GMV-PID Controller Design with Gradient Method for the Energy Weighting Factor in Nonlinear Plants

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Abstract: Since the PID tuning formula of Ziegler-Nichols step test, perhaps the best known, the process control literature is still showing lots of PID control schemes to improve the closed-loop stability. In this paper, using the relationship between the PID and generalized minimum variance (GMV) controller and adjusting online the control weighting factor by an adaptive learning algorithm of first-order (gradient method), a new control design is investigated. Besides, the control algorithm is gathering the direct estimation technique, with the covariance resetting procedure based on the weighted prediction error, to deal with nonlinear plants. Numerical and practical experiments are shown to evaluate the behavior of the proposed GMV-PID control design.

Keywords: PID controllers, minimum variance control, optimal search techniques, adaptive algorithms, numerical simulation, gradient methods, asymptotic stability.

1. INTRODUCTION

The importance of the PID controller in the industry is related to factors such as simplicity, stability and performance. At least three parameters must be set and sometimes they need an online retuning in complex environments, i.e., due to plant parameters variations (Åström and Hägglund, 2006; Visioli, 2006; Gude et al., 2006). Depending on the cleverness of the operator, the parameter tuning procedure can vary from seconds to hours, especially for slow dynamic or in nonlinear plants. To deal with these real loop situations, it is possible to find, in books and conference papers, many automatic tuning approaches where success and failure cases are reported in the following industries: chemical, petrochemical, metallurgy, pulp and paper, electrical, mechanical, etc. (Ang et al., 2005). In this scenario there are projects related to the PID tuning where the combination with other control strategies are applied in order to guarantee the robustness of the control loop. Internal Model Control (IMC) and Generalized Minimum Variance (GMV) control methodologies are of great importance because they can acquire different closed-loop characteristics such as model reference, detuned behavior and time-delay compensation (Gude et al. 2006; Veronesi and Visioli, 2011). A good example is the work of Cameron and Seborg (1983) hybridizing the GMV design with the PID tuning, where excessive actuator movements can be avoided and non-minimum phase plants can be controlled. However, due to the fixed tuning, the performance of GMV-PID controller can degrade or even become unstable when applied to nonlinear processes. In addition, self-tuning control strategies are becoming conventional methods and a considerable number of successful experimental studies have been carried out in PID and GMV over the last years (Doi and Mori, 2002; Hägglund and Åström, 2000; Kirecci et al., 2003). The idea of using direct parameter estimation is good because the GMV-PID linear controller design is based on a linear model that is obtained in the operating point vicinity and the performance specifications is not guaranteed in the whole operating range. To avoid the drawback of fixed GMV-PID parameters damaging the transient response, a gradient method is combined into the GMV-PID framework. Therefore in this paper, the adaptive character is implemented by means of two online adjustable ways: direct estimation of the GMV-PID parameters and learning method to optimize the energy weighting factor of the controller. The direct GMV-PID control design proposed in this paper differs from the existing GMV-PID controller of Cameron and Seborg (1983) and Yamamoto et al. (1999), where they only employ a recursive algorithm to identify the controlled plant parameters. The process response and control signal can be visualized from simulations with the proposed control-loop strategy. Comparison with conventional tuning sets is done.

2. GMV BASED PID CONTROLLER DESIGN

2.1 Process Description

Many experimental plants can be well described by the following First-Order Plus Dead-Time (FOPDT) model (Åström and Hägglund (2006):

$$\frac{Y(s)}{U(s)} = G_m(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \tag{1}$$

where K_p, τ and θ denote the process gain, time-constant and time-delay, respectively. In order to ensure a second-order model for controller design purpose, it is possible to use a first-order lag element for the time-delay and (1) can be rewritten as

$$G_m(s) \approx \frac{K_p}{(\theta s+1)(\tau s+1)} \tag{2}$$

The discrete-time model, obtained from (2), called Controlled Auto-Regressive (CAR) model, is given by

$$A(z^{-1})y(t) = z^{-1}B(z^{-1})u(t) + \xi(t) \tag{3}$$

where

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \;;\; B(z^{-1}) = b_0 + b_1 z^{-1} \tag{4}$$

and $y(t)$, $u(t)$ and $\xi(t)$ are the process output, control input and white noise. Furthermore, z^{-1} denotes the backward shift operator, namely, $z^{-1}y(t) = y(t-1)$.

