FOPID Controllers and Their Industrial Applications: A Survey of Recent Results [1]

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Abstract: The interest towards using Fractional-order (FO) PID controllers in the industry is mainly fueled by the fact that these controllers have two additional "tuning knobs" that can be used to adjust the control law in a way that would benefit the control loop. However, there are certain points that are rarely addressed in literature, namely: (1) What are the particular advantages (in concrete numbers) of FOPID controllers versus conventional, integer-order (IO) PID controllers in the light of complexities arising in the implementation of the former? (2) For real-time implementation of FOPID controllers, approximations are used that are equivalent to high order linear controllers. What, then, is the benefit of using FOPID controllers? In the present paper, we attempt to address these issues by reviewing recent literature in the field and by providing relevant analysis and recommendations.

Keywords: Fractional-order PID Control, Industrial applications, Frequency domain analysis

1. INTRODUCTION

PID controllers are widely used for industrial process control. While more powerful control techniques are readily available, the PID controller is still popular due to its relative simplicity and applicability to a wide range of industrial control problems (Åström and Hägglund (2006)). However, it is a commonly acknowledged fact that only a fraction of the existing PI/PID controller based loops are tuned to achieve optimal performance (O'Dwyer (2009)).

In recent years, the emergence of fractional calculus has made possible the transition from classical models and controllers to those described by differential equations of noninteger order. Thus, fractional-order dynamic models and controllers were introduced. The parallel form of the fractional-order PID controller has been proposed in Podlubny et al. (1997)

$$G_c(s) = \frac{U(s)}{E(s)} = K_P + K_I s^{-\lambda} + K_D s^{\mu}, \qquad (1)$$

where $(\lambda, \mu) > 0$. Such a controller has more tuning freedom and thus also a wider region of parameters that stabilize the plant under control, and offers improvements in control loop robustness. Corresponding studies have been carried out to confirm this (see, e.g., Xue et al. (2006); Čech and Schlegel (2006); Luo and Chen (2012)). Other FOPID control loop structures have been reviewed in, e.g., Tavakoli-Kakhki and Haeri (2011); Padula and Visioli (2013); Azarmi et al. (2015).

However, even though FOPID controllers offer advantages over IOPID controllers, the adoption of the former in the industry is slow (Chen (2006)). In this paper, we aim to explore the reasons for this by conducting a survey of recent results related to advantages of FOPID controllers, their implementation, and industrial applications. Furthermore, we focus on particular advantages of using FOPID controllers stemming from frequency domain analysis and also provide relevant conclusions. Furthermore, we also discuss the possible issues related to industrialization of FOPID controllers.

The rest of the paper is organized as follows. In Section 2, performance and implementation comparison of IOPID and FOPID controllers is done based on recent publications. In Section 3, recent research related to the prospective industrial use of FOPID controllers is reviewed. In Section 4, we attempt to cover the issue of comparing FOPID controller implementations to general high order integer-order controllers. In Section 5, heuristic tuning methods for FOPID controllers are reviewed. Then, in Section 6 the question of industrialization of FOPID controllers is addressed, and some related patents are reviewed. In Sec-

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tion 7 general items for discussion are provided. Finally, in Section 8 conclusions are drawn.

2. PERFORMANCE AND IMPLEMENTATION COMPARISON OF IOPID AND FOPID CONTROLLERS

Based on the feedback from industrial partners (Vansovits et al. (2012)), the main issue of adopting FOPID controllers in the industry can be formulated as follows: "Can the complexity of implementation of FOPID controllers outweigh the benefits of additional tuning flexibility?". Indeed, in the most basic case, to implement a conventional PID controller digitally one just implements the corresponding control law in software

$$u(k) = K_p e(k) + K_i \sum_{j=0}^{n} e(k) + K_d \left(e(k) - e(k-1)\right), \quad (2)$$

whereas for a FOPID controller one usually has to use approximations which are often more complicated and require considerable computational resources. Still, modern embedded software solutions have been found to easily handle the additional implementation complexity (Tepljakov (2017)). Thus, in what follows, we focus on the benefits of FOPID controllers with respect to achievable performance improvement.

