

## Fractional - order modelling and control for two parallel PWM rectifiers

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**Abstract:** For the purpose of improving control quality, fast response, stability and other quality indexes for parallel coupled PWM rectifiers, as well as derived from being able to determine the actual transfer function which is closer to the actual control object, the fractional-order transfer function of the three-phase PWM rectifiers. This paper used a fractional proportional Integral controller instead of a traditional PI controller. The use of the fractional-order controller object will help to design a higher precision fractional-order controller of the system. This paper analyzes the characteristics of the system, constructs the function of the fractional-order controller object, designs the current and voltage of the fractional-order controllers, performs the simulations and related experiments. Simulation and experiment results have demonstrated that using the fractional proportional integral controller to control the system of two parallel PWM rectifiers has resulted in ideal control, clearly improved control quality.

*Keywords:* Fractional-order, Modelling, parallel, rectifier

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### 1. INTRODUCTION

With a single Pulse Width Modulation (PWM) rectifier control system, the use of traditional PI controllers gives ideal control results, but when coupling two or more PWM rectifiers coupled in parallel, then the design parameters are not absolutely accurate and sensitive to changes in system parameters. Output voltages are too high, and the selection of parameters for the controller is difficult. To overcome these shortcomings, this paper proposes the use of fractional proportional integral controllers PI instead of using traditional PI controllers.

In recent years, the fractional controller has been increasingly used in various fields of engineering and technology, especially in systems requiring high-quality control such as rectifier control, inverter, motor, wind control, boiler control system, liquid level control and other fields, etc C.A. Monje (2010), Nie Bing (2012), Z. Zou (2013), Huang Li Lian(2013), Abdelfatah Charef (2013). For the use of fractional controllers to control PWM rectifiers, many scientists have also studied and published the results, Wu Zhao Jun (2014) and Nazir Rabia(2015), Zhifeng Pan (2017). All published results have demonstrated that the use of fractional controllers to replace traditional PI or PID controllers is an absolutely reliable scientific basis. However, the study of the application of the new fractional controller is limited to the application to single rectifier systems that are

almost not applicable to systems with two or more parallel rectifiers. This paper advances one step further and applies a fractional proportional integral controller for parallel PWM rectifier control systems.

For each control system and the specific controlled object, selecting the optimal controller is still important to determine the quality of control of the system. By specific research and analysis, we know that a 3-phase PWM rectifier integrated with a three-phase grid is a nonlinear control object, so it is possible to construct a transfer function for this object, which will be described accurately and in proximity to the actual controlled object. This is of great help for us to design the controller in the most accurate and close way to the controller we need to use in the most practical system. With the controlled object being a fractional object, the choice of a fractional controller would provide better control results than using the traditional PI controller as before. The analysis, calculations, and results obtained in the next step will prove the correctness of the choice of the paper.

### 2. OVERVIEW OF THE FRACTIONAL-ORDER CONTROLLER

The overview structure of the fractional order controller is shown in Fig. 1, where

$$F(s) = K_p + \frac{K_I}{s^\lambda} + K_D s^\mu \quad (1)$$

$F(s)$ : the fractional order controller;  $G(s)$ : the controlled object;  $X(s)$ : the input signal;  $Y(s)$ : the output signal;  $K_p, K_i, K_d$ : the factors of proportional, integral and derivative values respectively;  $\lambda, \mu$ : the positive real numbers.

The fractional order controller  $PI^\lambda D^\mu$  has two additional adjustable parameters  $\lambda, \mu$  compared to the traditional controller PID, so it increases the control range for the system and the slope of the logarithmic characteristic is not -20dB/dec but is  $-20\lambda$ dB/dec or  $+20\mu$ dB/dec. It is easy to see that traditional PID, PI and PD controllers are just one of the special cases of fractional  $PI^\lambda D^\mu$  controllers. Therefore, the choice of a reasonable set of parameters can improve the control quality of the system.

