fractional order transfer function (Chen et al. (2009); Xue and Chen (2015)).

Discrete realization of theoretical fractional order derivative is not so straightforward, because computations of ideal fractional order derivative are not localized at the current value of the function. To deal with high computational burden, integer-order approximate models have been widely utilized for fractional order system realization.

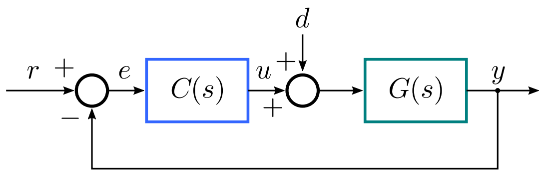


Fig. 1. Block diagram of a negative unity feedback control system with additive input disturbance

These are two fundamental methods for integer order approximation: continued fraction expansion (CFE) method (Vinagre et al. (2003); Chen et al. (2004)), and Oustaloup's approximation method (Oustaloup et al. (2000)).

The transfer function of a FOPID controller can be written in general form as

$$C(s) = k_p + \frac{k_i}{s^\lambda} + k_d s^\mu, \tag{3}$$

where parameters k_p, k_i and k_d are controller coefficients, and $\lambda > 0$ and $\mu > 0$ are fractional orders of controller function (Podlubny et al. (1997); Chen et al. (2009); Xu et al. (2016)). For realization of FOPID controllers, a retuning FOPID controller function, given by

$$C(s) = (C_R(s) + 1) C_{PID}(s), \tag{4}$$

where $C_R(s)$ is the transfer function of the retuning controller and $C_{PID}(s)$ is the transfer function of the original PID controller, was suggested to transform an existing PID-based control loops to FOPID control systems in Tepljakov et al. (2014, 2016).

3. PLANT INDEPENDENT FREQUENCY DOMAIN PERFORMANCE INDICATORS

Figure 1 depicts a fundamental block diagram of negative feedback closed loop control systems. Closed loop transfer function of this system is written by

$$T(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}. \tag{5}$$

To perform frequency response analysis, the open loop transfer function is given by,

$$H(s) = C(s)G(s). \tag{6}$$

When $H(s) \gg 1$, equation (5) states that $T(s) \to 1$ and hence $Y(s) \to R(s)$ (Ogata (2010)). This is an indication of improved control performance. Therefore, the maximization of $|H(j\omega)|$, leads to increase $\|H(s)\|_\infty$ improves control performance of system (Ogata (2010)). However, $|H(j\omega)|$is bounded by stability requirement of closed loop control system because unstable systems are useless in practical sense. Inside the stability bounds of the system, optimization of design parameters to maximize $|H(j\omega)|$ improves control performance. To avoid effects of plant function $G(s) = G_0(s)e^{-Ls}$ on performance evaluations, we assumed the plant function as an ideal communication channel modeled by $G(s) = 1e^{j0}$, which allows omitting contribution of plant function to amplitude and phase response by $G_0(s) = 1$ and the dead time delay of system by $L = 0$. In this case, one eliminates amplitude and phase contributions of the plant function because of $|G(j\omega)| = 1$ and $\angle G(j\omega) = 0$. Stability analyses of fractional order system can be carried out by considering phase and amplitude responses of open loop transfer function, which are expressed as

$$|H(j\omega)| = |C(j\omega)|\,|G(j\omega)| = |C(j\omega)| \tag{7}$$

and

$$\angle H(j\omega) = \angle C(j\omega) + \angle G_0(j\omega) - L\omega = \angle C(j\omega), \tag{8}$$

where $|H(j\omega)|$ and $\angle H(j\omega)$ are amplitude and phase responses of $H(j\omega)$ function and Bode diagrams are obtained by using these functions. For the stability analysis of transfer function, consideration of phase margin of closed loop control system with unit plant function is very useful, which is expressed by considering crossover frequency ω_c as

$$\theta_m = \angle C(j\omega_c) + k\pi \text{ for } |H(j\omega_c)| = 1, k = 1, 3, 5, ... \tag{9}$$

