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Active vibration control of two-mass flexible system using parametric Jordan form assignment

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Abstract: This paper deals with stage motion control system for scene manipulation during theater performance. Particular task of rope drum control is presented and solved. The system exhibits an oscillatory dynamics due to the elasticity of the rope with a hanging load. The goal was to find a simple control strategy based on a common cascade PID structure which is available in most of the industrial AC drives. Formerly developed method of parametric Jordan form assignment was used to solve the problem and obtain simple tuning rules.

Keywords: Active vibration control, motion control, PID control, parametric Jordan form assignment, two-mass model, flexible system

1. INTRODUCTION

Stage motion control system is an essential part of a modern theater. It consist of several technical devices including rope drums, moving walls, turntables and drop curtains which allow to create and change a shape of the stage during a break or ”on the fly” within a running performance. Rope drums serve for manipulation with various loads in form of scenes performing variety of motions ranging from simple rest-to-rest manoeuvres in one direction to complicated multidimensional trajectories with multiple synchronized ropes. Typical structure of the control system is shown on Fig. 1. The rope drum is driven by an electrical drive, usually an AC induction motor with corresponding frequency inverter. The axis controller is responsible for desired trajectory tracking and serves as position controller. The setpoint values for desired position, velocity and acceleration of each axis are received from motion planning level ensuring trajectory generation and proper synchronization during multi-axes motions. The system is parameterized and supervised by an operator using a human-machine-interface.

The increasing complexity of live performance in theaters and a call for advanced special effects on the stage results in higher desired bandwidth at the motion control level. Due to the fast movements, serious problems with unwanted oscillations caused by a flexible coupling may arise. This is especially true for the rope drum systems, which often exhibit an oscillatory dynamics due to the elasticity of the rope. This leads to difficulties with parameter tuning of the drive controllers and prolongs the commissioning of the technology. There are several methods for motion controller synthesis ranging from classical PID, predictive, sliding mode, robust or adaptive control [Vukosavić (2007); Kwon and Chung (2004); Ohnishi et al. (1996); Šabanovic (2011)] which work well for rigid mechanisms. However, their use is very limited in case of flexible systems. Several special methods for vibration damping have been developed. One way is to employ an additional instrumentation in form of position, speed, acceleration or driving torque sensors which are attached to the load side. The mechanical resonance can then be suppressed in an active way by closing the state feedback. In case of inaccessible state variables, a state observer may be used to acquire the information about the motion on both sides of the elastic coupling [Ji and Sul (1995)]. Such methods are known to be sensitive to modelling errors and nonlinear effects occurring in the mechanical subsystem. Moreover, it may be difficult to extract the required information from noise corrupted signals, especially when the noise frequency coincides with those of resonant modes of the system. Passive vibration damping can be used when the state variables can be neither measured nor reconstructed [Vukosavić (2007)]. In this case a series antiresonant compensator in form of IIR or FIR filter is placed inside the velocity control loop ensuring that the oscilatory modes cannot be excited by the driving torque in the vicinity

