An Improved Relay-based Identification Approach based on Asymmetric Oscillations

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Abstract: A new identification procedure has been developed based on the information obtained from the oscillations that a non-linear element as a simple relay introduces in the feedback loop. Features of the method are: (1) the procedure does not demand a priori process information, (b) non-iterative algorithms are needed to derive the process parameters, (c) only one test is needed, and (d) it allows identifying the process at a user-specified phase lag in the third quadrant.

Keywords: Describing function, relay, estimation, limit cycle, oscillations.

1. INTRODUCTION

The proposed approach allows the identification of model parameters from measurements derived of the asymmetric oscillations generated by a relay and a bias added to the output. These measures are the oscillation frequency, the main harmonics, and the steady gain (if the process is without integration). The identification procedure is relevant for four reasons:

(1) Unlike previous works based on relay-induced oscillations, the model parameters do not need to be calculated using a-priori information of the process (i.e., static gain).
(2) Iterative methods to solve non-linear equations are not necessary,
(3) The number of tests to obtain the measures is always one regardless the transfer function structure or order, and
(4) The procedure can be extended to estimate any type of transfer function.

The structure of the paper is as follows. First, the basis of the estimation approach is described in Section 2. In Section 3, the procedure is explained also, expressions to estimate the parameters of common transfer functions are derived and simulation results are presented. Section 4 discusses a way to estimate with user-defined phase angle specifications. The paper finishes with conclusions.

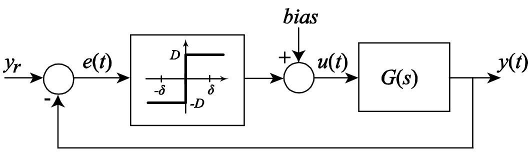

Fig. 1. Basic relay feedback control loop.

2. BASIS AND PROBLEMS OF THE ESTIMATION BASED ON RELAY-INDUCED OSCILLATIONS

The pioneering works on the use of relay feedback for identification purposes are from the 80's (Åström and Hägglund, 1984). The basis of the method is that a linear system under an ideal relay control (Figure 1) oscillates, approximately, at its ultimate frequency, that is, $\omega_{osc} \approx \omega_u$ and the critical gain K_u is derived from the describing function (DF) of the ideal relay. That is,

$$N(A) = \frac{4D}{\pi A} = K_u \tag{1}$$

where A is the oscillation amplitude and D is the relay output. Knowing that the oscillation happens when the expression

$$\frac{-1}{N(A)} = G(j\omega_{osc}) \tag{2}$$

is satisfied, it is feasible to derive the parameters of a transfer function model. One way to do that it is to use the magnitude and argument of the critical point $G(j\omega_{osc})$ in the Nyquist plot that corresponds to the intersection of $G(s)$ and $-1/N(A)$, separating both components to solve them. That is,

$$\left|\frac{-1}{N(A)}\right| = \left|G(j\omega_{osc})\right| \tag{3}$$

$$\arg\frac{-1}{N(A)} = \arg G(j\omega_{osc}) \tag{4}$$

To separate both parts is key to the estimation approach presented here. It is possible to apply iterative methods to solve directly (2), but when the transfer function to estimate has four or more unknowns (for example, K, T_1, T_2, and L), most of the time the iterative methods do not converge (Li *et al.*, 1991). It is due primarily to the inaccuracy of the describing function and the extreme nonlinearity of the equations with respect to the parameters. However, by separating (2) in magnitude and argument, it is possible to solve (3) to get the zeros and poles, and (4) to obtain the time delay.

After reviewing the literature on DF-based identification, the main problems found are summarized:

