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Model Order Reduction for Decentralized PID Control Design on TITO processes ⋆

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Abstract: The decentralized PID control design of multivariable processes is an appealing approach due to its practical applicability and the existence of a good theoretical basis. However, although industrial processes may be modeled using simple first order and second order models, in decentralized control, the models increase their complexity due to loop interaction, compromising the applicability of existing PID tuning rules. In fact, even inverse response behavior may appear due to loop interaction. The contribution of the article is twofold. First, we derive conditions to assess the inverse response behavior of reduced effective transfer functions. As a result, we know when tuning rules for first order and minimum phase second order models are suited for decentralized control tunning. Secondly, at the sight of matrix transfer function parameters, we derive simplified models in a straightforward way. The results are useful for decentralized MIMO process control design using simple tuning rules derived for first order and second order systems.

Keywords: Multivariable process, Model order reduction, Reduced effective transfer function, Inverse response, Dimensional analysis.

1. INTRODUCTION

Processes in industrial settings are normally multivariable. Multivariable control systems may be designed using multivariable control structures (Skogestad and Postlethwaite (1996)). However, multivariable controller design and tuning is far more complicated than the well established single loop control. As a result, one practical approach is to control multivariable processes by using single input/single output (SISO) control structures, that is, decentralized control.

Decentralized control is a valid approach because, in some cases, the performance increase obtained by using multivariable control structures does not justify its design and maintenance. Moreover, the controller (re)tuning for uncertain and non-linear industrial processes is simplified by decentralized control. There is also a well developed theory and tuning rules for SISO PID control (Astrom and Hagglund (2006)) available to be used in decentralized control. As a result, decentralized control is an appealing approach to be used in industrial process control.

On the other hand, even for industrial multivariable processes showing simple dynamics, the design of decentralized controllers have to deal with complex model dynamics due to loop interaction. The complexity of resulting models dramatically limits the applicability of PID tuning rules existing in the literature (O'Dwyer (2006)), because they are based on simple models.

We tackle the problem of model order reduction for systems resulting from multivariable processes. In particular, we focus on two inputs/two outputs (TITO) systems which entries are modeled by first order plus dead time (FOPDT) models. Even in this case, the models that capture loop interactions are of high order and may show inverse response behavior. We derive conditions to ascertain when we can expect inverse response behavior of reduced order effective transfer functions (RETF). As a result, we know when a tuning rule for a FOPDT model or a minimum phase SOPDT model can be used for PID tuning or, on the contrary, a non-minimum phase SOPDT tuning rule is required.

The second contribution of the article is to show that dimensionless numbers may be used to derive simpler models in an straightforward way without requiring other complex model reduction approaches. Thus, at the sight of the RETF we derive simple FOPDT and SOPDT models that capture the most important dynamics for control and they enable the use of PID tuning rules derived for these model structures. By using dimensional analysis theory we set the problem into a dimensionless framework (Balaguer et al. (2009)). In this way we are able to exploit simplification assumptions at the sight of dimensionless number values and their relations.

2. PRELIMINAIRES

2.1 Effective and reduced effective transfer functions

The effective transfer function (ETF) is defined as the relation between an input-output pair of a MIMO system when the rest of input-output pairs are closed loop by

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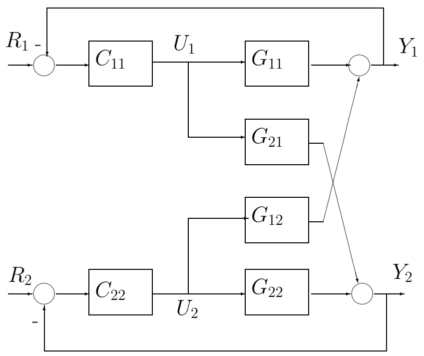

Fig. 1. Decentralized control structure of a TITO system.

their respective decentralized controllers. For instance, considering the TITO system of figure 1, the ETF between output Y_1 and input U_1 is

$$G_{11}^e = \frac{Y_1}{U_1} = G_{11} - \frac{C_{22}G_{12}G_{21}}{1 + C_{22}G_{22}} \tag{1}$$

where for the sake of readability the complex variable s is not shown. Equation (1) shows that the ETF depends on the existing controllers of closed loop pairs.