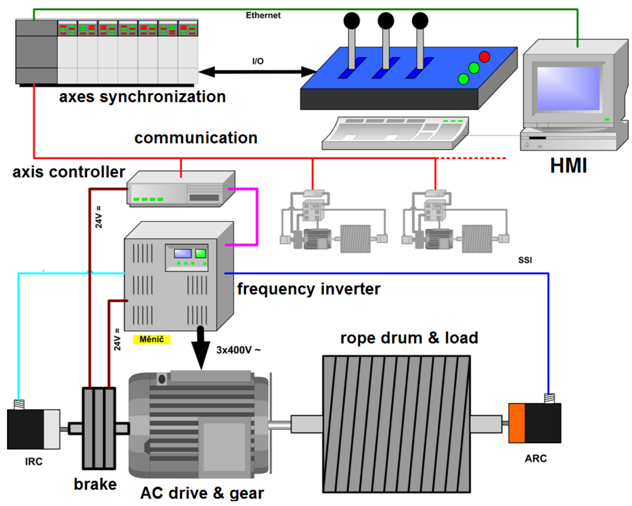

Fig. 1. Typical structure of stage motion control system

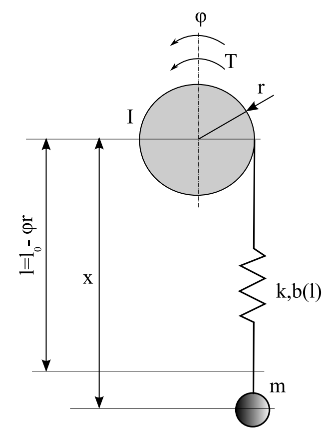

Fig. 2. Simplified model of the rope drum system

of the resonant frequency. Alternatively, proper filter can shape the setpoint values for the motion controller in the feedforward path [Goubej and Schlegel (2010)]. The main disadvantage of passive damping approach is that the resonant mode remains uncontrollable after the cancelation with the series compensator and any outer disturbance will cause the undesirable oscillations.

Many modern methods for optimal controller synthesis (LQG, H_2, H_∞) generally produce high-order compensators which are difficult to implement due to their sensitivity to rounding and modelling errors. Only a numerical solution to the problem is often available and set of complex mathematical routines is needed to obtain some results. The parameter tuning is achieved by modifying some weighting functions without a clear physical meaning and several iterations may be needed to get a suitable design. There is a gap between the academic research and industrial practice where the PID controller is still prevailing. Therefore, our goal was to find a simple method based on partial pole placement technique for low-order compensator synthesis which may be used in most standard industrial drives equipped with common cascaded PID control structure.

2. MATHEMATICAL MODEL OF THE ROPE DRUM SYSTEM

The system consists of rotational drum, flexible rope and a hanging load (Fig. 2). To obtain a finite-order model suitable for control law design, some simplifications have to be made. We assume that the elastic rope with load can be modelled as an mass-spring-damper system. The values of spring constant k and viscous damping coefficient b are generally time varying and they depend on the length of the rope according to Hooke's law:

$$k = \frac{k_0}{l}, b = \frac{b_0}{l} \tag{1}$$

where k_0 corresponds to the stiffness of the unit length of the rope and l is the actual length in dependence on drum position φ

$$l = l_0 - \varphi r \tag{2}$$

The equations of motion can be derived using the Newton-Euler method.

From the total torque T_t acting on the drum we get:

$$T_t = T - k(x - l)r - br(\dot{x} - \dot{l}) = I\ddot{\varphi} \tag{3}$$

Sum of forces acting F_t on the load:

$$F_t = mg - k(x - l) - b(\dot{x} - \dot{l}) = m\ddot{x} \tag{4}$$

Substituting from (1,2) we can write:

$$T - \frac{k_0}{l_0 - \varphi r}(x - l_0 + \varphi r)r - br(\dot{x} + r\dot{\varphi}) = I\ddot{\varphi} \tag{5}$$

$$mg - \frac{k_0}{l_0 - \varphi r}(x - l_0 + \varphi r) - b(\dot{x} + r\dot{\varphi}) = m\ddot{x} \tag{6}$$

By introducing state variables $x = [x_1\ x_2\ x_3\ x_4]^T = [x\ \varphi\ \dot{x}\ \dot{\varphi}]^T$ we get nonlinear state space representation in form:

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= -\frac{1}{m}\frac{k_0}{l_0 - rx_2}(x_1 - l_0 + rx_2) - \frac{b}{m}(x_3 + rx_4) + g \\
\dot{x}_4 &= -\frac{1}{I}\frac{k_0}{l_0 - rx_2}(x_1 - l_0 + rx_2)r - \frac{br}{I}(x_3 + rx_4) + \frac{1}{I}T \\
b &= \frac{b_0}{l_0 - rx_2}
\end{aligned} \tag{7}$$