2.2 PID Controller Structure

The following digital PID control law, in parallel form, is considered (Bobál et al., 2005; Visioli, 2006):

$$\begin{aligned}\Delta u(t) = K_c[e(t)-e(t-1)] + K_i e(t) + K_d[e(t) - \\ 2e(t-1)+e(t-2)]\end{aligned} \tag{5}$$

where K_c, K_i and K_d are the tuning parameters and represent the proportional, integral and derivative gains, respectively. In addition, $e(t) = y_r(t) - y(t)$ is the system error and $y_r(t)$ is the reference signal. Assuming that the reference appears only in the integral part and the output is filtered, $y_f(t)$, then for the GMV control design, (5) is modified and can be rewritten as

$$\begin{aligned}\Delta u(t) = K_i y_r(t) - (K_c + K_i + K_d)y_f(t) + \\ (K_c + 2K_d)y_f(t-1) - K_d y_f(t-2)\end{aligned} \tag{6}$$

The closed-loop system performance strongly depends on PID parameters. These parameters can be computed with the GMV-PID control tuning scheme, which have been already proposed by Cameron and Seborg (1983) and Yamamoto et al. (1998), using the trial and error procedure. However, due to the difficult to tune a design parameter of the GMV, that weights the energy of the control signal, and to avoid a fixed value that can be unfeasible in nonlinear essays, this paper proposes an on-line adaptive learning procedure to estimate it that is desirable as a systematic way for practical applications.

2.3 GMV Based PID Parameter Tuning

The derivation of GMV-PID controller proposed in Cameron and Seborg (1983) is reviewed. To design the GMV controller it is assumed that the monovariable controlled system is modelled by (3) and, additionally, an auxiliary output is defined as follows:

$$\phi(t) = P(z^{-1})y(t) + Q(z^{-1})u(t-1) - R(z^{-1})y_r(t-1) \tag{7}$$

with $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$ being weighting polynomials. If the input-output data are known until time t, the $\phi(t+1)$ can be regarded as estimating $\phi_y(t+1)$ by

$$\phi_y(t+1) = P(z^{-1})y(t+1)$$

$$\phi_y(t+1) = \frac{F(z^{-1})}{P_d(z^{-1})}y(t) + E(z^{-1})B(z^{-1})u(t) + E(z^{-1})\xi(t+1) \tag{8}$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained by the identity

$$\frac{P_n(z^{-1})}{A(z^{-1})P_d(z^{-1})} = E(z^{-1}) + z^{-1}\frac{F(z^{-1})}{A(z^{-1})P_d(z^{-1})}$$

with $P_n(z^{-1})/P_d(z^{-1}) = P(z^{-1})$. Assuming $G(z^{-1}) = E(z^{-1})B(z^{-1})$ then (8) is rewritten as follows:

$$\phi_y(t+1) = F(z^{-1})y_f(t) + G(z^{-1})u(t) + \varepsilon(t) \tag{9}$$

where $\varepsilon(t) = E(z^{-1})\xi(t+1)$ and $y_f(t)$ is the filtered output defined by

$$y_f(t) = \frac{1}{P_d(z^{-1})}y(t) = \frac{(1-p_{d1})}{(1-p_{d1}z^{-1})}y(t) \;;\; 0 \le p_{d1} < 1$$

Therefore, from (7) and (9) and forcing to zero the deterministic part of $\phi(t+1)$, the GMV control law is given by

$$u(t) = \frac{R(z^{-1})y_r(t) - F(z^{-1})y_f(t)}{G(z^{-1}) + Q(z^{-1})} \tag{10}$$