2.2 Reduced effective transfer functions

The EFT dependence on existing controllers may be eliminated under the perfect control assumption. For a TITO system, if we assume perfect control of controller C_{22}, that is (Skogestad and Postlethwaite (1996)),

$$\frac{Y_2}{R_2} = \frac{C_{22}G_{22}}{1 + C_{22}G_{22}} = 1 \tag{2}$$

the ETF can be simplified as follows

$$G_{11}^{re} = G_{11} - \frac{G_{12}G_{21}}{G_{22}} \tag{3}$$

Equation (3) is known as reduced effective transfer function (RETF), and it depends on system transfer functions only.

2.3 Problem Statement

We consider the decentralized PID control design of TITO systems composed by FOPDT processes, as can be seen in (4). The objective is to tune decentralized PID controllers using existing tuning rules relations designed for FOPDT and SOPDT systems, which are pervasive in industrial settings (O'Dwyer (2006)).

$$G(s) = \begin{bmatrix} \dfrac{K_{11}e^{-T_{11}s}}{\tau_{11}s+1} & \dfrac{K_{12}e^{-T_{12}s}}{\tau_{12}s+1} \\ \dfrac{K_{21}e^{-T_{21}s}}{\tau_{21}s+1} & \dfrac{K_{22}e^{-T_{22}s}}{\tau_{22}s+1} \end{bmatrix} \tag{4}$$

Unfortunately, even for FOPDT TITO systems the RETF (3) is no longer a FOPDT or SOPDT transfer function. This fact limits the applicability of simple tuning methods. Moreover, the approximation of RETF by FOPDT or SOPDT models is not obvious. Consider for instance the RETF of a FOPDT TITO system, given by

$$G_{11}^{re} = \frac{K_{11}e^{-T_{11}s}}{(\tau_{11}s+1)} - \frac{K_{12}K_{21}(\tau_{22}s+1)e^{-(T_{12}+T_{21}-T_{22})s}}{K_{22}(\tau_{12}s+1)(\tau_{21}s+1)} \tag{5}$$

At the sight of transfer function (5) it is not clear if it can be accurately approximated by a FOPDT or a SOPDT model. In fact transfer function (5) may even show inverse response behavior that cannot be captured by a minimum phase model.

The first objective of the article is to characterize the feasibility of approximating RETF by means of FOPDT/SOPDT systems and deciding when there is inverse response behavior, so other model structures should be used. The second objective of the article is to provide approximate models in the general case and in some particular cases that yield to simplifications.

The model reduction of the RETF is performed by representing the RETF (5) in dimensionless form (Balaguer et al. (2009)). The equation (5) is represented in dimensionless form as

$$\bar{G}_{11}^{re} = \frac{e^{-\bar{T}_{11}\bar{s}}}{(\bar{s}+1)} - \bar{K}\frac{(\bar{\tau}_{22}\bar{s}+1)e^{-\bar{T}_{\Sigma}\bar{s}}}{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)} \tag{6}$$

with the dimensionless numbers $\bar{G}_{11}^{re} = G_{11}^{re}/K_{11}$, $\bar{s} = \tau_{11}s$, $\bar{K} = (K_{12}K_{21})/(K_{11}K_{22})$, $\bar{\tau}_{12} = \tau_{12}/\tau_{11}$, $\bar{\tau}_{21} = \tau_{21}/\tau_{11}$, $\bar{\tau}_{22} = \tau_{22}/\tau_{11}$, $\bar{T}_{11} = T_{11}/\tau_{11}$, and $\bar{T}_{\Sigma} = (T_{12} + T_{21} - T_{22})/\tau_{11}$.

The benefit of dimensionless representation is twofold. On the one hand we reduce the number of parameters dependence from 8 dimensional parameters to 6 dimensionless parameters. Secondly, and more important, the dimensionless parameters enable the interpretation in terms of limiting values, what facilitates the analysis of particular cases.