The only outputs available for measurement are motor speed and position. The system can be linearized around any chosen equilibrium point corresponding to a fixed rope length l. The obtained LTI model matrices:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_0}{ml} & -\frac{r(k_0 + gm)}{ml} & -\frac{b_0}{ml} & -\frac{b_0 r}{ml} \\ -\frac{k_0 r}{Il} & -\frac{r^2(k_0 + gm)}{Il} & -\frac{b_0 r}{Il} & -\frac{b_0 r^2}{Il} \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I} \end{bmatrix} \tag{9}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

The transfer function from motor torque to motor speed:

$$\begin{aligned}
P(s) &= \frac{mls^2 + b_0 s + k_0}{s(Jlms^2 + (b_0 r^2 m + Jb_0)s + (r^2 k_0 m + r^2 gm^2))} \\
&= \frac{K}{s}\frac{s^2 + 2\xi_z\omega_{nz}s + \omega_{nz}^2}{s^2 + 2\xi_p\omega_{np}s + \omega_{np}^2}
\end{aligned} \tag{11}$$

We can see that the linearized model has the structure of well known two-mass system which is usually used to describe the dynamics of two rotating inertial loads connected by flexible shaft. Typical resonance and antiresonance phenomenon can be observed in case of weakly damped complex poles and zeros in (11).

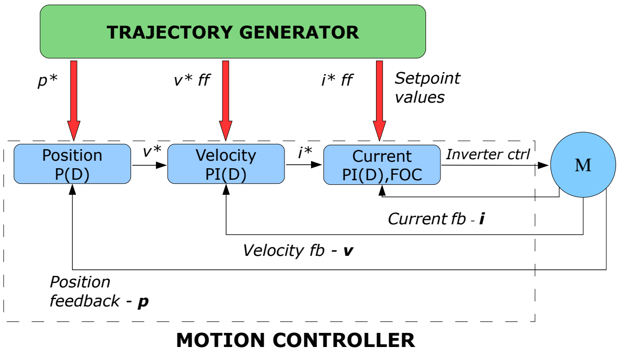

Fig. 3. Motion controller in industrial servo drive

3. CONTROL LAW DESIGN

Typical control structure which is used in most of the industrial electrical drives consists of three cascaded feedback loops. Current loop controls the mechanical torque generated by the drive. Usually the Field oriented control scheme along with PI(D) algorithm and space vector modulation or Direct torque control method is used for driving the voltage source three-phase frequency inverter. On the next level, PI or PID velocity controller is employed. The last layer is formed by position controller which most frequently runs in proportional mode. Lowpass filters are commonly used to attenuate the measurement noise in feedback signals, notch filter, lowpass or lead-leg compensator may be present to deal with an oscillatory dynamics.Setpoint values are acquired from trajectory generator (interpolator) which computes desired motion for the given axis of the machine.

The most difficult part of drive commissioning in case of flexible mechanisms is the velocity controller setting because of the oscillatory dynamics, time varying parameters and various nonlinear effects occurring in the system. Therefore, our goal was to develop simple tuning rules which may be used for design of active antiresonant PID velocity controller. To accomplish that, partial Jordan form assignment method was used which is briefly explained in the next section.

3.1 Parametric Jordan form assignment

We consider a linear time invariant system defined by state space representation:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

$$y(t) = Cx(t) \tag{13}$$

where $x(t) \in \mathbb{R}^n$ is state vector, $u(t) \in \mathbb{R}^m$ is input vector, $y(t) \in \mathbb{R}^p$ is measurement output vector, A, B, C are constant real matrices with corresponding dimensions and the pair (A, B) is controllable.