To ensure that the GMV in (10) has the PID structure of (6) it is necessary to define the polynomials $F(z^{-1})$ and $R(z^{-1})$ as

$$F(z^{-1}) = f_0 + f_1 z^{-1} + f_2 z^{-2} \tag{11}$$

$$R(z^{-1}) = H(1) = \frac{F(1)}{P_d(1)} = \sum_{i=0}^{2} f_i \tag{12}$$

The derivation of $R(z^{-1}) = H(1)$ is to ensure steady-state agreement between output and reference signals. As the final design step, the integral action must be introduced by setting the $Q(z^{-1})$ polynomial as

$$Q(z^{-1}) = \frac{(1-z^{-1})}{\nu} - G(z^{-1}) \tag{13}$$

where ν is a design parameter. If (12) and (13) are substituted into (10), then the corresponding settings of the GMV-PID control law are expressed as

$$\Delta u(t) = \nu[Hy_r(t) - F(z^{-1})y_f(t)] \tag{14}$$

$$K_c = -\nu(f_1 + 2f_2)\ ;\ K_i = \nu(f_0 + f_1 + f_2)\ ;\ K_d = \nu f_2 \quad (15)$$

The choice of ν modifies the closed-loop behavior of the controlled system. Large values of ν will tend to result in more vigorous control and underdamped responses while small values of ν will tend to provide more sluggish control and overdamped responses. Therefore for real applications, it is desirable to have a systematic approach for obtaining an adequate value of ν, avoiding the time-consuming trial and error procedure, that sometimes sets an inadequate value which may lead to an unstable dynamic.

2.4 Optimizing the Weighting Factor of the GMV-PID Controller

To tune the design parameter ν of (14), which is constrained between *0* and *1*, a sigmoid function is employed to map the space *[0 1]* to the entire real number space as follows:

$$\nu(t) = \frac{1}{1 + e^{-\rho(t)}} \quad (16)$$

with $\rho(t)$ being a real number. Using (16) the parameter $\nu(t)$ is calculated updating $\rho(t)$ according to the gradient method, such that

$$\rho(t+1) = \rho(t) - \mu(t)\frac{\partial J}{\partial \rho(t)} \quad (17)$$

where the parameter $\mu(t) = \mu_0/t\ (0 < \mu_0 < \mu_{max})$ regulates the stability, convergence velocity and must be selected for each application. The cost function J has the form

$$\begin{aligned} J(t+1) = {} & [P(z^{-1})y_r(t+1) - \phi_y(t+1)]^2 + \\ & \lambda[u(t) - u(t-1)]^2 \end{aligned} \quad (18)$$

with λ as the weighting parameter. In order to calculate the gradient $\partial J/\partial \rho(t)$ the chain rule is applied as follows:

$$\frac{\partial J}{\partial \rho(t)} = \frac{\partial J}{\partial u(t)}\frac{\partial u(t)}{\partial \nu(t)}\frac{\partial \nu(t)}{\partial \rho(t)} \quad (19)$$

where

$$\begin{aligned} \frac{\partial J}{\partial u(t)} = {} & -2[P(z^{-1})y_r(t+1) - F(z^{-1})y_f(t) - \\ & \overline{G}(z^{-1})u(t-1)]\frac{\partial \phi_y(t+1)}{\partial u(t)} + 2\lambda[u(t) - u(t-1)] \end{aligned} \quad (20)$$

$$\overline{G}(z^{-1}) = z[G(z^{-1}) - g_0]\ ;\ \frac{\partial \phi_y(t+1)}{\partial u(t)} = g_0$$

$$\frac{\partial u(t)}{\partial \nu(t)} = (\sum_{i=0}^{2} f_i)y_r(t) - (\sum_{i=0}^{2} f_i z^{-i})y(t) \quad (21)$$

$$\frac{\partial \nu(t)}{\partial \rho(t)} = \nu(t)[1 - \nu(t)] \quad (22)$$

2.5 Updating the Weighting Factor with Cameron-Seborg Tuning

The scalar parameter $\nu(t)$, in (14), provides a convenient tuning parameter for the GMV-PID controller. For practical applications, it is desirable to have a satisfactory initial estimation procedure before actually implementing the digital control signal. So, for the purpose of comparison, the Cameron and Seborg (1983) tuning method is employed as the standard control design. Then, $\nu(t)$ can be well tuned based on the discrete model of (1) such as

$$G_p(z) = \frac{z^{-(d+1)}b_0}{1 - a_1 z^{-1}} \quad (23)$$

where $b_0 = K_p(1 - a_1)$, $a_1 = exp(-T_s/\tau)$, $d = \theta/T_s$ and T_s is the sampling period. Cameron and Seborg (1983) have demonstrated that, based on the characteristic equation and the Jury stability method, the maximum value of $\nu(t)$, to guarantee closed-loop stability for $d = 0$ and $d = 1$, is given, respectively, by

$$\nu_{max} = \frac{2(1 + a_1)}{K_p a_1(1 - a_1)}$$
$$\nu_{max} = \frac{1}{K_p a_1^2} \quad (24)$$