In general, to make a valid comparison between the performance of IOPID and FOPID controllers one can turn to *global optimization* based methods for tuning both controllers because that way the best possible controller gains and orders are assumed to be obtained (Beschi et al. (2015)). We expand on this issue in Section 5. In Merrikh-Bayat and Jamshidi (2013), IOPID and FOPID controllers are designed for the control of a nonlinear boost converter using an artificial bee colony algorithm. Based on simulation results, the authors conclude that the "proposed FOPID controller can improve the startup response of the boost converter by using less on-off switching actions compared to the optimal PID controller" and stress the practical importance of this result. This essentially means an improvement in the control law. Moreover, better rejection of disturbances and better output voltage regulation are also cited as advantages. Further, authors of the conference paper de Castro et al. (2016) applied both IOPID and FOPID controllers to a liquid level control problem while tuning both using genetic programming. The results show that "... the $\mathrm{PI}^{\lambda}\mathrm{D}^{\mu}$ has performed *slightly* better for the response signal ...". However, one remarkable result is that a more desirable control law is obtained in this case as well by measuring the *variance* of $u(t)$. For IOPID it measures 3.44, while for FOPID the value obtained is 1.39. Furthermore, the same conclusion regarding the reduction of the control effort is reached in Marinangeli et al. (2018). This result can be considered very important in cases where reducing energy due to control effort matters, like process systems and precision positioning systems where the generation of heat reduces precision.

In Badri and Tavazoei (2015), the design of a FOPD motion controller was investigated. One important item in the conclusions reads "... the inefficiency of PID controllers for simultaneously ensuring specifications [was shown] ... in the cases that these specifications are simultaneously achievable by an FOPD controller ..." thus showcasing the tuning flexibility of FOPID controllers compared to IOPID controllers.

Let us also consider related critical research. One interesting critical work is that of Padula and Visioli (2016) which uses the term *fragility* with respect to FOPID controllers designed for FOPDT plants. In the paper, the authors assume that the parameters of the controller are subject to variation and thus devise measures to study this scenario. One practical example is when the parameters of a tuned FOPID controller are changed manually. In this case, having an idea about the *fragility* of the FOPID controller would be useful. There is also a critical paper published in a Russian journal (Zhmud and Zavorin (2013)) the title of which can be translated to English as "On the Inadvisability of using Fractional-order PID Controllers". The main conclusion of the paper is that by applying a proper optimization procedure it is possible to obtain IOPID controllers that are superior to FOPID controllers obtained in Bettou and Charef (2009). However, the premise itself is based on insufficient evidence, so the conclusion, taken more generally, is questionable.

The advantages of FOPID controllers can be seen most clearly by invoking frequency domain analysis and control design methods. Seminal works on the subject include Monje et al. (2008, 2010); Sabatier et al. (2015). Results on H_∞ based design methods were reported by Padula et al. (2013). A metric for measuring reference-to-disturbance ratio (RDR) was proposed in Alagoz et al. (2015) which can be used to compare the performance of IOPID and FOPID controllers. Robust design for parametric variations of control systems can be achieved also through the assessment of maximum sensitivity properties thereof. A related study is conducted in Ranganayakulu et al. (2016).

A clear theoretical demonstration of the advantages of fractional-order control was provided by Vinagre et al. (2003) by considering Bode's ideal transfer function. The system under study is composed of a fractional controller in the form $C(s) = s^\gamma$ and double integrator plant function in the form of $G(s) = A/s^2$. The plant function is used to model a fundamental system that represents single-degree-of-freedom translational and rotational motion in the field of robotics. The corresponding closed-loop system is in the form of Bode's ideal transfer function $G_{CL}(s) = A/(s^{2-\gamma} + A)$. The main advantages of this closed loop system are:

(1) *Gain margin is infinite.* This property provides the advantage of the system being insensitive to gain changes;
(2) *The phase margin is constant at* $\varphi_m = \pi(1-(2-r)/2)$. The variations of the gain result in the change of the crossover frequency ω_c, but the phase margin of the system is unchanged.