In the rest of the article we consider the following assumptions:

Assumption 1. All the previous dimensionless parameters are positive. The implication is stability of FOPDT transfer functions and that negative gain signs appear in pairs.

Assumption 2. For causality considerations, $\bar{T}_{\Sigma} \geq 0$, that is $T_{12} + T_{21} - T_{22} \geq 0$.

Assumption 3. By proper selection of input output pairs, we have that $\bar{K} < 1$. For instance consider pairing following the relative gain array (RGA) procedure.

3. INVERSE RESPONSE ANALYSIS

Given the dimensionless reduced effective transfer function (DRETF) (6), the first point of study is to characterize when (6) shows an inverse response behavior. In this case a non minimum phase SOPDT model might be used. On the contrary, if the DRETF does not show inverse response, then a FOPDT or a minimum phase SOPDT model may be more appropriate to capture the fundamental dynamics of DRETF. In what follows we provide conditions for inverse response behavior, first in the general case and, secondly, in some particular cases.

3.1 General case

Assume without loss of generality that $T_\Sigma > T_{11}$. On the contrary, if $T_\Sigma < T_{11}$, an analogue analysis may be performed. The case $T_\Sigma = T_{11}$ is analyzed in the next section as a particular case.

In case that $T_\Sigma > T_{11}$, the DRETF (6) may be rewritten as

$$\bar{G}_{11}^{re} = \left(\frac{1}{(\bar{s}+1)} - \bar{K}\frac{(\bar{\tau}_{22}\bar{s}+1)e^{-\bar{T}_x \bar{s}}}{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)} \right) e^{-\bar{T}_{11}\bar{s}} \quad (7)$$

where $\bar{T}_x := \bar{T}_\Sigma - \bar{T}_{11} > 0$.

Transfer function (7) shows inverse response if the difference of transfer functions between parentheses has zeros on the right half plane (RHP), that is, positive zeros (Skogestad and Postlethwaite (1996)). Note that the time delay $\bar{T}_{11}$ has no influence on the system inverse response.

In order to characterize the inverse response property of transfer function (7), we approximate the time delay $\bar{T}_x$ by a first order Padé approximation, yielding

$$\bar{G}_{11}^{re} = \left(\frac{1}{(\bar{s}+1)} - \bar{K}\frac{(\bar{\tau}_{22}\bar{s}+1)(\frac{-\bar{T}_x}{2}\bar{s}+1)}{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)(\frac{\bar{T}_x}{2}\bar{s}+1)} \right) e^{-\bar{T}_{11}\bar{s}} \quad (8)$$

which can be rewritten as

$$\bar{G}_{11}^{re} = \frac{a\bar{s}^3 + b\bar{s}^2 + c\bar{s} + d}{(\bar{s}+1)(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)(\frac{\bar{T}_x}{2}\bar{s}+1)} e^{-\bar{T}_{11}\bar{s}} \quad (9)$$

where the third order polynomial of the numerator $P(\bar{s}) := a\bar{s}^3 + b\bar{s}^2 + c\bar{s} + d$ has parameters defined as

$$\begin{aligned}
a &= (\bar{\tau}_{12}\bar{\tau}_{21} + \bar{K}\bar{\tau}_{22})\frac{\bar{T}_x}{2} \\
b &= \bar{\tau}_{12}\bar{\tau}_{21} + \frac{\bar{T}_x}{2}(\bar{\tau}_{12} + \bar{\tau}_{21}) - \bar{K}(\bar{\tau}_{22} - (1+\bar{\tau}_{22})\frac{\bar{T}_x}{2}) \\
c &= \bar{\tau}_{12} + \bar{\tau}_{21} + \frac{\bar{T}_x}{2} - \bar{K}(\bar{\tau}_{22} - \frac{\bar{T}_x}{2} + 1) \\
d &= 1 - \bar{K}
\end{aligned}$$

Moreover, we define the discriminant Δ of $P(\bar{s})$ as $\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$ (Weisstein (2003)).