In practical applications, it is desirable that the estimated value of $\nu(t)$ be conservative to avoid closed-loop dynamics with oscillatory or unstable behavior. Hence Cameron and Seborg (1983) have recommended that $\nu(t) = \beta\nu_{max}\ (\forall t)$ be used where $0 < \beta < 1$. A value of $\beta = 0.1$ or $\beta = 0.2$ can provide a satisfactory calibration for $\nu(t)$ if the process is open-loop stable.

2.6 Direct Adaptive Estimation of the GMV-PID Controller

For the GMV-PID control law synthesis, in the direct adaptive framework, a parametric estimation scheme is based on the recursive least-squares algorithm with covariance resetting. The estimator is derived from (9) assuming that $F(z^{-1}) = f_0 + f_1 z^{-1} + f_2 z^{-2}$ and $G(z^{-1}) = g_0 + g_1 z^{-1}$. Parameters and measurements vectors are given by

$$\theta^T(t) = [f_0\ f_1\ f_2\ g_0\ g_1]$$
$$\varphi^T(t) = [y(t-1)\ y(t-2)\ y(t-3)\ u(t-1)\ u(t-2)] \quad (25)$$

and the estimated parameters are updated using the standard recursive least-squares estimator

$$\theta(t)=\theta(t-1)+K(t)[y(t)-\varphi^T(t)\theta(t-1)]$$

$$K(t)=\frac{P(t-1)\varphi(t)}{1+\varphi^T(t)P(t-1)\varphi(t)} \tag{26}$$

$$P(t)=P(t-1)+\frac{P(t-1)\varphi(t)\varphi^T(t)P(t-1)}{1+\varphi^T(t)P(t-1)\varphi(t)}$$

where *K(t)* and *P(t)* are the estimator gain and covariance matrix, respectively. To deal with inaccuracy, robustness, interactions between the estimator-controller and time-varying plants, the covariance resetting procedure is employed. Hence, the covariance matrix is calculated in the traditional way, i.e., $P(t) = P(t) + Q(t)$. The reset of *P(t)* is based on the weighted prediction error which is determined as follows:

$$\text{error}_{\text{reset}}(t)=\frac{\sum_{i=1}^{N_e}\gamma(i)e_{est}(t-i+1)}{\sum_{i=1}^{N_e}\gamma(i)} \tag{27}$$

with $e_{est}(t)=y(t)-\varphi^T(t)\theta(t-1)$. At every discrete-time instant, a moving window of N_e values is used to calculate the weighted prediction error and the matrix *Q(t)* is added only when the weighted prediction error value exceeds a tolerance error, e_{tol}, given by the user. This kind of behavior is appropriate not only to reduce undesirable covariance resetting due to process noise but also to provide good accuracy for tracking time-varying parameters.

3. NUMERICAL AND EXPERIMENTAL SIMULATIONS

The effectiveness of the proposed adaptive GMV-PID controller is assessed on two simulation examples. As the first essay, a linear time-invariant plant is given by the following equation:

$$G_p(s)=\frac{1}{(s+1)^4}$$

User specified parameters are set to $P(0) = 200I_{5x5}$, $\theta(0) = 0.01$, $P(z^{-1}) = 1$, $\nu(0) = 1$, $\rho(0) = 0$, $\mu_{max} = 20$, $\lambda = 2$ and $T_s = 1\ s$. The behavior of the closed-loop system, for two setpoint values, is shown in Figure 1. Figure 2 shows the trajectory of $\nu(t)$, with final value of 0.1097. PID parameters at the end of the simulation become $K_c = 0.0235$, $K_i = 0.0836$, $K_d = 0.0255$.