If the phase margin of the system is greater than zero ($\theta_m > 0$), the system becomes stable. By using equations (7) and (8), plant independent frequency response of FOPID controller can be written as

$$H(j\omega) = C(j\omega) = k_p + k_i \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + k_d \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu} + jk_d \sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - jk_i \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}, \tag{10}$$

$$|H(j\omega)| = \left(\left(k_p + k_i \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + k_d \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu}\right)^2 + \left(k_d \sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - k_i \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}\right)^2\right)^{\frac{1}{2}} \tag{11}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{k_d \sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - k_i \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}}{k_p + k_i \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + k_d \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu}}\right). \tag{12}$$

For a numerical assessment of disturbance rejection capacity of closed loop negative feedback control systems, Reference to Disturbance Rate (RDR) indices was proposed for unknown additive input disturbance model (Deniz et al. (2014); Alagoz et al. (2015a,b)). It has shown that disturbance rejection capacity of a negative feedback control system only depends on energy spectral density of controller transfer function (Alagoz et al. (2015a,b)). It is a plant independent measure that indicates strength of controller in a closed loop control. For improved disturbance rejection control, FOPID providing a high value of RDR should be designed,

$$RDR = |C(j\omega)|^2 \geq M, \quad M \gg 1. \tag{13}$$

For a quantitative evaluation of disturbance rejection capability of closed-loop FOPID control systems, the RDR index derived for unknown additive disturbance model can be expressed in the form (Alagoz et al. (2015a))

$$RDR(\omega) = \left(k_p + k_i \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + k_d \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu}\right)^2 + \left(k_d \sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - k_i \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}\right)^2. \tag{14}$$

Equation (14) provides a tool for the spectral analysis of disturbance rejection control performance of FOPID (Deniz et al. (2014); Alagoz et al. (2015a,b)). Recently, RDR objective can be employed for improvement of disturbance rejection control performance in a multi-objective controller tuning problem (Ates et al. (2017)).

Table 1 presents a summary of frequency domain robust performance indicators, which are considered in this study.

Table 1. List of some frequency domain robust performance indicators

Indicators	**Improvement direction**	**Explanation**
$\lvert H(j\omega)\rvert$	Increase	Improves control performance due to $T(s) \to 1$ for $\omega < \omega_c$
θ_m	Increase	Improves robust stability
RDR	Increase	Improves the disturbance and noise rejection performance

4. FREQUENCY DOMAIN PERFORMANCE EVALUATION OF FRACTIONAL-ORDER PID CONTROLLERS

To isolate effects of plant function and common design parameters in analysis of FOPID and classical PID, the following assumptions are applied:

(1) An ideal communication channel model of plant function is applied by unit plant function $G(s) = 1$.
(2) To neutralize effect of common coefficients of FOPID and PID controllers, the gain coefficients is set to one, which is $k_p = 1$, $k_i = 1$and $k_d = 1$.

These assumptions allow to evaluate pure and explicit contributions of fractional-order dynamics of the controller to the control loop. The dynamic contributions of FOPID control to open loop frequency response can be written as

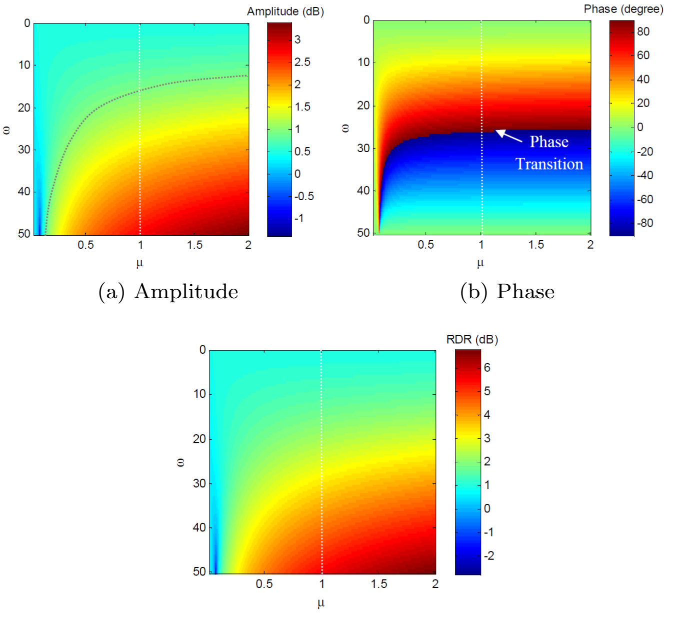
$$H(j\omega) = C(j\omega) = 1 + \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu} + j\left(\sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}\right). \tag{15}$$