Theorem 1. Assume that $P(\bar{s})$ has only real valued zeros (i.e. $\Delta \geq 0$). Transfer function (9) is minimum phase (i.e. all zeros are left half plane (LHP) zeros) if and only if

$$\bar{\tau}_{22} < \min\{ \frac{\bar{\tau}_{12}\bar{\tau}_{21} + \frac{\bar{T}_x}{2} + \bar{K}\frac{\bar{T}_x}{2}}{\bar{K}(1 - \frac{\bar{T}_x}{2})}, \frac{\bar{\tau}_{12} + \bar{\tau}_{21} + \frac{\bar{T}_x}{2}(1+\bar{K})}{\bar{K}} - 1 \} \quad (10)$$

Proof: By Descartes' rule of signs we know that i) the maximum number of positive zeros is equal to the changes of sing of $P(\bar{s})$, and ii) the minimum number of negative zeros is equal to the changes of sign of $P(-\bar{s})$.

Note that term a is always greater than zero because we are adding positive values. Moreover, d is also greater than zero because, by proper pairing, $\bar{K} < 1$. As a result, the number of sign changes is determined by parameters b and c. If $b > 0$ and $c > 0$ then the number of sign changes of $P(\bar{s})$ is zero, thus there are no positive zeros (i.e. no inverse response). In fact, in this case, the number of sign changes of $P(-\bar{s})$ is equal to three, the number of negatives zeros.

If any of the parameters b or c, or both, are negative, then the number os sign changes of $P(\bar{s})$ is two, whereas the number of sign changes of $P(-\bar{s})$ is one. Two possibilities are then open: there are two positive zeros and one negative zero; or there are two complex conjugate zeros and one negative zero. The existence of complex zeros is discarded by assumption $\Delta \geq 0$. Therefore, there are two positive real zeros and the system is non-minimum phase.

Finally, $b > 0$ is equivalent to

$$\bar{\tau}_{22} < \frac{\bar{\tau}_{12}\bar{\tau}_{21} + \frac{\bar{T}_x}{2} + \bar{K}\frac{\bar{T}_x}{2}}{\bar{K}(1 - \frac{\bar{T}_x}{2})} \quad (11)$$

and $c > 0$ is equivalent to

$$\bar{\tau}_{22} < \frac{\bar{\tau}_{12} + \bar{\tau}_{21} + \frac{\bar{T}_x}{2}(1+\bar{K})}{\bar{K}} - 1 \quad (12)$$

As $b > 0$ and $c > 0$ must be fulfilled at the same time, the result follows.

□

Corollary 1. Under the assumption $\Delta \geq 0$, a necessary condition for minimum phase is that

$$\frac{\bar{K} - \bar{\tau}_{12} - \bar{\tau}_{21}}{1 + \bar{K}} < \frac{\bar{T}_x}{2} < 1$$

3.2 Time delay factorization

Assuming that $T_\Sigma = T_{11}$, both delays in (6) may be factored, then the RETF may be written as

$$\bar{G}_{11}^{re} = \left(\frac{1}{(\bar{s}+1)} - \bar{K}\frac{(\bar{\tau}_{22}\bar{s}+1)}{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)} \right) e^{-\bar{T}_{11}\bar{s}} \quad (13)$$

In this case, the time delay has no influence on the inverse behavior of the DRETF and the inverse response behavior of transfer function (13) may be analyzed equivalently by transfer function

$$\bar{G}_{11}^{re} = \frac{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1) - \bar{K}(\bar{\tau}_{22}\bar{s}+1)(\bar{s}+1)}{(\bar{s}+1)(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)} e^{-\bar{T}_{11}\bar{s}} \quad (14)$$

The inverse response behavior occurs when transfer function (14) has, at least, one right half plane zero (RHP). The zeros of the numerator polynomial are given by

$$a\bar{s}^2 + b\bar{s} + c = 0 \tag{15}$$

with $a = \bar{\tau}_{12}\bar{\tau}_{21} - \bar{K}\bar{\tau}_{22}$, $b = \bar{\tau}_{12} + \bar{\tau}_{21} - \bar{K}(\bar{\tau}_{22} + 1)$, and $c = (1 - \bar{K})$.