It is possible to observe that the GMV-PID control system provides a good performance with a short settling time, without overshoot and the control variable presents a smooth response that is important from the viewpoint of practical implementations.

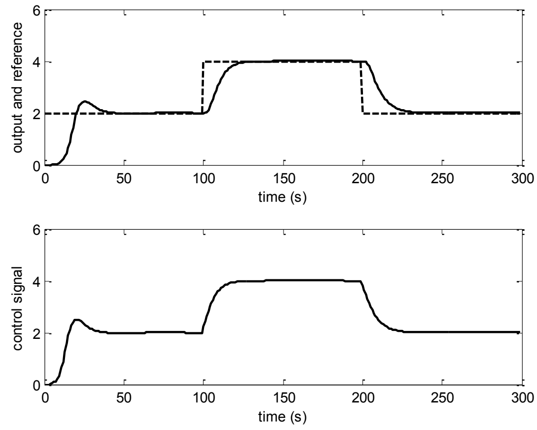

Fig. 1. Process output for the proposed GMV-PID controller.

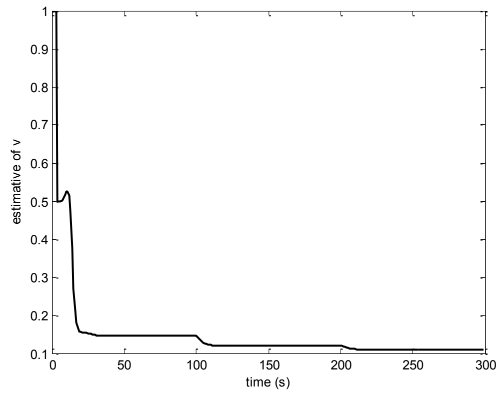

Fig. 2. Behavior of the design parameter $\nu(t)$.

To evaluate the behavior of the controlled forth-order process with the adaptation of the weighting factor with Cameron-Seborg procedure, a FOPDT model is employed as

$$G_m(s)=\frac{e^{-s}}{3.2s+1}$$

Parameters are set to $P(0) = 200I_{5x5}$, $\theta(0) = 0.01$ and $\beta = 0.2$. The behavior of the closed-loop system, under the same setpoint values, is shown in Figure 3, with $\nu(t) = 0.3736\ (\forall t)$. PID parameters at the end of the simulation are $K_c = 0.1201$, $K_i = 0.2975$, $K_d = 0.0503$. Now the response is oscillatory, exhibits overshoot and the settling time is a little bit higher.

For the purpose of comparison, if the Ziegler-Nichols PID setting is employed then an unstable dynamic is achieved. Figure 4 illustrates the response with the AMIGO tuning rule for the PID controller where the closed-loop system has sluggish behavior with a longer settling time (O'Dwyer, 2000; Åström and Hägglund, 2004; Ang et al., 2005). So, the numerical results indicate that the proposed GMV-PID control provides the best responses.

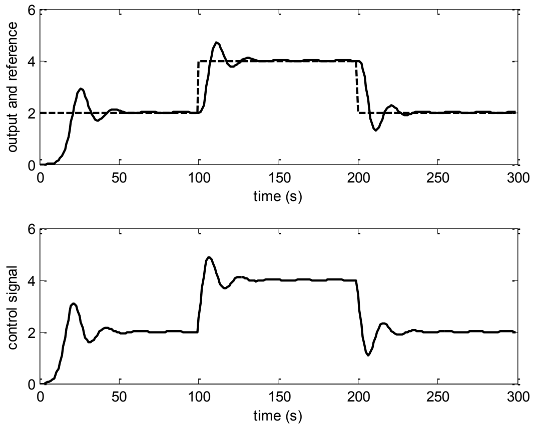

Fig. 3. Output for the GMV PID tuned by Cameron-Seborg.