- The reciprocal of the DF provides an approximation of the Nyquist point at the frequency where the process oscillates. It is fundamental to obtain an exact value of $G(j\omega_{osc})$ during a test.
- In process without integration, it is necessary to obtain $G(0)$ as a first parameter. If the transfer function template to fit is exactly equal to the real process to identify, it is not necessary, as the identification will provide exact results at any frequency. However, if the true process has higher order or different structure to the template and $G(0)$ is not known, this will produce that the result is good around the critical frequency $\omega_{\approx-180^\circ}$ but with discrepancies at $\omega = 0$. It is due to the fact that the template is fitted with lesser degrees of freedom that the true process. So, the fitting will be exact around ω_{osc} but will present discrepancies in frequencies close to zero.
- If the process has an integrator, as the identification is based on the behavior in the third quadrant, the results around $\omega_{\approx-180^\circ}$ will be good even when the transfer function has different structure than the real process. However, if the structure is different, the results will differ at low frequencies, improving the results if the identification is done at frequencies around ω_{-135°.
- It is necessary to design a procedure to get in just one test as many points of the Nyquist plane as unknowns $|G(s)|$ owns to solve the system of equations. One point will always be the oscillation point situated in the third quadrant of the Nyquist map. A second point must be $G(0)$ when needed depending of the template to fit.
- To be able of identifying at a user-defined phase angle. In (Åström and Hägglund, 2006), authors recommend estimating the process at ω_{-135° or ω_{-180° depending if PI or PID control is applied, respectively.

3. BASIS OF THE APPROACH

As said before, the reciprocal of the relay DF is just an approximation and can introduce errors in the estimation. The solution adopted to get accurately $G(j\omega_{osc})$ during a test is presented in (Vivek and Chidambaram, 2005). As $y(t)$ and $u(t)$ are periodic and piecewise, using the Laplace transform of both, it can be written

$$G(j\omega_{osc}) = \frac{Y(j\omega_{osc})}{U(j\omega_{osc})} = \frac{\int_0^{2\pi/\omega_{osc}} y(t)e^{-j\omega_{osc}t}\,dt}{\int_0^{2\pi/\omega_{osc}} u(t)e^{-j\omega_{osc}t}\,dt} \quad (5)$$

and following (5), and as indicated in (Wang *et al.*, 2007), it is possible to obtain the harmonics

$$G(jn\omega_{osc}) = \frac{\int_0^{2\pi/\omega_{osc}} y(t)e^{-jn\omega_{osc}t}\,dt}{\int_0^{2\pi/\omega_{osc}} u(t)e^{-jn\omega_{osc}t}\,dt}, n = 1,2,3,... \quad (6)$$

Expression (6) lets solving two problems: (i) to get the value of $G(j\omega_{osc})$ that represents the exact point in the Nyquist plot where the intersection with $-1/N(A)$ is happening and, (ii) to obtain the additional points $G(jn\omega_{osc})$ needed in one test to work out the equations.

The last problem to solve is the estimation of the steady gain. Expression (5) cannot be applied as the oscillations produced by a relay are symmetric and the integration of the semi periods will be zero. However, if an asymmetry is introduced in the oscillations by adding a small bias to the relay output (see Figure 1), the process static gain can be derived from (5) (Shen *et al.*, 1996) as

$$G(0) = \frac{\int_0^{2\pi/\omega_{osc}} y(t)\,dt}{\int_0^{2\pi/\omega_{osc}} u(t)\,dt} \quad (7)$$

It is important to notice that without bias, the signals are symmetric and even harmonics become cero.

3.1 Models

The identification procedure can be adapted to most of the process found in process industry just obtaining the magnitude and argument expressions of the transfer function to fit. Expressions for a first order plus time delay (FOPTD), an overdamped second-order with time delay (SOPTD-1), a second-order with time delay (SOPTD-2) to identify underdamped processes, and an integrating process with inverse response and time delay (IPIRTD) are given.

Model 1: FOPTD

$$G_1(s) = \frac{Ke^{-Ls}}{Ts+1} \quad (8)$$

Model 2: SOPTD-1

$$G_2(s) = \frac{Ke^{-Ls}}{(Ts+1)^2} \quad (8)$$

Model 3: SOPTD-2

$$G_3(s) = \frac{Ke^{-Ls}}{as^2+bs+1} \quad (10)$$

Model 4: IPIRTD

$$G_4(s) = \frac{K(1-T_1 s)e^{-Ls}}{s(T_2 s+1)} = \frac{(1-T_1 s)e^{-Ls}}{as^2+bs} \quad (11)$$

where $K = 1/b$ and $T_2 = a/b$.

From each of these models, the expressions of magnitude and argument of $G(j\omega_{osc})$ are derived.

Model 1: FOPTD

$$|G_1(j\omega_{osc})| = \frac{K}{\sqrt{T^2\omega_{osc}^2+1}} \quad (12)$$

$$\arg G_1(j\omega_{osc}) = -\arctan(T\omega_{osc}) - \omega_{osc}L \quad (13)$$

where $\arg G_1(j\omega_{osc}) \in [-2\pi, 0]$.