The zeros of polynomial (15) are given by $\bar{s} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. For real zeros, by imposing the condition $-b + \sqrt{b^2 - 4ac} < 0$, we assure that the zeros are LHP zeros and no inverse response is possible. By arranging and squaring previous condition we have that $b^2 - 4ac < b^2$, yielding the condition $ac > 0$. This is written in terms of dimensionless numbers as

$$(\bar{\tau}_{12}\bar{\tau}_{21} - \bar{K}\bar{\tau}_{22})(1 - \bar{K}) > 0 \tag{16}$$

which is a necessary and sufficient condition to discard non minimum phase behavior in (13). As by assumption $\bar{K} < 1$, condition (16) is simplified to

$$\bar{K} < \frac{\bar{\tau}_{12}\bar{\tau}_{21}}{\bar{\tau}_{22}} \tag{17}$$

In case that poles are complex conjugates, the condition for LHP zeros is that $b > 0$, that is

$$\bar{K} < \frac{\bar{\tau}_{12} + \bar{\tau}_{21}}{\bar{\tau}_{22} + 1} \tag{18}$$

3.3 Pole-zero cancelation

The case of pole-zero cancelation occurs when $\bar{\tau}_{12} = \bar{\tau}_{22}$ (or $\bar{\tau}_{21} = \bar{\tau}_{22}$) and yields to further simplifications. In this case transfer function (7) may be reduced to

$$\bar{G}_{11}^{re} = \left(\frac{1}{\bar{s}+1} - \bar{K}\frac{(\frac{-\bar{T}_x}{2}\bar{s}+1)}{(\bar{\tau}_{21}\bar{s}+1)((\frac{\bar{T}_x}{2}\bar{s}+1))}\right)e^{-\bar{T}_{11}} \tag{19}$$

The inverse response behavior occurs when transfer function (19) has a RHP. The zeros of the numerator polynomial are given by

$$a\bar{s}^2 + \bar{s} + c = 0 \tag{20}$$

with $a = (\bar{\tau}_{21} + \bar{K})\frac{\bar{T}_x}{2}$, $b = \bar{\tau}_{21} + \frac{\bar{T}_x}{2} - \bar{K}(1 + \frac{\bar{T}_x}{2})$, and $c = (1 - \bar{K})$.

By an analogous procedure as the one performed in the previous section, we arrive at the condition for minimum phase for real zeros, given by

$$(\bar{\tau}_{21} + \bar{K})\frac{\bar{T}_x}{2}(1 - \bar{K}) > 0 \tag{21}$$

which by assumptions 1 and 3, we have that $0 < \bar{K} < 1$, and the non-minimum phase behavior is discarded for any RETF.

For complex zeros the condition to discard inverse response is that $b > 0$, that yields

$$\bar{K} < \frac{\bar{\tau}_{21} + \frac{\bar{T}_x}{2}}{\frac{\bar{T}_x}{2} + 1} \tag{22}$$

3.4 Time delay factorization and pole-zero cancelation

The case of time delay factorization and pole-zero cancelation yields to further simplifications. In this case transfer function (7) may be reduced to

$$\bar{G}_{11}^{re} = \left(\frac{1}{\bar{s}+1} - \bar{K}\frac{1}{\bar{\tau}_{21}\bar{s}+1}\right) \tag{23}$$

The difference of two first order systems yields an inverse response if $\frac{1}{\bar{\tau}_{21}} > \frac{1}{\bar{K}} > 1$ (Linoya and Altpeter (1962)). As by assumptions 1 and 3, we have that $0 < \bar{K} < 1$, it is always true that $\frac{1}{\bar{K}} > 1$. Thus, there is inverse response behavior if and only if

$$\bar{K} > \bar{\tau}_{21} \tag{24}$$

3.5 Summary of inverse response analysis

In table 1 we present a summary of inverse response conditions for each case considered. Recall that the discriminant for the general case is defined as $\Delta = 18abcd - 4b^3d + b^2c^2 - 4ac^3 - 27a^2d^2$, whereas the discriminant for the rest of cases is defined as $\Delta = b^2 - 4ac$.