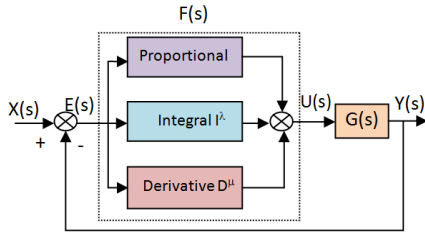


Fig. 1. The overview structure of the fractional order controller

### 3. DESIGN OF FRACTIONAL PROPORTIONAL INTEGRAL CONTROLLER FOR THE CONTROL SYSTEM COUPLING TWO PARALLEL THREE-PHASE PWM RECTIFIERS

In the system (Fig. 2), we use two identical coupled parallel PWM rectifiers and individually controlled to increase the system's performance and minimize damage caused by the faults. When coupling two PWM rectifiers in the system, there will be some problems such as balanced current or average current method. However, due to the length constraints, we do not discuss these issues, but rather only discuss the design of a controller for the system. For simplicity of design when building a fractional order controller for a system, because two PWM rectifiers are in parallel, theoretically the controller design for two rectifiers is the same, so we only need to design a fractional controller for one set, then the other will be used the same controller as the first one. Meanwhile, in order to improve the quality control, we design a current controller and a voltage controller independently.

#### 3.1 Fractional order modelling

For the three-phase PWM rectifier AC/DC system is structured as, we realize that our controlled object is a nonlinear object because of the collapsing characteristics of the IGBT, dead-time, R coil resistor and L inductor are nonlinear elements. so the control object of the closed-loop current can not only use the first order system  $\frac{1}{R+Ls}$

express. In addition, the design of the controller relies heavily on the parameters of the controlled object, so finding the exact controlled object closest to the actual object is essential. The actual models of the three-phase PWM rectifier system after decoupling, including coordinate transformation, SVPWM vector modulation, dead-time influence are all nonlinear, therefore, defining the transfer function of the actual controlled object is relatively difficult. We assume that the controlled object model of the current closed loop is unknown (Fig. 3). Based on the input/output characteristics of the real object, it is possible to construct a replacement controlled object for the actual object. To verify the accuracy of this object it can be done through deviation  $e$  (the difference between the observed current of the constructed object and the current of the actual system). The fractional order transfer function of the three-phase PWM rectifiers is expressed as (2).

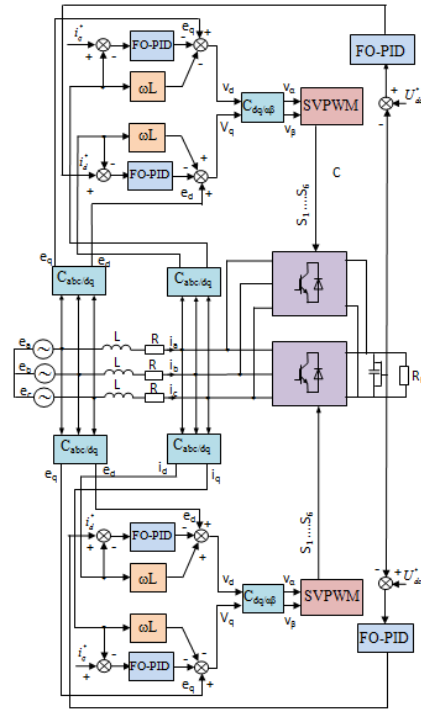


Fig. 2. The structure of the control system coupling two parallel three-phase PWM rectifiers by using direct current controller

$$B(s) = \frac{1}{sT_p + 1} \frac{K_s}{s^\delta T_s + 1} \quad (2)$$

Where  $T_p$  is the time constant of the rectifier.

Based on the experimental method described above, we can determine the fractional order transfer function function defined as (3) is very close the controlled object:

$$B(s) = \frac{0.635}{0.0001s + 1} \frac{0.2}{0.0002s^{1.38} + 1} \quad (3)$$

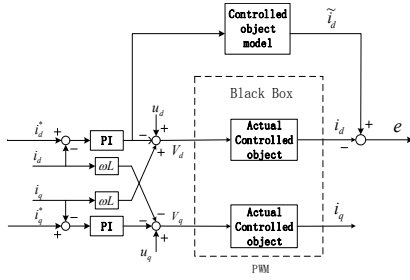


Fig. 3. Scheme of the closed-loop current for controlled object to be built and actual controlled object