Phase and amplitude response contributions of FOPID controller can be formulated as

$$|H(j\omega)| = \left(\left(1 + \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu}\right)^2 + \left(\sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}\right)^2\right)^{\frac{1}{2}} \tag{16}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}\mu\right)\omega^{\mu} - \sin\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda}}{1 + \cos\left(\frac{\pi}{2}\lambda\right)\omega^{-\lambda} + \cos\left(\frac{\pi}{2}\mu\right)\omega^{\mu}}\right). \tag{17}$$

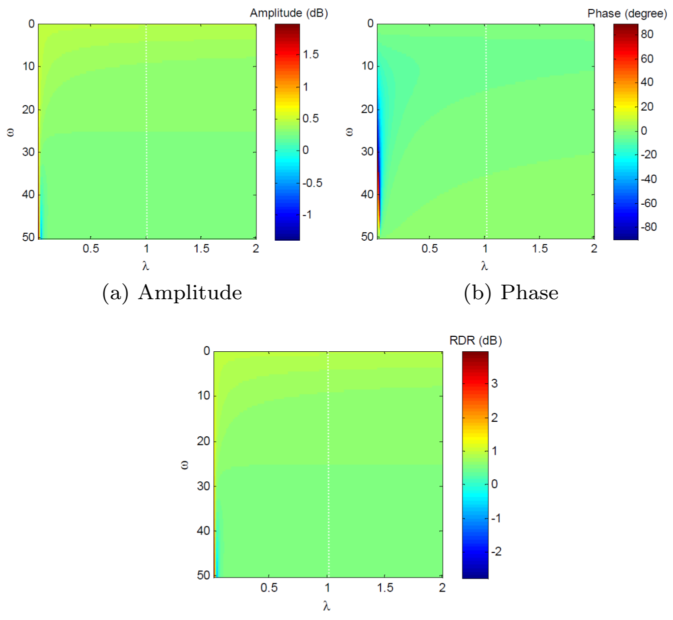
It is obvious that by configuring fractional orders to one ($\lambda = 1$ and $\mu = 1$) in equations (15), (16), and (17), one obtains the frequency response of a classical PID controller. Therefore, we carry out comparison of FOPID and classical PID controller performance by considering order ranges in the form of $\lambda \in [1 - \frac{\Delta\lambda}{2}, 1 + \frac{\Delta\lambda}{2}]$ and $\mu \in [1 - \frac{\Delta\mu}{2}, 1 + \frac{\Delta\mu}{2}]$. Thus, effects of additional two parameters of FOPID



(a) Amplitude (b) Phase

(c) RDR spectrum

Fig. 2. Frequency characteristics of the controllers for $\omega \in [0, 50]$ with an increment of 0.1rad/s versus $\mu \in [0, 2]$ with 0.002 increments