As K, ω_{osc} and $G_1(j\omega_{osc})$ are obtained directly from the test using (6) and (7) by adding a bias, T can be obtained directly from (12). After that, L is derived from (13).

Model 2: SOPTD-1

$$|G_2(j\omega_{osc})| = \frac{K}{T^2\omega_{osc}^2+1} \quad (14)$$

$$\arg G_2(j\omega_{osc}) = -2\arctan(T\omega_{osc}) - \omega_{osc}L \quad (15)$$

The unknowns are the same that in the previous one, so the procedure to apply is the same.

Model 3: SOPTD-2

$$|G_3(j\omega_{osc})| = \frac{K}{\sqrt{a^2\omega_{osc}^4 + b^2\omega_{osc}^2 - 2a\omega_{osc}^2 + 1}} \quad (16)$$

$$\arg G_3(j\omega_{osc}) = \arctan(-b\omega_{osc}, -a\omega_{osc}^2+1) - \omega_{osc}L \quad (17)$$

There are three unknowns K, a, and b in (10). As K is obtained directly from the test using (7) by adding a bias, to get the other two unknowns is necessary the expression corresponding to the second harmonics, that is, $|G_3(j2\omega_{osc})|$. This expression is derived from (16) just replacing ω_{osc} by $2\omega_{osc}$ and its experimental value is obtained from the test (see (6)). Once a and b are known, the dead-time is derived from (17).

Model 4: IPIRTD

$$|G_4(j\omega_{osc})| = \sqrt{\frac{T_1^2\omega_{osc}^2+1}{\omega_{osc}^2(a^2\omega_{osc}^2+b^2)}} \quad (18)$$

$$\arg G_4(j\omega_{osc}) = -\arctan\left(\frac{-(T_1 a\omega_{osc}^2 - b)}{-\omega_{osc}(T_1 b + a)}\right) - \omega_{osc}L \quad (19)$$

The unknowns in (18) are T_1, a, and b. As it is a process with integration, the velocity gain can directly be obtained solving the system of equations. So, the expressions of $|G_4(j2\omega_{osc})|$ and $|G_4(j3\omega_{osc})|$ are needed and its experimental values are got from the test (see (6). The dead-time is directly got from (19) once the other three unknowns are obtained.

The following expressions are the result of solving the previous equations for the four models. For the sake of simplicity, C_n represents $|G(jn\omega_{osc})|$ and $\arg C_1$ corresponds to $\arg G(j\omega_{osc})$.

Model 1: FOPTD

$$T = \frac{\sqrt{K^2 - C_1^2}}{\omega_{osc}C_1} \quad (20)$$

$$L = -\frac{\arg C_1 + \arctan(\omega_{osc}T)}{\omega_{osc}} \quad (21)$$

Model 2: SOPTD-1

$$T = \frac{\sqrt{C_1(K - C_1)}}{\omega_{osc}C_1} \quad (22)$$

$$L = -\frac{2\arctan(T\omega_{osc}) + \arg C_1}{\omega_{osc}} \quad (23)$$

Model 3: SOPTD-2

$$aux = \sqrt{(3C_1^2 - 12C_2^2)K^2 + 9C_1^2C_2^2}$$

$$a = \frac{aux}{6\omega_{osc}^2 C_1 C_2} \quad (24)$$

$$b = \frac{\sqrt{-3C_1^2K^2 + 48C_2^2K^2 - 45C_1^2C_2^2 + 12C_1C_2\,aux}}{6\omega_{osc}C_1C_2} \quad (25)$$

$$L = \frac{\arctan(-b\omega_{osc}, -a\omega_{osc}^2+1) - \arg C_1}{\omega_{osc}} \quad (26)$$

Model 4: IPIRTD

$$T_1 = \frac{1}{\omega_{osc}}\sqrt{\frac{C_1^2C_2^2 - 6C_1^2C_3^2 + 15C_2^2C_3^2}{-9C_1^2C_2^2 + 24C_1^2C_3^2 - 15C_2^2C_3^2}} \quad (27)$$

$$a = \frac{1}{\omega_{osc}^2} \sqrt{\frac{-5C_1^2 + 32C_2^2 - 27C_3^2}{-108C_1^2C_2^2 + 288C_1^2C_3^2 - 180C_2^2C_3^2}} \quad (28)$$

$$b = \frac{1}{\omega_{osc}} \sqrt{\frac{5C_1^2 - 128C_2^2 + 243C_3^2}{-108C_1^2C_2^2 + 288C_1^2C_3^2 - 180C_2^2C_3^2}} \quad (29)$$

$$L = -\frac{\arctan\left(-(T_1 a\omega_{osc}^2 - b), -\omega_{osc}(T_1 b + a)\right) + \arg C_1}{\omega_{osc}} \quad (30)$$