Our first goal is to find all state feedbacks $F \in \mathbb{R}^{m\times n}$ in form

$$u(t) = Fx(t) \tag{14}$$

which assign a chosen Jordan form $L \in \mathbb{R}^{s\times s}$ and therefore fulfill condition $A+BF \sim L$. Such feedback matrices form a set $\mathcal{F}_s$

$$\mathcal{F}_s(A,B,L) \triangleq \left\{ F \in \mathbb{R}^{m\times n} : \; (A+BF) \sim \begin{bmatrix} L & * \\ 0 & * \end{bmatrix} \right\} \tag{15}$$

where $*$ denotes an arbitrary real matrix of proper dimension. If $s < n$ we call it partial Jordan form assignment. From definition of similar matrices it follows:

$$\begin{aligned} A + BF &= TMT^{-1} \\ \Rightarrow AT - TM + BFT &= 0 \end{aligned} \tag{16}$$

where

$$M = \begin{bmatrix} L & R \\ 0 & S \end{bmatrix}$$

and R, S are matrices of proper dimensions. Next we consider $T = [X, V]$, $X \in \mathbb{R}^{n\times s}$, $V \in \mathbb{R}^{n\times(n-s)}$.

From (16) we get

$$AX - XL + BH = 0 \tag{17}$$

where $H \triangleq FX \in \mathbb{R}^{m\times s}$. It clearly holds that for $F \in \mathcal{F}_s(A,B,L)$ there exist the matrices H and X which fulfill the equation (17). Now the process can be reversed to derive an algorithm for computation of state feedback F. If we choose the matrix H we can solve the Sylvester equation (17). Supposing that the eigenvalues of matrices A and L are different, general solution exists in form

$$F = H\left[X^T(H)X(H)\right]^{-1} X^T(H) + F_0 \tag{18}$$

where F_0 is an arbitrary matrix fulfilling condition $F_0X(H) = 0$. It can be shown that the solution (18) holds for almost any chosen H. In case of a multi-input system ($m > 1$), there is an infinite number of state feedbacks which assign a specified Jordan form. Thus, there is a freedom in choice of H, which may be used to fulfill some additional design specification e.g. robustness in stability or control effort. Moreover, the number of free parameters in H can be reduced by replacing it by so called parametric matrix $Q(\alpha)$ where the parametric vector α contains a minimum set of design parameters.

Now the problem of Jordan form assignment using static output feedback is to find a set of all the matrices $\mathcal{K}_s$ fulfilling

$$\mathcal{K}_s(A,B,C,L) \triangleq \left\{ K \in \mathbb{R}^{m\times p} : \; (A+BKC) \sim \begin{bmatrix} L & * \\ 0 & * \end{bmatrix} \right\} \tag{19}$$

Again, if $s < n$ we call it partial Jordan form assignment. From previous section it follows that for any $K \in \mathcal{K}_s(A,B,C,L)$ there has to be $F \in \mathcal{F}_s(A,B,L)$ such that $F = KC$. Thus, there exists $H \in \mathbb{R}^{m\times s}$ and F_0 and relation

$$H\left[X^T(H)X(H)\right]^{-1} X^T(H) + F_0 = KC, \tag{20}$$

where $X(H)$ is solution of Sylvester equation (17). By multiplying (20) with $X(H)$ from the right we get

$$H = KCX(H). \tag{21}$$

Again, we can replace H by a parametric matrix $Q(\alpha)$ with minimum set of free parameters and obtain a bilinear system of equations

$$Q(\alpha) = KCX(\alpha) \tag{22}$$

for the unknown α and K. An analytical method which can solve a set of polynomial equations is needed in order to find all the real solutions. To accomplish that, Gröbner basis method was used. Freely available toolbox for Maple software was developed. More details about the above mentioned method can be found in [Schlegel and Königsmarková (2011)].

3.2 Two-mass system control

The method of partial Jordan form assignment is especially useful for many practical control problems where a low-order fixed structure compensator needs to be designed. The method provides all the solutions in a symbolic form parameterized by a minimum set of free parameters, which may be used to meet any additional design requirements.