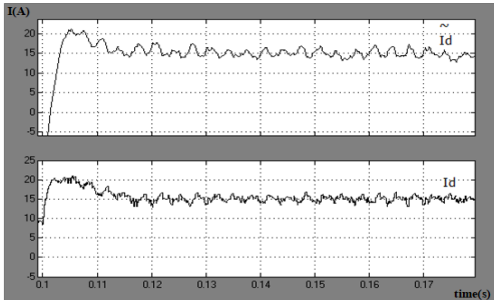


Fig. 4. The observed current  $\tilde{i}_d$  and the actual current  $i_d$  curve

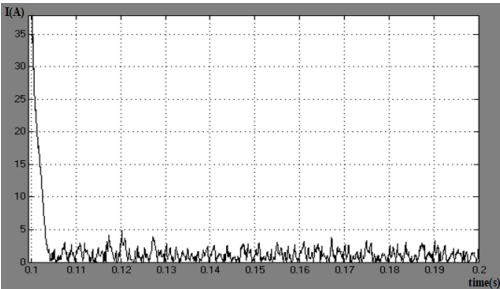


Fig. 5. The difference between  $i_{d_s}$  and  $i_d$

From Fig. 4,5, the fractional order transfer function is defined as (3) which is very close to the actual characteristics of the system. The difference between  $i_d$  and  $\tilde{i}_d \approx 0$ . So (3) is the fractional order transfer function of the controlled object we need to find.

To validate the correctness of the constructed control object model, as well as to design two PI controllers by using traditional method through first order hold  $\frac{1}{R+Ls}$  and the

fractional control object  $B(s) = \frac{0.635}{0.0001s+1} \cdot \frac{0.2}{0.0002s^{1.38}+1}$ , then we use the two controllers that were designed for the actual system and compare the results.

From the results obtained as shown in Fig. 6 and Fig. 7 we find that using the control object model

$$B(s) = \frac{0.635}{0.0001s+1} \cdot \frac{0.2}{0.0002s^{1.38}+1}$$

for controller design gives better control quality, fast response time. This again demonstrates that building a fractional order model for control object is effective and necessary. In the next step this paper will use this result to design a fractional order controller for the system.

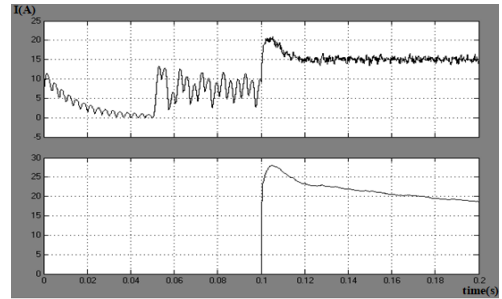


Fig. 6. The current characteristics of the real system (above) and the system using the control object  $\frac{1}{R+Ls}$  (below) when using PI controller designed for control object

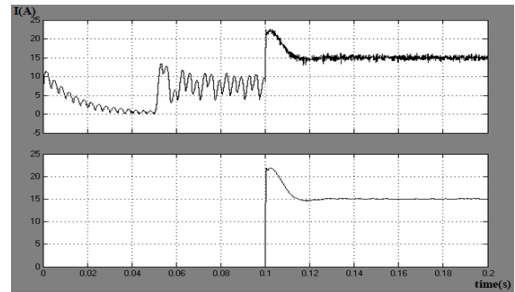


Fig. 7. The current characteristics of the real system (above) and the system using new constructed control object B(s) (below) when using PI controller designed for control object

From Schematic diagram of the three-phase PWM rectifier we have

$$C = \frac{du_{dc}}{dt} = i_{dc} - i_L \quad (4)$$

$$i_{dc} = S_a i_a + S_b i_b + S_c i_c \quad (5)$$

$$\text{With : } i_L = \frac{u_{dc}}{R_L}$$

From (4) we can determine the active power of the system

$$P = Cu_{dc} \frac{du_{dc}}{dt} + \frac{u_{dc}^2}{R_L} \quad (6)$$

With  $U_{drc} = U_{dc} + \Delta U_{dc}$  we obtain

$$p = Cu_{dcr} \left( \frac{du_{dc}}{dt} + \frac{du_{dc}}{R_L C} \right) \quad (7)$$

$$\frac{u_{dc}(s)}{P(s)} = \frac{K}{(R_L C s + 1)} = \frac{K}{T_c s + 1} \quad (8)$$