ASSUMPTIONS	NO INVERSE RESPONSE IF...
$\Delta \geq 0$	$\bar{\tau}_{22} < \min\{\frac{\bar{\tau}_{12}\bar{\tau}_{21} + \frac{T_x}{2} + \bar{K}\frac{T_x}{2}}{\bar{K}(1-\frac{\bar{T}_x}{2})}, \frac{\bar{\tau}_{12}+\bar{\tau}_{21}+\frac{\bar{T}_x}{2}(1+\bar{K})}{\bar{K}} - 1\}$
$\bar{T}_{11} = \bar{T}_{\Sigma}$, $\Delta \geq 0$ $\bar{T}_{11} = \bar{T}_{\Sigma}$, $\Delta < 0$	$\bar{K} < \frac{\bar{\tau}_{12}\bar{\tau}_{21}}{\bar{\tau}_{22}}$ $\bar{K} < \frac{\bar{\tau}_{12}+\bar{\tau}_{21}}{\bar{\tau}_{22}+1}$
$\bar{\tau_{12}} = \bar{\tau_{22}}$, $\Delta \geq 0$ (or $\bar{\tau_{21}} = \bar{\tau_{22}}$), $\Delta < 0$	No inverse response $\bar{K} < \frac{\bar{\tau}_{21}+\frac{\bar{T}_x}{2}}{\frac{\bar{T}_x}{2}+1}$
$\bar{T}_{11} = \bar{T}_{\Sigma}$ and $\bar{\tau_{12}} = \bar{\tau_{22}}$ (or $\bar{\tau_{21}} = \bar{\tau_{22}}$)	$\bar{K} < \bar{\tau}_{21}$

Table 1. Summary of no inverse response conditions for each case considered.

4. REDUCED ORDER MODEL: GENERAL CASE

One general approach to obtain reduced order models of the RETF is to perform Maclaurin approximations, as presented in (Vu and Lee (2010)). The idea is to expand the RETF in its Maclaurin series as follows

$$\bar{G}_{11}^{re} = a_{11} + b_{11}\bar{s} + c_{11}\bar{s}^2 + O(\bar{s}^3) \tag{25}$$

where the polynomial coefficients are

$$a_{11} = \bar{G}_{11}^{re}(0)$$
$$b_{11} = \frac{d\bar{G}_{11}^{re}(\bar{s})}{d\bar{s}}\Big|_{\bar{s}=0}$$
$$c_{11} = \frac{1}{2}\frac{d^2\bar{G}_{11}^{re}(\bar{s})}{d\bar{s}^2}\Big|_{\bar{s}=0}$$

Next, we obtain the analytic polynomial coefficients of the Maclaurin series for the dimensionless FOPDT model

$$\bar{G}(s) = \frac{K}{K_{11}} \frac{e^{-\bar{T}\bar{s}}}{\bar{\tau}\bar{s}+1} \tag{26}$$

that is given by

$$\bar{G}^{re} = \frac{K}{K_{11}} - \frac{K}{K_{11}}(\bar{\tau}+\bar{T})\bar{s} + \frac{K}{K_{11}}\left(\frac{1}{2}\bar{T}^2 + (\bar{T}+\bar{\tau})\bar{\tau}\right)\bar{s}^2 + O(\bar{s}^3) \tag{27}$$

For SOPDT processes, the same procedure may be repeated. However, the SOPDT parameters are not easily obtainable as before (Vu and Lee (2010)). Next section shows how SOPDT models may be approximated without requiring the Maclaurin approximation.

5. REDUCED ORDER MODEL: PARTICULAR CASES

In this section, at the sight of dimensionless numbers values, we may obtain reduced order models in a straightforward manner. In fact, if some dimensionless numbers are small or there is certain relationship among them, simpler models may be easily obtained.