(a) Amplitude (b) Phase

(c) RDR spectrum

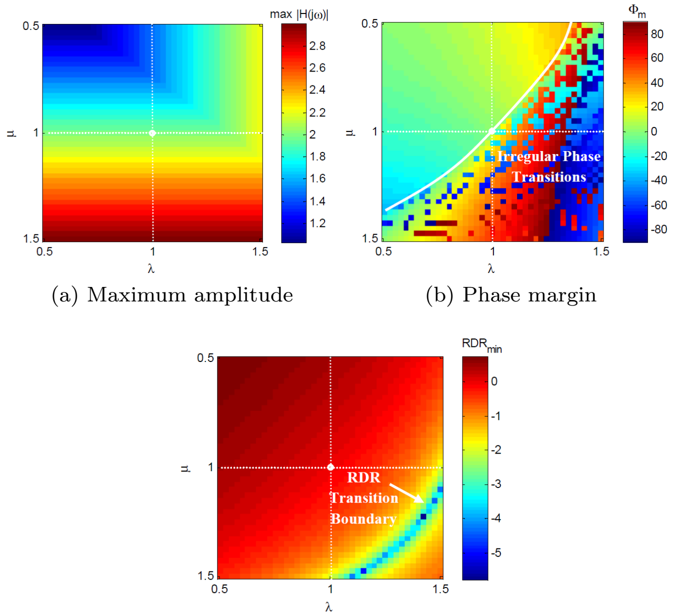
Fig. 3. Frequency characteristics of the controllers for $\omega \in [0, 50]$ with an increment of 0.1rad/s versus $\lambda \in [0, 2]$ with 0.002 increments

controller on the control performance can be evaluated and one can observe that contributions of fractional order dynamics to classical PID control.

Figure 2 shows values of $|H(j\omega)|$, $\angle H(j\omega)$ and $RDR(\omega)$ functions in $\omega \in [0, 50]$ with an increment of 0.1rad/s versus $\mu \in [0, 2]$ with 0.002 increments, while $\lambda = 0$ was configured to observe only the contribution of μ order. The white dash line indicates the responses for $\mu = 1$, which refers the case of a classical PID controller differentiator. The gray dash curve shows the crossover frequency of FOPID controller, which is approximately drawn and reveals the frequencies collection satisfying to$|H(j\omega)| = 1$. Amplitude response in Figure 2a reveals that $|H(j\omega)|$ can be increased by μ and this offers the potential of control performance improvement for FOPID. Phase response in Figure 2b shows dependence of phase transition region to μ parameter. This dependence can be utilized to configure an appropriate phase to controller so that it can stabilize control system. For stabilization of system, the phase requirement, $\angle C(j\omega^*) + \angle G_0(j\omega^*) - L\omega^* > -k\pi$, $|H(j\omega^*)| = 1$,$k = 1, 3, 5, ...$, should be satisfied, which infers a positive phase margin of closed loop control system. The line of a sharp phase transition in the figure increases the risk of instability in realization due to increase of sensitivity to μ parameter and makes fractional-order derivative critical for robust stability. Sensitivity of robust stability to fractional order derivative was previously reported in Padula and Visioli (2016) and it is consistent with our expectation related to instability risks arisen in the configurations in the vicinity of phase transition line. However, for the high values of μ, RDR performance can be further improved in Figure 2c.

Figure 3 shows values of $|H(j\omega)|$, $\angle H(j\omega)$ and $RDR(\omega)$ functions in $\omega \in [0, 50]$ with an increment of 0.1rad/s versus $\lambda \in [0, 2]$ with 0.002 increments, while $\mu = 0$ was set to observe only the contribution of λ order to frequency response. The white dash line also indicates the response for the case of $\lambda = 1$, which refers to an integer of PID controller. One can observe that fractional-order integrator can increase $|H(j\omega)|$ and $RDR(\omega)$ at low frequency region, and contributes to set-point control performance as value of λ decreases. The analyses did not indicate a transition region formation in the parameter ranges, which can lead to a high sensitivity to λ order.