Active antiresonant PI(D) controller

Our first goal is to design PI velocity controller with transfer function

$$C(s) = K_p + \frac{K_i}{s} \tag{23}$$

The problem can be easily formulated by means of static output feedback design. By choosing the output matrix (10) we get a controller

$$T(t) = KCx(t) = K_i\varphi + K_p\omega \tag{24}$$

where T is torque setpoint for the lower current loop, $\omega = \dot{\varphi}$ is actual motor velocity. The coefficients of the feedback matrix K directly correspond to the proportional and integral gain of the PI controller. The dynamics of the inner current loop is omitted because of significantly smaller time constants with respect to those of the mechanical subsystem.

The controlled system is of fourth order and only the motor feedback providing two parameters K_i, K_p is available. Therefore, only two closed loop poles can be freely assigned. A suitable choice is to assign a pair of complex poles

$$s_{1,2} = -\xi\omega_c \pm \omega_n\sqrt{(1-\xi^2)}i = -p \pm qpi;\ \xi \in <0,1> \tag{25}$$

In case that those poles are assigned to be dominant, the resulting dynamics of the closed loop is directly determined by the setting of natural frequency w_c and damping ξ. The corresponding real Jordan form for the choice (25) is

$$L = \begin{bmatrix} -p & qp \\ -qp & -p \end{bmatrix} \tag{26}$$

By solving the set of polynomial equations (22) we get analytical solution for controller parameters in form of rational functions

$$K = \begin{bmatrix} K_i \\ K_p \end{bmatrix} = \begin{bmatrix} \dfrac{n_i(p,q)}{d_i(p,q)} \\ \dfrac{n_p(p,q)}{d_d(p,q)} \end{bmatrix} \tag{27}$$

where numerator and denominator functions $n(p,q), d(p,q)$ are two dimensional polynomial functions of fourth order with coefficients depending on parameters of the system (8,9) and design parameters p, q. The exact print-out of the complete solution is omitted due to the limited space. Numerical example is given in the next section of the paper.

Similar procedure can be used for PID controller design. The compensator transfer function has the form

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{\tau s + 1} \tag{28}$$

The system model needs to be extended to include a next state variable $x_5(s) = \widehat{\epsilon} = \frac{\ddot{\varphi}}{\tau s+1}$ representing the derivative action of the controller. The state space representation for the extended model is

$$\overline{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -\dfrac{k_0}{ml} & -\dfrac{r(k_0+gm)}{ml} & -\dfrac{b_0}{ml} & -\dfrac{b_0 r}{ml} & 0 \\ -\dfrac{k_0 r}{Il} & -\dfrac{r^2(k_0+gm)}{Il} & -\dfrac{b_0 r}{Il} & -\dfrac{b_0 r^2}{Il} & 0 \\ -\dfrac{k_0 r}{Il\tau} & -\dfrac{r^2(k_0+gm)}{Il\tau} & -\dfrac{b_0 r}{Il\tau} & -\dfrac{b_0 r^2}{Il\tau} & -\dfrac{1}{\tau} \end{bmatrix} \tag{29}$$

$$\overline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{I} \\ \dfrac{1}{I\tau} \end{bmatrix}, \overline{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

Now the static output feedback has the form

$$T(t) = KCx(t) = K_i\varphi + K_p\omega + K_d\widehat{\epsilon} \tag{31}$$

In this case, three closed loop poles can be freely assigned. There are numerous options for the choice of their location. A suitable method is selection of Butterworth pattern for third order polynomial which minimizes the number of free parameters to desired closed loop bandwidth ω_c

$$\begin{aligned} s_{1,2} &= \frac{-\omega_c \pm \sqrt{3}\omega_c i}{2} = -p \pm \sqrt{3}pi \\ s_3 &= -\omega_c = -2p \end{aligned} \tag{32}$$