Where,  $K = R_L / U_{dcr}$ ;  $T_c = R_L \cdot C$

However, we also see that capacitors C and resistors  $R_L$  are nonlinear elements, so the control object of the current loop circuit can also be considered a fractional order object, from which we find the mathematical transfer function of the circuit (R//C) as expressed in (9)

$$C(s) = \frac{K}{s^{\lambda} T_c + 1} \quad (9)$$

Using the same method with the current loop, consequently, we figure out the object of the voltage loop, below (10)

$$C(s) = \frac{47.33}{0.01s^{0.93} + 1} \quad (10)$$

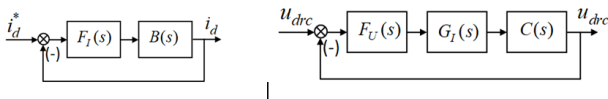
Based on the schematic digram of the PWM rectifier through DCC and Fig. 2, we have three-phase PWM rectifier using the fractional order controller as follows:

$$F_I(s) = K_{p_i} + \frac{K_{I_i}}{s^{\lambda_i}} \quad (11)$$

Equation (11) is the transfer function of the current controller.

$$F_U(s) = K_{p_u} + \frac{K_{I_u}}{s^{\lambda_u}} \quad (12)$$

Equation (12) is the transfer function of the voltage controller,  $G_I(s)$  is the transfer function of the current loop circuit.



a) Scheme of closed-loop current      b) Scheme of closed-loop voltage

Fig. 8. Current closed loop, voltage closed loop

### 3.2 Determine the parameters of the fractional controller for the current loop circuit $K_{p_i}, K_{I_i}, \lambda_i$

For the three-phase PWM rectifiers coupled in parallel, select the expected phase  $\varphi_m$  and the cut-off frequency  $\omega_c$ , the designed system must meet

the following conditions:

$$\varphi_m = \arg[F_I(j\omega_c)B(j\omega_c)] + \pi \quad (13)$$

$\omega_c$  satisfies

$$|F_I(j\omega_c)B(j\omega_c)| = 1 \quad (14)$$

Using the current controller transfer function in (11) we have

$$K_{p_i} + K_{I_i} \frac{\cos \frac{\pi \lambda_i}{2}}{\omega_c^{\lambda_i}} = R_{mc} \quad (15)$$

$$-K_{I_i} \frac{\sin \frac{\pi \lambda_i}{2}}{\omega_c^{\lambda_i}} = I_{mc} \quad (16)$$

Where

$$\frac{-\cos \varphi_m - j \sin \varphi_m}{B(j\omega_c)} = R_{mc} + jI_{mc} \quad (17)$$

For three-phase PWM rectifier system, we choose the expected phase  $\varphi_m = 45^\circ$ ;  $\omega_c = 2730 \text{ rad/s}$ ; let  $\lambda_i$  vary in (0.1:1) with a step of 0.1, we obtain a combination of 9 output characteristics corresponding to different  $\lambda_i$  values.

From (15), (16) we can determine the values of  $K_{p_i}, K_{I_i}$

$$K_{p_i} = R_{mc} + I_{mc} \cot \frac{\pi \lambda_i}{2} \quad (18)$$

$$K_{I_i} = -I_{mc} \frac{\omega_c^{\lambda_i}}{\sin \frac{\pi \lambda_i}{2}} \quad (19)$$

Use integrated time and absolute error method, we determine the values of  $K_{p_i} = 91.6; K_{I_i} = 1.6; \lambda = 0.9$

$$\Rightarrow F_I(s) = 91.6 + \frac{1.6}{s^{0.9}} \quad (20)$$

### 3.3 Determine the parameters of the fractional controller for the voltage loop: $K_{p_u}, K_{I_u}, \lambda_u$

As for the voltage loop, as discussed above, the capacitors C and its resistance R are nonlinear elements, so the transfer function of capacitor C and resistor R parallel coupling is

$C(s) = \frac{47.33}{0.01s^{0.93} + 1}$ . In addition, from Fig. 8(b) we find that the current loop is also part of the controlled object of the voltage loop, and the current loop is a fractional element, from which we have a fractional control object of the voltage loop