Figure 4 shows values of $\max |H(j\omega)|$, ϕ_m and $\min RDR(\omega)$ functions of FOPID controllers in $\omega \in [0, 30]$ with an increment of 0.01rad/s versus $\lambda \in [0.5, 1.5]$ with 0.025 increments and $\mu \in [0.5, 1.5]$ with 0.025 increments. Intersection of $\lambda = 1$ and $\mu = 1$ lines (white dash lines) represents the performance of a classical PID control dynamics. Figure 4a shows contribution of λ and μ couple to $\max |H(j\omega)|$, where a high value of $\max |H(j\omega)|$ indicates improved set-point performance. This figure clearly shows that FOPID control dynamics can improve control performance. Figure 4b shows contribution of λ and μ together to phase margin ϕ_m. Increase in option of phase margin provided by FOPID controller enhances stabilization skill of control systems in controller tuning task. However, an irregular phase transition region below to white curve can lead to a fragility of system stability for non-ideal FOPID implementation cases because approximate implementation of FOPID causes imprecise configuration of fractional orders and this may lead to occurrence of an undesired phase and hence serious performance degradation in this irregular region. For realization of FOPID controller coinciding in this region, approximate implementation of FOPID controllers should be carried out with the highest accuracy around the crossover frequency ($|H(j\omega_c)| = 1$) to avoid undesired phase errors. These irregular transition regions can be a reason of high robustness fragility index reported in Padula and Visioli (2016). This study sug-



(a) Maximum amplitude (b) Phase margin

(c) Minimum RDR spectrum

Fig. 4. Frequency characteristics of the controllers for $\omega \in [0, 30]$ with an increment of 0.01rad/s

gested that FOPID controller is much more fragile in the sense of robustness than the PID controller, therefore fine tuning of FOPID is more critical. Irregular phase transition region can be a source of high sensitivity of robustness to FOPID controller coefficients. Figure 4c reveals that RDR performance can be further improved by FOPID controller. Besides, there is a transition region of RDR, which may cause very poor RDR performance and this region should be considered in FOPID controller designs.

5. ADDITIONAL DYNAMICS OF FRACTIONAL-ORDER PID CONTROLLERS COMPARED TO CLASSICAL PID CONTROLLERS

The additional dynamics introduced by FOPID controllers relative to classical PID controllers can be expressed as

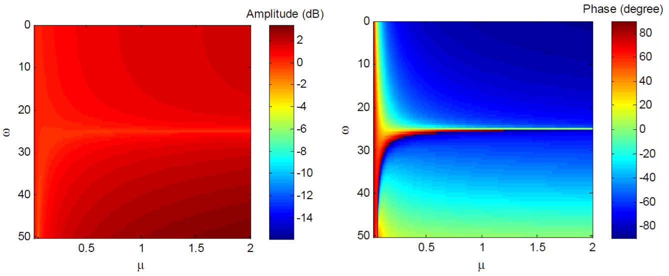
$$\Delta C(s) = C_{fopid}(s) - C_{pid}(s). \tag{18}$$

Then, the additional dynamics of FOPID with respect to classical PID can be written by

$$\Delta C(s) = k_i(s^{-\lambda} - s^{-1}) + k_d(s^{\mu} - s). \tag{19}$$

By using $s = j\omega$, the frequency response can be expressed as

$$\Delta C(j\omega) = k_i\omega^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + k_d\omega^{\mu}\cos\left(\frac{\pi}{2}\mu\right) + j\left(k_i\omega^{-1} - k_d\omega + k_d\omega^{\mu}\sin\left(\frac{\pi}{2}\mu\right) - k_i\omega^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right)\right), \tag{20}$$



(a) Amplitude (b) Phase

Fig. 5. Frequency characteristics of the additional controller dynamics for $\omega \in [0, 30]$ for $\mu \in [0, 2]$

$$|\Delta C(j\omega)| = \left(\left(k_i\omega^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + k_d\omega^{\mu}\cos\left(\frac{\pi}{2}\mu\right)\right)^2 + \left(k_i\omega^{-1} - k_d\omega + k_d\omega^{\mu}\sin\left(\frac{\pi}{2}\mu\right) - k_i\omega^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right)\right)^2\right)^{\frac{1}{2}}, \tag{21}$$

$$\angle\Delta C(j\omega) = \tan^{-1}\left(\frac{C_b(j\omega)}{C_a(j\omega)}\right), \tag{22}$$

where

$$C_b(j\omega) = k_i\omega^{-1} - k_d\omega + k_d\omega^{\mu}\sin\left(\frac{\pi}{2}\mu\right) - k_i\omega^{-\lambda}\sin\left(\frac{\pi}{2}\lambda\right), \tag{23}$$