The corresponding real Jordan form is

$$L = \begin{bmatrix} -p & \sqrt{3}p & 0 \\ -\sqrt{3}p & -p & 0 \\ 0 & 0 & -2p \end{bmatrix} \tag{33}$$

The solution leads to scalar rational functions for each of the controller gains:

$$K = \begin{bmatrix} K_i \\ K_p \\ K_d \end{bmatrix} = \begin{bmatrix} \dfrac{n_i(p)}{d_i(p)} \\ \dfrac{n_p(p)}{d_p(p)} \\ \dfrac{n_d(p)}{d_d(p)} \end{bmatrix} \tag{34}$$

We get numerator and denominator polynomials of eighth or ninth order.

The obtained controllers are able to actively damp the oscillatory dynamics of the system which may be excited due to external disturbance or setpoint change. The overall desired closed-loop bandwidth w_c is limited due to the fixed order of the compensator. When approaching the values of the antiresonant frequency w_{nz} (11), the rest of the poles which are not assigned are moving towards the right half-plane and cause a degradation of performance or even instability of the closed loop. However, the obtained symbolic expressions for the controller parameters can be very useful for precise manual tuning in practical applications, because they give a systematic guide for adjustment of the PID gains.

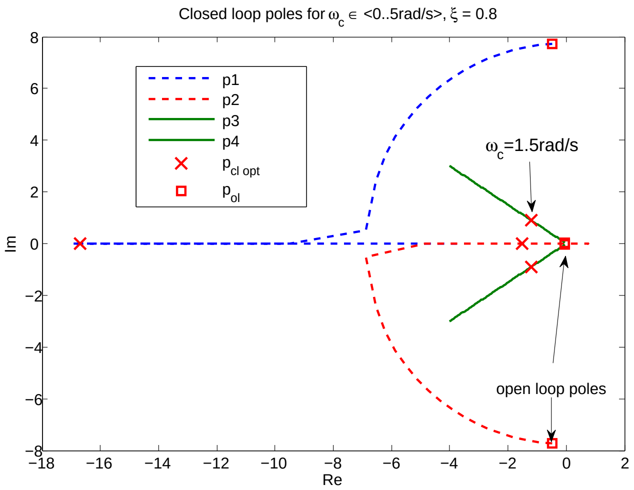

Fig. 4. Root locus for closed loop system

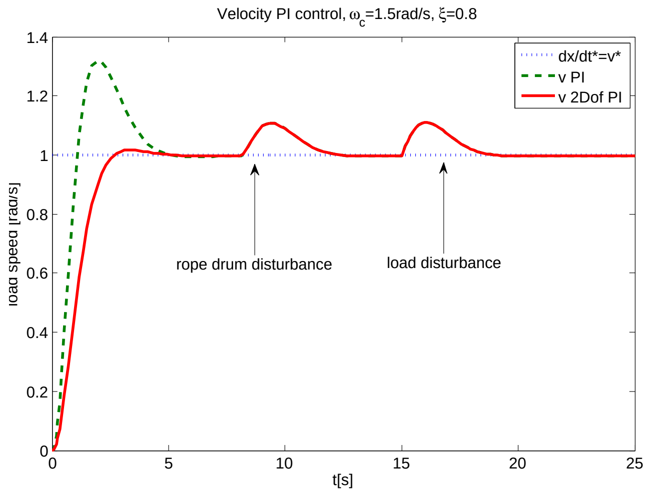

Fig. 5. Load speed control and disturbance rejection

4. NUMERICAL EXAMPLE & SIMULATION

We consider following parameter values for the rope drum system:

$$\begin{aligned} I &= 0.4\ kg.m^2, b_0 = 10, k_0 = 10000, \\ m &= 100\ kg, r = 0.2\ m, g = 10\ \frac{m}{s^2} \end{aligned} \tag{35}$$

The linearized model for the equilibrium in $l = 20m$ is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5 & -\frac{11}{10} & -\frac{1}{200} & -\frac{1}{1000} \\ -250 & -55 & -\frac{1}{4} & -\frac{1}{20} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{5}{2} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{36}$$