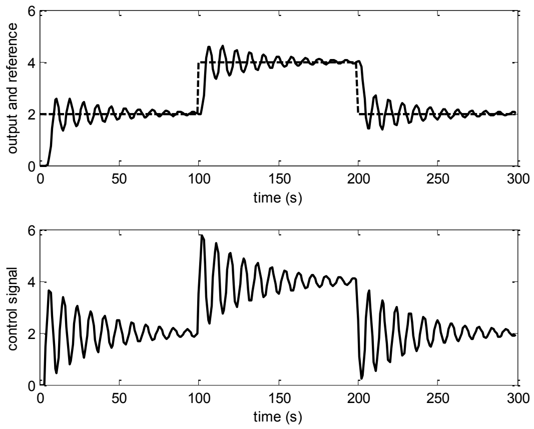

Fig. 4. Output for the PID controller tuned by AMIGO.

As a second essay, an experimental plant, called brightness process, is used to evaluate the proposed GMV-PID controller and it was built at the Department of Automation and Systems/Federal University of Santa Catarina. The closed-loop control system is composed of a light sensor, a signal conditioning circuit for the output measurement, a data acquisition board A/D-D/A and a power drive circuit, as shown in Figure 5.

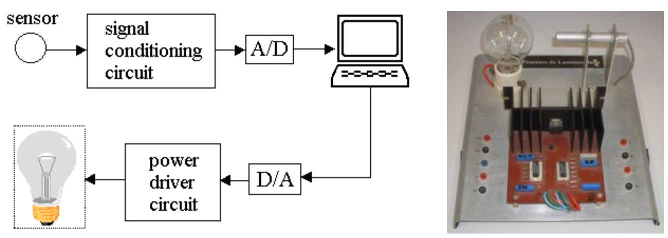

Fig. 5. Diagram and photo of the brightness process.

The process is characterized by a nonlinear behavior for both, the time constant and static gain, and exhibits saturation for the control signal, ranging from $0\ V$ to $5\ V$. These complex features motivate the application of the GMV-PID controller in a practical scenario.

Response to setpoint changes for a step input of magnitude of $2\ V$ and $3\ V$ are shown in Figure 6. Figure 7 illustrates the behavior of the PID gains in the whole simulation for the proposed GMV-PID controller. Tuning parameters are set to $P(0) = 300I_{5x5}$, $Q(0) = 3I_{5x5}$, $\theta(0) = 0.01$, $P(z^{-1}) = 1$, $\nu(0) = 1$, $\rho(0) = 0$, $\mu_{max} = 10$, $\lambda = 3$ and $T_s = 0.1\ s$. The controller provides good setpoint tracking and the control signal exhibits smooth dynamic. The final value of $\nu(t)$ is 0.1195. For (27), it is set $N_e = 3$, with $\gamma(1) = 0.6$, $\gamma(2) = 0.3$, $\gamma(3) = 0.1$ and $e_{tol} = 0.035$.

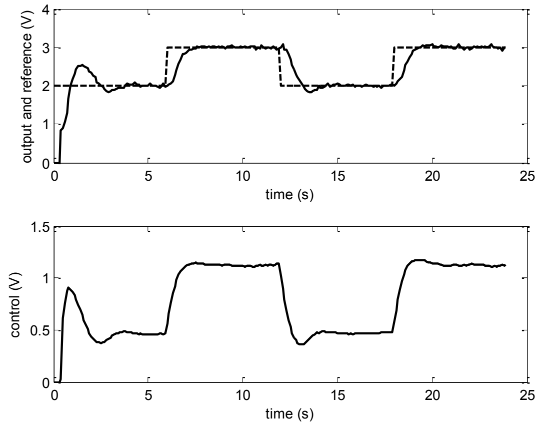

Fig. 6. Output and control for the GMV-PID controller.

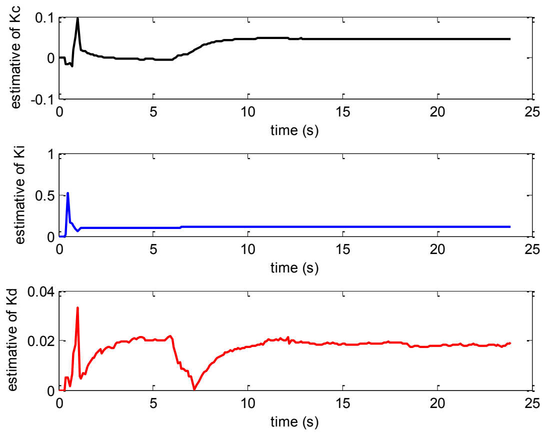

Fig. 7. Evolution of the GMV-PID gains.