3.2 Outline of the procedure

The procedure is based on producing a limit cycle oscillating at ω_{osc} as consequence of the relay control actions generate when $G(s)$ crosses zero. As shown before, mathematically, the limit cycle is explained as the consequence of the intersection of $G(s)$ with $-1/N(A)$ at ω_{osc}. The intersection point corresponds to the value of $G(j\omega_{osc})$.

The estimation procedure can be divided into the following steps:

1. Add a small bias, i.e, $\approx 0.1D$ to the relay output.
2. Once the oscillation is stable, annotate ω_{osc}, and the harmonics $G(jn\omega_{osc})$ needed to solve the equations. In models without integration, it is necessary *K*.
3. Use the expressions:
   - (20) and (21) for FOPTD model.
   - (22) and (23) for SOPTD-1 model.
   - (24), (25), and (26) for SOPTD-2 model.
   - (27), (28), (29), and (30) for IPIRTD model.

Remark 1: Model 3 lets detecting if the true process corresponds to a dynamics of first order instead of second order. In this case, the procedure will generate a value for *a* close to zero.

Remark 2: Model 2 and 3 can produce the same result if the true process corresponds to an overdamped second order system. In case of being underdamped (non-monotone), models 1 and 2 can produce a complex value for the time lag or an overdamped solution, and the correct results are produced only by Model 3.

3.3 Simulation examples

The simulations have been run in Matlab/Simulink with a sampling time *h*=0.001. The parameters used in the tests have been *D*=1 and *bias*=0.1*D*.

Table 1 shows the results when the structure of the true process and the model are the same. Models 2 and 3 produce similar solutions for Cases 2 and 3 as both real processes correspond to overdamped systems. However, in Case 4, model 2 produces complex values as the true process is underdamped. The identification in Case 5 provides a complex value for T_1 close to zero (0.08i) due to the numerical errors; and it is ignored and equated to zero. Accuracy in the estimated process model is computed using integral of absolute error (IAE) criterion

$$IAE = \int_0^{\omega_{osc}} \left| \frac{G_m(j\omega) - G(j\omega)}{G(j\omega)} \right| d\omega$$

where $G_m(j\omega)$ is the identified model, $G(j\omega)$ is the actual process and ω_{osc} is the critical frequency, respectively.

Table 2 corresponds to the results when the structure of the true process is different to the transfer function to fit. Cases 7 and 8 correspond to the fitting of FOPTD models; cases 9, 10, and 11 to second order processes, and cases 12 and 13 to processes with integration and inverse response. In Case 9, it is not possible to present solutions for models 1 and 2 as the true process is an underdamped system (non-monotone).

Table 1: Solutions for processes with same order and structure than the transfer function fitted.

	Process	Proposed method	IAE
1	$\frac{e^{-2s}}{10s+1}$	$\frac{1.003e^{-2.0008s}}{10.0364s+1}$ $\varphi_{osc} \approx -0.78^\circ$ $\omega_{osc} \approx 0.85$	0.012
2	$\frac{e^{-2s}}{(2s+1)^2}$	$\frac{1.0003e^{-2.004s}}{4.024s^2+4.007s+1}$ $\frac{1.0003e^{-1.996s}}{(2.0043s+1)^2}$ $\varphi_{osc} \approx -0.61^\circ$ $\omega_{osc} \approx 0.656$	0.002 0.008
3	$\frac{e^{-0.2s}}{0.5s^2+1.5s+1}$	$\frac{1.001e^{-0.2s}}{0.5004s^2+1.502s+1}$ $\frac{1.001e^{-0.189s}}{(0.7188s+1)^2}$ $\varphi_{osc} \approx 2.27^\circ$ $\omega_{osc} \approx 3.646$	0.0018 0.0992
4	$\frac{e^{-s}}{2s^2+s+1}$	$\frac{1.002e^{-0.98s}}{2.013s^2+0.9946s+1}$ $\varphi_{osc} \approx 0.6^\circ$ $\omega_{osc} \approx 0.917$	0.007