Resulting transfer function from motor torque to load velocity

$$P(s) = \frac{K}{s}\frac{s^2 + 2\xi_z\omega_{nz}s + \omega_{nz}^2}{s^2 + 2\xi_p\omega_{np}s + \omega_{np}^2} = \frac{2.5}{s}\frac{s^2 + 0.005s + 5}{s^2 + 0.055s + 60}$$

$$\omega_{nz} = 2.24\frac{rad}{s}, \xi_z = 0.001, \omega_{np} = 7.75\frac{rad}{s}, \xi_p = 0.004$$

By computing the output feedback for the PI controller with parameterization of the closed loop poles given by (25), we get the equations for proportional and integral gain:

$$\begin{aligned} K_i = {} & (2/5)(40000p^4 + 120000q^4p^4 + 120000q^2p^4 + \\ & +40000q^6p^4 - 400q^4p^3 - 800q^2p^3 - 400p^3 - \\ & -2599989q^4p^2 - 4399978q^2p^2 - 1799989p^2 - 22000q^2p - \\ & -22000p + 12000000q^2 + 12000000)p^2/(40000q^4p^4 + \\ & +400001p^2 - 2000p + 80000q^2p^4 + 1000000 + \\ & +40000p^4 - 400p^3 - 400q^2p^3 - 399999q^2p^2) \\ K_p = {} & (4/5)p(-2400q^2p^3 + 12000000 - 22500p - 500q^2p - \\ & +1000q^4p^3 - 1400p^3 + 400011p^2 + 40000p^4 + \\ & +80000q^2p^4 - 399989q^2p^2 + 40000q^4p^4)/ \\ & /(40000q^4p^4 + 400001p^2 - 2000p + 80000q^2p^4 + \\ & +1000000 + 40000p^4 - 400p^3 - 400q^2p^3 - 399999q^2p^2) \end{aligned}$$

Functionality of the resulting design may be viewed in terms of location of the closed loop poles for varying choice of closed loop bandwidth ω_c. Figure (4) shows the root locus for the closed system and $\omega_c \in < 0..5 > \frac{rad}{s}$. The value of desired damping was fixed to $\xi = 0.8$ for sake of clear interpretation. As we can see in the figure, the pair of assigned poles ($p_{3,4}$ - solid green plot) tracks the the line with a constant slope which corresponds to chosen damping and parameterization (25). The second pair of weakly damped open loop poles ($p_{1,2}$ - blue and red dashed plot) are moving in complex plane from their initial open loop position (red squares) towards the imaginary axis and their damping increases. Once they reach the imaginary axis, they split and move to the left and right side. As the ω_c is further increasing, one of the poles is becoming dominant and the bandwidth of the closed loop is decreasing. For large values of $\omega_c > \omega_z$, the pole enters right half-plane and the closed loop becomes unstable. Optimal setting of $\omega_c \approx 1.5\frac{rad}{s}$ gives the dominant pair of well damped poles and couple of faster real poles.

Figure (5) shows the simulation results obtained with proposed PI controller and linear model (36). Setpoint change and disturbance rejection test was performed. The first disturbance in form of load torque in time $t = 8s$ is acting on the rope drum, the second one occurring in $t = 15s$ is a force applied on the load. The compensator