Figure 8, from the top to the bottom, it is shown the simulation results with Cameron-Seborg, Ziegler-Nichols and AMIGO tuning rules. Used PID gains are listed in Table 1.

It is possible to observe that the closed-loop system results are degrading with oscillatory output and ringing in the control that is not good from the viewpoint of the actuator. On the other hand, the investigated GMV-PID control strategy, excluding the initial operating condition, regulates the plant output in a conservative way, as a first-order response (slower control variance, faster settling time and exhibits no overshoot).

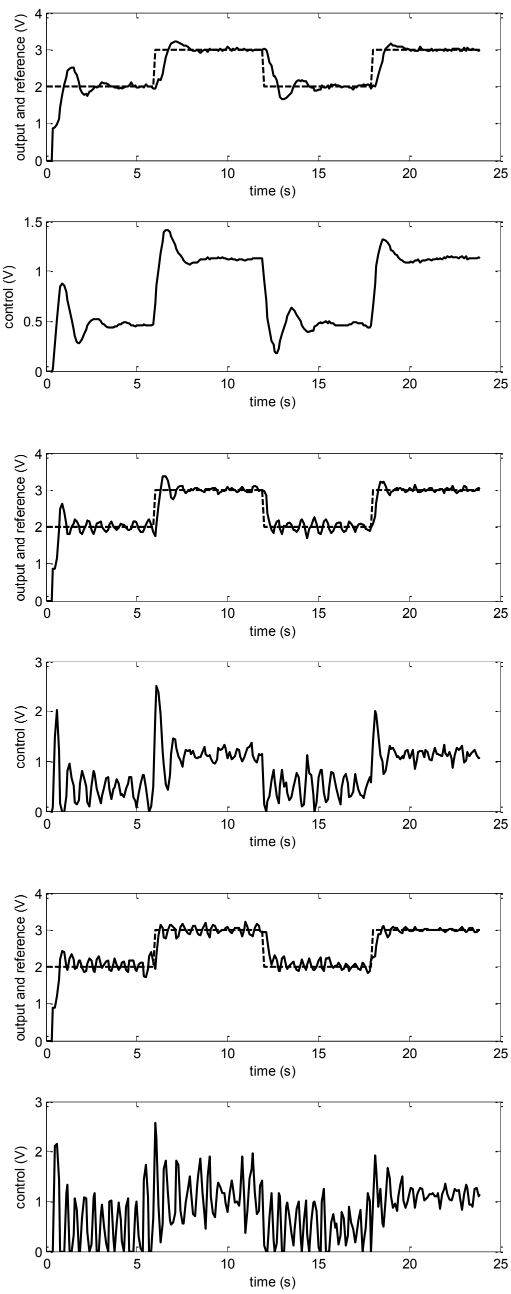

Fig. 8. Output and control for classical PID tuning rules.

Table 1. PID gains for the experimental simulation

Tuning Rule	K_c	K_i	K_d
Cameron-Seborg	0.0841	0.2123	0.0440
Ziegler-Nichols	1.1067	1.1067	0.2667
AMIGO	2.5333	1.3412	0.6104

4. CONCLUSIONS

The PID tuning procedure is, in many situations, nontrivial and sometimes difficult to find an adequate set of parameters to guarantee a desired closed-loop dynamic.

The idea described in this paper, for the design of the GMV-PID controller, which is based on a combination of the least squares estimation and the parameter optimization method for weighting the control signal, is good to improve the behavior of the system and to be used in time-varying plants. The resulting GMV-PID controller was applied in two simulation examples and has shown an improved performance over popular PID tuning rules.

Although the gradient method idea, to penalize excessive variations of the control signal, which is impractical, has been employed as part of the GMV-PID controller, it is possible to insert it on a variety of digital controller design types. This idea is been investigated as future works for the dynamic matrix controller and GMV state-space form.

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