The desired robustness motivated the use of fractional-order controllers in classical control systems to enhance their performance (Oustaloup (1995); Chen et al. (2009); Pommier-Budinger et al. (2008); Banos et al. (2008)). A comparitive introduction of CRONE and TID type controllers was provided in Xue and Chen (2002). Chen et al. demonstrated that several fractional-order controllers can be tuned to meet the criterion of robustness to gain variations which is given by

$$\left(\frac{\mathrm{d}\arg\left(C(j\omega)P(j\omega)\right)}{\mathrm{d}\omega}\right)_{\omega=\omega_c} = 0. \tag{3}$$

This property is known as *iso-damping* and has been frequently cited as one of the most attractive features of FO control (Monje et al. (2010)) and forms in part the basis of the so called *fractal robustness* (Oustaloup (1995)). It was further investigated in, e.g., Pommier-Budinger et al. (2008); Feliu-Batlle et al. (2009); Sabatier et al. (2015); Beschi et al. (2015); Azarmi et al. (2016); Bettayeb et al. (2016).

3. INDUSTRIAL APPLICATIONS OF FOPID CONTROL

With respect to industrial use of FOPID controllers, a survey paper is immediately found (Efe (2011)). The main content of the paper comprises the description of various classical control techniques (PID control, sliding mode control, backstepping, MRAC) extended to make use of fractional-order calculus. There is a single example of a control problem provided where a control system is developed for a cement mill.

In Ghasemi et al. (2014), a fractional controller is designed for a *wind turbine generator*. Here, based on simulations, the authors claim the following: (1) "... while the fractional order PI controller ... properly tracks the input command, the simple integer order PI controller is not capable to cope with the nonlinearity due to backlash phenomenon." and (2) "... the fractional order control system accurately tracks the reference input [under plant parameter variation] ... however, the integer order control system becomes unstable [under the same conditions]" once again highlighting advantages of FOPID controllers.

In Jiangbo and Junzheng (2015), a FOPI controller is developed for an *electro-hydraulic system* with a particular emphasis on the energy saving aspect. To validate the performance of the obtained control system, a PI controller is designed and experiments with both PI and FOPI controllers are conducted (including variants thereof where an *orifice compensation* (OC) system is added). The authors claim that the FOPI+OC method "has the minimum tracking error, and common PI controller has the biggest one."

The authors of Azarmi et al. (2015) design FOPID controllers for *twin rotor systems*. Here, the authors evidently show that "... disturbance rejection by applying the [setpoint weighted FOPID] controller is always better done in comparison with the designed [setpoint weighted PID], [filtered FOPI] and [filtered PI] controllers ...".

In Hassan and Zolotas (2017) the impact of FO methods on *tilt control of rail vehicles* is discussed. Here, both the design of FOPID controllers, and the reduction of corresponding approximations are investigated. The results of FOPID control are compared to that achieved with conventional PID controllers. To cite the authors, "[Relevant figure] illustrates the immense benefit of fractional order based control on improving tilt following (with full order control)."

In Lino et al. (2017), FOPI controllers were designed for *industrial electrical drives*. The papers deals with a particular design method. To cite the authors directly, "It is remarkable that the fractional pre-filters almost cancel the oscillations. The improvement is even more relevant in the case of speed control. The overshoot is greatly reduced and the settling and rise times are also reduced with respect to the PI-controlled system."

Finally, in Krijnen et al. (2017), a *precision positioning system* based of FO control is designed and analyzed. Two cascaded control loops with decoupled SISO controllers are implemented for the moving mass controlled on a mass-spring system which can be modeled as a fourth order system. Applying fractional order derivative compared to integer order one, the bandwidths are extended by 14.6% and 62%, for the inner and outer loop respectively. A closed-loop positioning bandwidth of the wafer of 60Hz is achieved, resulting in a positioning error of 104nm, which is limited by sensor noise and pressure disturbances.