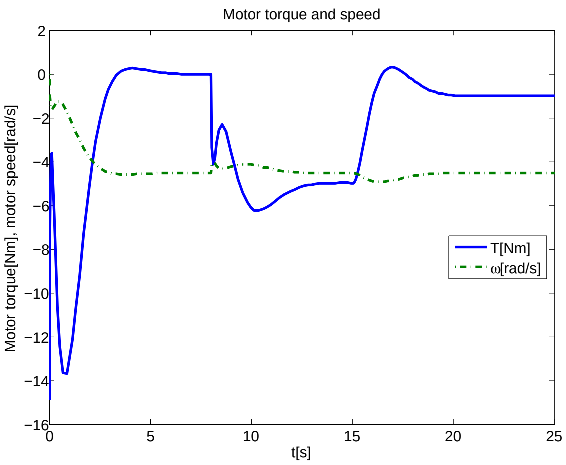

Fig. 6. Motor torque and speed

actively damps the oscillations even in the presence of disturbances. This is the main advantage compared to common method of passive notch-filter control. Similar results can be obtained with the proposed PID controller. The introduction of derivative action does not bring any special improvement for this particular case, because the achievable bandwidth is still limited by the value of w_z. However, it serves as an example of how a fixed-structure compensator can be designed.

Two degrees of freedom PI(D) controller can be used to reduce the initial overshoot which is caused by stable real closed loop zero. With the control law in the form

$$T(s) = K_p(b\omega^*(s) - \omega(s)) + \frac{K_i}{s}(\omega^*(s) - \omega(s)), \qquad (37)$$

where b is setpoint weghting coefficient, the location of the closed loop zero can be changed to reduce the overshoot while the disturbance response remains unaltered. Motor torque and speed for the same experiment is shown in figure 6.

The obtained controller is designed for the linearized model of the rope drum system. Significant change in rope length or load mass may lead to a shift in resonance and antiresonance frequencies and detuning of the compensator. In such case, gain-scheduling technique can be employed. Common proportional controller can be used in the position loop, once the velocity controller is properly tuned. Figure 7 shows an experiment with nonlinear system (7). The load tracks desired jerk-limited position profile during a rest-to-rest movement. Combination of gain-scheduling and input shaping method was used to damp any unwanted oscillations.

5. CONCLUSION

The paper deals with problem of rope drum control for stage motion control systems. Oscillatory dynamics is observed due to the elasticity of the rope with hanging load. Derivation of mathematical model leads to time varying two mass flexible system. Active antiresonant PI(D) velocity controller is designed for the linearized model using partial Jordan form assignment method. Simple tuning rules are derived in a symbolic form for systematic adjustment of the PID gains and rapid commissioning of the drive.

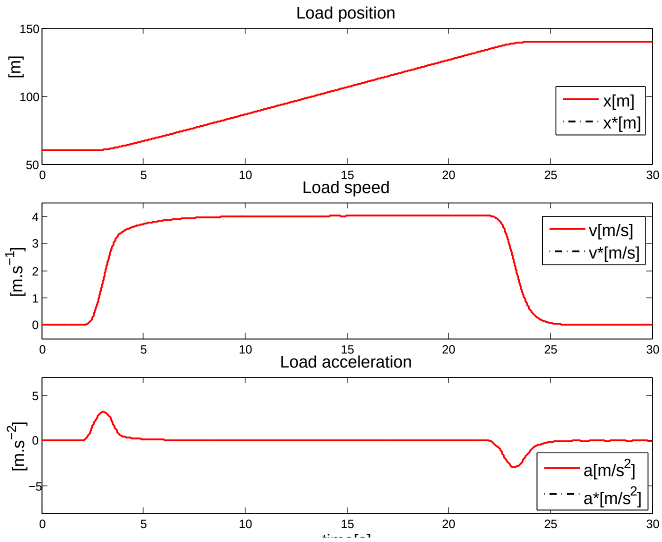

Fig. 7. Position tracking of nonlinear system

The results can be applied to a problem of motion control of any mechanical system with flexible coupling and one dominant resonance mode. The proposed algorithm for output feedback Jordan form assignment can be very useful for many practical control problems where a low order fixed structure compensator needs to be designed. The true potential of this method reveals itself in case of MIMO systems. Here the non-redundant parameterization with minimum number of free parameters offers a large degree of freedom for fulfilment of various design requirements while guaranteeing that all solutions have been found. The only limitation is computational burden which grows exponentially with the number of free parameters.

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