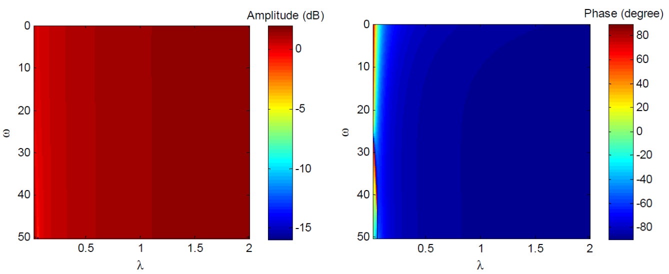
and

$$C_a(j\omega) = k_i\omega^{-\lambda}\cos\left(\frac{\pi}{2}\lambda\right) + k_d\omega^{\mu}\cos\left(\frac{\pi}{2}\mu\right). \tag{24}$$

Contributions of additional dynamics of FOPID controller to frequency domain properties of classical PID controller are presented in Figures 5 and 6. To eliminate gain coefficients in analyses, k_i and k_d coefficients were set to one. Figure 5a shows $|\Delta C(j\omega)|$ and Figure 5b shows $\angle\Delta C(j\omega)$ in $\omega \in [0, 50]$ with an increment of 0.1rad/s and $\mu \in [0, 2]$ with 0.002 increments, where $\lambda = 0$. We observed that increase of μ adds additional amplitude on the amplitude response of classical PID in high and low frequency parts. It adds negative phases at the both sides of $\omega = 24$rad/s in Figure 5b. Figure 6a shows $|\Delta C(j\omega)|$ and Figure 6b shows $\angle\Delta C(j\omega)$ in $\omega \in [0, 50]$ with an increment of 0.1rad/s and $\lambda \in [0, 2]$ with 0.002 increments, where $\mu = 0$. Increase of λ adds additional amplitude on the amplitude response of classical PID uniformly. It removes phases from the phase response.

6. CONCLUSIONS

This paper demonstrates contribution of FOPID controller dynamics to frequency response of a control system by isolating effects of FO dynamics from effects of other parameters on the frequency response performance indicators. This type of analysis can be helpful for numerically proving, classifying and evaluating the contributions of FOPID control to control loop performance. In phase margin analyses, accuracy in finding crossover frequency by solving $|H(j\omega_c)| = 1$ depends on sampling period of an-



(a) Amplitude (b) Phase

Fig. 6. Frequency characteristics of the additional controller dynamics for $\omega \in [0, 30]$ for $\lambda \in [0, 2]$

gular frequency ω. Therefore, for more accurate calculation of phase margin, angular frequency ω should be refined.

In this work, an ideal communication channel model is used for the model of plant function. This can be seen as a weakness in the overall discussion, since from a practical perspective real-life process control loops rarely enjoy this favorable property. Therefore, one important future direction of this study is related to the practical verification of the proposed approach. Meanwhile, the results of the present work can be summarized as follows:

- The order of FO differentiation μ allows further increase $\|H(s)\|_\infty$ and thus can contribute to improvement of set-point control performance. It also allows for further adjustment of phase transition region to improve stability of closed loop control systems by providing positive phase margin.
- The set-point control performance of FOPID can be further improved by tuning λ and μ for higher $\max|H(j\omega)|$.
- According to analysis results, it is evident that tuning λ and μ of FOPID controller provides more options to stabilize a control system, however there are regular and irregular transition regions where sensitivity to λ and μ increase sharply. The designs coinciding in sharp phase transition region can cause a fragility of control performance. In these regions, accuracy of approximation method should be maximized. These regions can be related to high robustness fragility of FOPID controller compared to classical PID controller reported in Padula and Visioli (2016).
- The disturbance rejection control performance of FOPID can be further improved by tuning λ and μ for higher RDR values. However, the transition region in min RDR characteristics should be taken into account to avoid poor RDR performance designs.

A further study can be conducted for considering other performance indices to have a complete view of behavior of FOPID controllers.

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