4. FOPID CONTROLLERS VERSUS HIGH ORDER INTEGER ORDER CONTROLLERS

For real-life implementation of FOPID controllers high order IO transfer function approximations are generally used (Monje et al. (2010)). This brings about the question: "If integer-order approximations are used anyway, why not just use high order integer-order controllers instead of FOPID approximations?" Although this is a very important issue from the modeling standpoint, surprisingly few publications deal with this matter. A few (almost) relevant papers are reviewed below.

In Merrikh-Bayat (2012), the author proposes some rules for selecting the parameters for approximating FOPID controllers using the Oustaloup recursive filter method. This can potentially help to reduce the order of the resulting controller. However, this does not solve the "FOPID versus high order IO controller" issue that is the topic of this section.

In one instance, the authors of Mansouri et al. (2010) suggest to use FO models for "compressing" high order integer-order models. The proposed method has certain limitations (e.g., it works only for real transfer function poles). In Barbé et al. (2013) this approach is extended. Here, the authors use fractional-order models for creating compact models obtained by (1) identifying a high order integer-order model; (2) converting the model to a fractional-order one; (3) optimizing the fractional-order model. Both of these contributions propose the basic idea that FO models can be used for compact description of high order systems but offer little with respect to the FO control discussion.

Clearly, more relevant research devoted to this issue is required. As a preliminary conclusion, we consider the following:

(1) FOPID controllers are *not* generally speaking equivalent to high order integer-order controllers, and we also assume that an ideal implementation of a FOPID controller is available.

(2) We focus on the fact that a FOPID controller is an extension to an IOPID controller and treat its two additional parameters as "tuning knobs".

It is also apparent that managing (e.g., tuning) all of the parameters of a high order integer-order controller is more difficult than in case of a FOPID controller.

5. HEURISTIC OPTIMIZATION METHODS IN THE TUNING OF FOPID CONTROLLERS

Heuristic optimization techniques are commonly used for optimization of high complexity problems and perform set-trail search to find out an optimal solution. This property makes it a straightforward solution even for online tuning of parameters of real systems. Since industry looks for shortcut techniques to fix the problems, heuristic optimization techniques can provide a low-cost solution for FOPID tuning of real time control systems. Control literature has extensively demonstrated utilization and advantage of heuristic optimization techniques (Tepljakov et al. (2012); Ateş and Yeroglu (2016)). Many recent works have addressed several heuristic optimization methods for FOPID controller tuning in an offline (simulation based) and online (experimental based) manners. Some studies, presenting with its application, are as follows.

Zamani et al. (2017) used a multi-objective cuckoo search optimization approach for FOPID tuning to adjust the contact force of piezoelectric friction dampers for semi-active control of base-isolated structures during far-field and near-field earthquake excitations. In Li et al. (2017b), a gravitational search algorithm combined with the Cauchy and Gaussian mutation, named as CGGSA, is proposed and used to optimize the FOPID controller parameters to control a pump turbine governing system (PTGS). From the perspective of multi-objective optimization, Zeng et al. (2015) proposed a novel FOPID controller design method based on an improved multi-objective external optimization (MOEO) algorithm for an automatic voltage regulator (AVR) system. The effectiveness of the proposed MOEO-FOPID algorithm on AVR system was also demonstrated. In Xu et al. (2016), gravitational search algorithm was improved for FOPID design to control pumped storage hydro unit. The results are compared with PSO (particle swarm optimization), GSA (gravitational search algorithm) and BCGSA (bacterial-foraging chemotaxis gravitational search algorithm). In Haji and Monje (2017), further enhancement of the PSO algorithm's rate of convergence and the minimization of the fitness function was achieved based on dynamic control parameters selection. The method was applied to fractional order fuzzy-PID control of a combined cycle power plant. The results are compared with several different evolutionary algorithms. In Yeroğlu and Ateş (2014), an online auto-tuning of FOPID controllers on a real time running experimental test system (Twin Rotor MIMO System-TRMS) was performed using Stochastic Multi-parameter Divergence Optimization (SMDO) algorithm. In Alagoz et al. (2013), SMDO algorithm was used for tuning FOPID parameters to track the response of the Bode's ideal control loop. The results are used to control DC Rotor of an experimental test system (TRMS). Many other heuristic optimization algorithms were proposed for FOPID tuning for different industrial applications (Li et al. (2016); Aghababa (2015); Wang et al. (2017); Li et al. (2017a)).