$D(s) = G_I(s) \cdot C(s)$ . Calculations are the same as for the

current loop, from the controlled object's transfer function, let  $\lambda_u$  vary in (0.1:1) with  $\varphi_m = 80^\circ$  and  $\omega_c = 100 \text{ rad/s}$ , we obtain a combination of output characteristics corresponding to different  $\lambda_u$  values, from expressions (15), (16) we can determine the values of  $K_{pu}; K_{Iu}$

From the combination of the obtained characteristics, we determine the values of  $K_{pu} = 0.244; K_{Iu} = 28.83; \lambda_u = 0.95$

$$\Rightarrow F_U(s) = 0.244 + \frac{28.83}{s^{0.95}} \quad (21)$$

#### 4. COMPARATIVE ANALYSIS OF SIMULATION RESULTS

We simulate and compare the results of the system that includes two parallel PWM rectifiers using the traditional PI controller and the fractional order controller  $PI^\lambda$ , the system is simulated in Matlab Simulink software with the following parameters: 220V grid voltage, 50HZ frequency, the input inductor of the rectifier has a value of  $L_L = 5 \text{ mH}$  and the output capacitor is  $C = 3300 \mu\text{F}$ , load resistor  $R_L = 30 \Omega$ , We have the simulation results as follows:

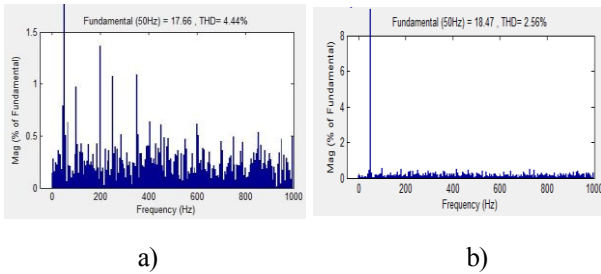


Fig. 9. Harmonic FFT of grid current under IO-PI controller (a) and FO-PI controller (b)

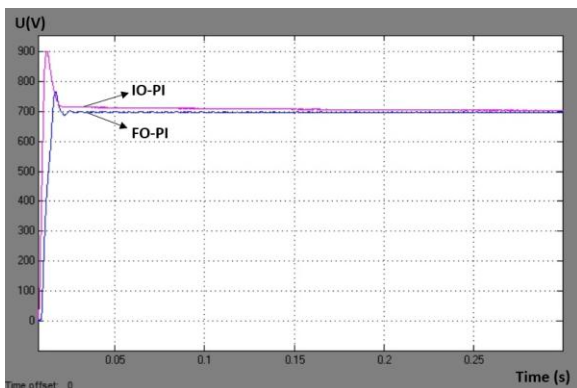


Fig. 10. DC voltage with IO-PI and FO-PI controller

Simulate and compare the results of the system that includes two parallel PWM rectifiers using the traditional PI controller and the fractional order controller  $PI^\lambda$ , the system is simulated in Matlab Simulink software with the following parameters: 220V grid voltage, 50HZ frequency, the input

inductor of the rectifier has a value of  $L_L = 5 \text{ mH}$  and the output capacitor is  $C = 280 \mu\text{F}$ , load resistor  $R_L = 70 \Omega$ , We have the simulation results as shown in Fig. 9~11.

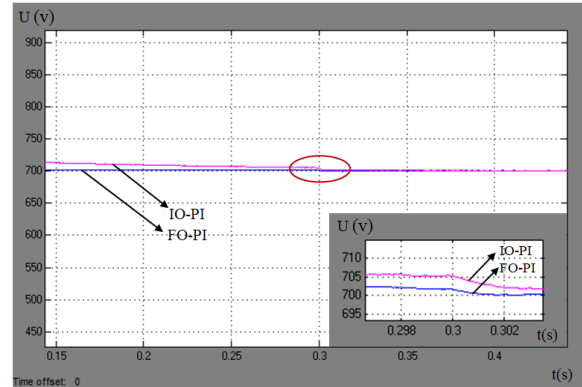


Fig. 11. Partial enlarges of DC voltage under IO-PI and FO-PI controller when load change