5	$\frac{e^{-10s}}{s(20s+1)}$	$\frac{0.999e^{-10.003s}}{s(19.99s+1)}$ $\varphi_{osc}\approx 2.88^o$ $\omega_{osc}\approx 0.0624$	3*10^-5
6	$\frac{(-2.4s+0.6)}{s(s+1)}$	$\frac{(-2.39s+0.6)e^{-0.0026s}}{s(0.99s+1)}$ $\varphi_{osc}\approx 21.5^o$ $\omega_{osc}\approx 0.3109$	8*10^-4

Table 2: Solutions for processes with higher order or different structure than the model fitted.

	Process	Proposed method	IAE
7	$\frac{e^{-2s}}{(10s+1)(s+1)}$	$\frac{1.002e^{-2.8632s}}{11.6998s+1}$	0.04
		$\frac{1.002e^{-1.2824s}}{(4.129s+1)^2}$	0.12
		$\frac{1.002e^{-2.0011s}}{10.013s^2+11.02s+1}$	5*10^-4
		$\varphi_{osc}\approx 0.62^o$ $\omega_{osc}\approx 0.594$	
8	$\frac{e^{-0.5s}}{(s+1)^5}$	$\frac{1.002e^{-3.1787s}}{3.3228s+1}$	0.071
		$\frac{1.002e^{-2.254s}}{(1.8206s+1)^2}$	0.0333
		$\varphi_{osc}\approx -0.01^o$ $\omega_{osc}\approx 0.633$	
9	$\frac{e^{-6s}}{(s+3)(s^2+s+1)}$	$\frac{0.3336e^{-6.28s}}{0.9992s^2+1.05s+1}$ $\varphi_{osc}\approx 3.58^o$ $\omega_{osc}\approx 0.413$	4*10^-4
10	$\frac{(-s+1)e^{-4s}}{(s+1)^4}$	$\frac{1.0008e^{-7.164s}}{1.855s+1}$	0.0011
		$\frac{1.0008e^{-6.305s}}{1.934s^2+2.609s+1}$	0.0027
		$\frac{1.0008e^{-6.4526s}}{(1.2479s+1)^2}$	0.0017
		$\varphi_{osc}\approx -5.12^o$ $\omega_{osc}\approx 0.367$	
11	$\frac{(-10s+1)e^{-s}}{s(s+1)^5}$	$\frac{(-10.36s+0.98)}{s(2.65+1)}e^{-3.12s}$ $\varphi_{osc}\approx 12.5^o$ $\omega_{osc}\approx 0.097$	2*10^-4
12	$\frac{(-5s+1)e^{-0.5s}}{s(s+1)(s^2+s+1)}$	$\frac{(-5.167+0.977)}{s(0.552s+1)}e^{-1.83s}$ $\varphi_{osc}\approx 14^o$ $\omega_{osc}\approx 0.207$	4*10^-4

4. SELECTING THE IDENTIFICATION PHASE ANGLE

The identification approach using a relay without hysteresis produces a model that corresponds exactly with the dynamics of the true process at the oscillation frequency, that is $G(j\omega_{osc}) = \hat{G}(j\omega_{osc})$. If the order of the process is equal to the transfer function model to fit, the identification will be exact and the behaviour of $\hat{G}(s)$ will be equal to $G(s)$ in all the frequencies range. However, if the order of process and model is different, it can be necessary to modify the oscillation frequency to find a more suitable approximation to the pursued control purposes.

In the previous examples, the phase angle is not 0º and changes depending of the features of the process. Theoretically, the bias added to the output cannot be used for this purpose as it does not affect to the phase angle as not provide phase shifting (Gelb and Van der Velde, 1968),

$$\frac{-1}{N(A)+bias} = -\frac{\pi A}{\pi A\cdot bias + 4D} \qquad (31)$$

To obtain a point in the Nyquist curve different to the obtained with the simple relay can be done by introducing hysteresis in the relay (Åström and Hägglund, 1984). The negative reciprocal of the DF of a biased relay with hysteresis is

$$\frac{-1}{N(A,\delta)} = -\frac{\pi}{4D}\sqrt{A^2-\delta^2} - i\frac{\pi\delta}{4D} \qquad (32)$$

where A is the oscillation output, D the relay amplitude, and δ the hysteresis (it must be noticed that the reciprocal is independent of the bias). As happens in the simple relay, this function is represented in the Nyquist map as a straight line parallel to the real axis. However, by increasing δ, the line can be move down along the imaginary axis of the Nyquist map. As the intersection of $G(s)$ with this line means the existence of an oscillation, the critical point where the intersection happens can be modified, increasing the phase angle to fulfill the user specifications.