As results of random search nature of heuristic optimization methods, the objectivity of assessments based on a single test is somewhat questionable because of unrepeatability and possibility of being stuck at a local minimum. To make findings of these studies more solid, a statistical evaluation of results is necessary. Hence, in comparison of controller, consideration of an average value of performance indices and revealing standard deviation of results for repeated tests is important in publications to increase objectivity and consistency of findings of heuristic optimization algorithms.

Consequently, heuristic optimization algorithms are low computational complexity tools for multi-objective and constrained optimization, that is, it is relatively easy to implement these algorithms. As a future perspective, heuristic optimization can be utilized in the realization of plug&play type FOPID controller devices implemented on low-cost programmable control cards for industrial applications.

6. INDUSTRIALIZATION OF FOPID CONTROLLERS

Industrialization of controllers is much of a challenge nowadays in a competitive world where companies tend to invest resources in order to bring down the cost of goods and improve the products' benefits to the customers. As it was demonstrated, FOPID controllers have technical advantages over its integer-order counterparts, but the cost of producing such controllers and the cost-benefit the end user would get are still something that should be investigated. Where a FOPID controller shows better technical performance in a twin rotor system in Azarmi et al. (2015), it does not necessarily mean that the cost of creating such FOPID controllers for commercial or industrial use is similar to ordinary PID controllers and its families. In such a case, it is suggested to look how to industrialized FOPID controllers considering financial factors. This is an open question that needs to be addressed in future research.

Patents can be seen as important factors in the industrialization process. To conclude this section, we investigate patents related to fractional-order control and implementation thereof.

The TID controller patent (Lurie (1994)) describes a *tilt-integral-derivative* controller. It is similar to a PID controller, but the proportional gain is replaced with the *tilt* component that has a transfer function of $s^{-1/n}$. The advantage of this controller was explained in Chen et al. (2009) as "The resulting transfer function of the entire compensator more closely approximates an optimal loop transfer function, thereby achieving improved feedback control performance. Further, as compared to conventional PID compensators, the TID compensator allows for simpler tuning, better disturbance rejection, and smaller effects of plant parameter variations on the closed-loop response".

Apart from that, only a few US-based patents can be found. In Abbisso et al. (2004), an invention implementing *noninteger* (i.e., fractional) systems is described and is based on artificial neural networks. In Chen (2009), a tuning method for fractional controllers is proposed. Finally, in Almadhoun et al. (2016) a fractional-order capacitor is described. This latter patent could be very important in the sense of achieving an ideal implementation of FO operators, and, as a direct consequence, also of FOPID-type controllers.

7. DISCUSSION

Due to the lack of coherent research results related to comparing high order IO approximations of FOPID controllers and high order IO controllers, this item is left for discussion. In addition, any feedback related to industrial adoption of FOPID controllers and corresponding issues is welcome.

8. CONCLUSIONS

Based on the evidence reviewed in the present paper, the following conclusions can be drawn:

- FOPID controllers offer clear advantages over IOPID controllers as seen from both simulations and experiments with real-life objects, the comparison is solidly based on global optimization based tuning for both types of controllers.
- One significant advantage of FOPID controllers when applied to industrial problems is the potential reduction of the control effort which also results in reduction of wasted energy.
- The most common design method for fractional-order controllers is based on frequency domain analysis. The properties of Bode's ideal transfer function and the iso-damping property are essential characteristics that define "fractal" robustness and showcase the advantage of fractional-order controls.
- Heuristic FOPID tuning methods can be seen as attractive due to the relative simplicity of implementing the corresponding algorithms, but do suffer from issues related to the assessment of results.
- An ideal, "basic building block" implementation of FO operators should be sought to compensate for the high order approximation issue. This implementation must be cost effective to facilitate industrial adoption of FO controllers.

These items form the basis for future research.

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