From the simulation results it can be seen that when using the fractional proportional integral controller (FO-PI), the simulation result given in the form of voltage and phase current A is ideal sinusoidal, the output voltage of the PWM rectifier quickly reaches to the desired value of 700V, the overshoot value is small (7%), the high order harmonic current is very small (2.59%), the power factor is approximately 1, when the load is changed after a 0.3s second interval, the system using the  $PI^\lambda$  controller (FO-PI) returns to a stable state with an output voltage about 700V. While the output voltage of the converter using a conventional PI controller (IO-PI) is asymptotic to the desired output voltage of 700V, the overshoot value is relatively high (25%), the high order harmonic current remains 6.8%, the three-phase current waveform obtained is only close to the ideal sinusoidal waveform.

#### 5. COMPARATIVE ANALYSIS OF EXPERIMENTAL RESULTS

Experiments are done with both coupling parallel PWM rectifier system using conventional PI controller and system using fractional controller  $PI^\lambda$ , the experimental results are as follows:

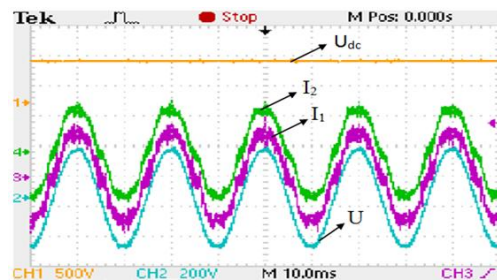


Fig. 12. Current and voltage of two parallel PWM rectifiers under IO-PI controller

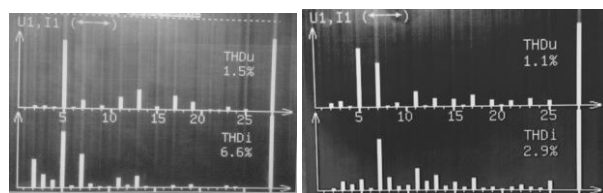


Fig. 13. IO-PID control (a) and FO-PID control (b) parameters under parallel programs of electric meter record experimental data

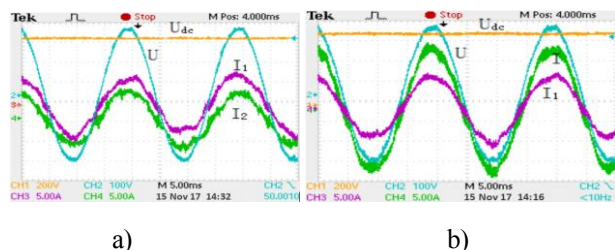


Fig. 14. Test Current and voltage of two parallel PWM rectifiers under FO-PI controller.

From the experimental results shown in Fig. 12, the current waveform of the two rectifiers is close to the sinusoidal shape, but there is a relatively high current harmonic component of 6.6% (Fig. 13a). When using the  $PI^\lambda$  controller, the obtained three-phase current of the two rectifiers is an ideal sinusoidal current, the harmonic current drops dramatically by 2.9% (Fig. 13b), the total current of the two rectifiers ( $I$ ) is also the ideal sinusoidal current (Fig. 14b) and in both cases, the rectifier DC voltage  $U_{dc}$  is stable at the desired voltage value (Fig. 12, Fig. 14). The power factor is factorized  $\cos \varphi \approx 1$ .

Experimental results demonstrate that the designed controllers not only meet the control requirements of the parallel PWM rectifier system but also improve many of the system's quality control specifications. This results demonstrate the correctness of the arguments presented above.

## 6. CONCLUSIONS

From the results of the simulation and the experiments, it was found that the fractional order  $PI^\lambda$  controller broke the viewpoint of the traditional PI controller. Building a fractional-order controller object that replaces the previous mathematical transfer function of the controlled object is of great significance in building a more accurate and efficient controller for the actual system. The system is more adaptable, the control quality is enhanced, the harmonic current is reduced, and the power factor is improved. Moreover, if the system parameters are controlled in a certain range, the fractional order  $PI^\lambda$  controller can still be controlled without adjusting the parameters of the controllers. This is the strength of the fractional order controller.

## ACKNOWLEDGMENT

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