From (32), the theoretically phase angle obtained by applying hysteresis to the relay is defined by

$$\varphi_m = \arctan\left(\frac{\delta}{\sqrt{A^2-\delta^2}}\right) = \arcsin\left(\frac{\delta}{A}\right) \qquad (33)$$

where $A \geq \delta$. Obviously, this expression provides just an approximation that depends on the features of the true process.

Example 1: The result of the identification of $G(s)=e^{-s}/s(s+1)$ using a relay with D=1, $\delta=0$, and *bias*=0.1*D* is

$$\hat{G}1(s)=\frac{(-0.1614s+1.051)e^{-0.781s}}{s(1.139s+1)}$$

with $\varphi_{osc}\approx 2.66^{\circ}$. If the hysteresis is increased to $\delta=2$, the result of the identification is

$$\hat{G}2(s)=\frac{(-0.08321s+1.001)e^{-0.929s}}{s(1.005s+1)}$$

with $\varphi_{osc}\approx 46^{\circ}$. In Figure 2, it can be appreciated the differences between the two results.

4.1 Automatic generation of the hysteresis

In the previous example, the hysteresis was fixed by trial and error in successive simulations. A solution proposed in (Åström and Hägglund, 1984) for autotuning of PID controllers can be adapted to adjust iteratively the hysteresis during the tests using the information obtained from the experimental measure of $G(j\omega_{osc})$. The formula is

$$\delta_{n+1}=\delta_n+\lambda(\varphi_{ref}-\varphi_n)\frac{\delta_n-\delta_{n-1}}{\varphi_n-\varphi_{n-1}} \quad (34)$$

where φ_{ref} is the desired phase angle and $\lambda\leq 1$ is a convergence speed factor.

Example 2: Applying the iterative method to the identification of the process of Example 1 using (34), fixing $\varphi_{ref}=45^{\circ}\pm 1^{\circ}$ and $\lambda=0.5$, it is obtained $\delta=1.897$ and the new identified model is

$$\hat{G}3(s)=\frac{(-0.02286s+1)e^{-0.979s}}{s(s+1)} \quad (35)$$

with $\varphi_{osc}\approx 45.32^{\circ}$.

6. CONCLUSIONS

The basis of the method is the oscillations that a relay generates in the feedback control loop. Instead of using the approximations of the intersection points that the describing function of the relay provides, on-line measurements of the process and control signals are used to obtain the harmonics needed to solve the linear equations and generate the estimations. In this way, the number of tests to make the estimations is always one regardless the order or structure of the model transfer function to fit. Moreover, the identification procedure is not iterative so the computational cost is very low. With the simple relay, the identification procedure is done near the ultimate frequency. By using a relay with hysteresis and modifying its value, the identification will be possible at a user-specified phase lag.

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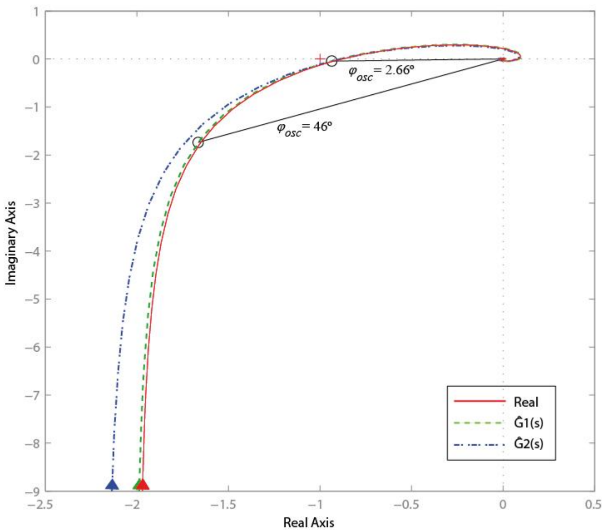

Figure 2: Example of the differences in the identification results depending of the phase angle.

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