5.1 Dimensionless gain

In case that $\bar{K} << 1$, an approximation to the dimensionless reduced effective transfer function is simply

$$\frac{1}{\bar{s}+1} e^{-\bar{T}_{11}\bar{s}} \tag{28}$$

As $\bar{K} = (K_{12}K_{21})/(K_{11}K_{22})$, the condition $\bar{K} << 1$ implies that the cross gains are small and we have low interaction. As a result we may consider the problem as no coupled.

5.2 Slow Model

Previous model may be improved in the case of two time scales. We have two time scales if $\bar{\tau}_{12} << 1$ and $\bar{\tau}_{21} << 1$. In this case we may disregard fast model dynamics. The DRETF (5) is given by the slow model with FOPDT transfer function given by

$$\frac{(1-\bar{K})}{\bar{s}+1} e^{-T_{11}\bar{s}} \tag{29}$$

However, note that when $\bar{\tau}_{12} << 1$ and $\bar{\tau}_{21} << 1$, it might be possible to perform time scale decoupling (Ogunnaike and Ray (1994)), and the original loop pairing might not be the most adequate, despite RGA pairing.

5.3 Time delay factorization and pole-zero cancelation

Recall that under the assumption $T_\Sigma = T_{11}$, equivalently $T_{11} + T_{22} = T_{12} + T_{21}$, the delay can be factored so the dimensionless reduced effective transfer function is given by (13). In this case the exact dimensionless reduced effective transfer function is given by the following transfer function

$$\bar{G}_{11}^{re} = \frac{(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1) - \bar{K}(\bar{\tau}_{22}\bar{s}+1)(\bar{s}+1)}{(\bar{s}+1)(\bar{\tau}_{12}\bar{s}+1)(\bar{\tau}_{21}\bar{s}+1)} e^{-\bar{T}_{11}\bar{s}} \tag{30}$$

that is too complex to be used for control design with commonly used PID tuning rules.

The case of pole-zero cancelation by $\bar{\tau}_{12} = \bar{\tau}_{22}$ (or $\bar{\tau}_{21} = \bar{\tau}_{22}$) yields to further simplifications. In this case transfer function (30) may be reduced to

$$\bar{G}_{11}^{re} = \frac{(\bar{\tau}_{12}-\bar{K})\bar{s} + (1-\bar{K})}{(\bar{s}+1)(\bar{\tau}_{12}\bar{s}+1)} e^{-\bar{T}_{11}\bar{s}} \tag{31}$$

Transfer function (31) is exactly modeled by second order transfer function. We have the following cases:

- $(\bar{\tau}_{12} - \bar{K}) < 0$. In this case a non-minimum phase SOPDT model is able to exactly represent the dimensionless reduced effective transfer function. There is inverse response.
- $(\bar{\tau}_{12} - \bar{K}) > 0$. In this case a minimum phase with LHP zero SOPDT model is able to exactly represent the dimensionless reduced effective transfer function.
- $(\bar{\tau}_{12} - \bar{K}) = 0$. In this case a SOPDT model is able to exactly represent the dimensionless reduced effective transfer function.

5.4 Summary of model order reduction

In table 2 we present a summary of reduced order models.

ASSUMPTIONS	MODEL
$\bar{K} << 1$	$\frac{1}{\bar{s}+1}e^{-T_{11}\bar{s}}$
$\bar{\tau_{12}} << 1$ and $\bar{\tau_{21}} << 1$	$\frac{(1-\bar{K})}{\bar{s}+1}e^{-\bar{T}_{11}\bar{s}}$
$\bar{T_{11}} = \bar{T_\Sigma}$ and $\bar{\tau_{12}} = \bar{\tau_{22}}$ (or $\bar{\tau_{21}} = \bar{\tau_{22}}$)	$\frac{(\bar{\tau}_{12}-\bar{K})\bar{s}+(1-\bar{K})}{(\bar{s}+1)(\bar{\tau}_{12}\bar{s}+1)}e^{-\bar{T}_{11}\bar{s}}$
$\bar{T_{11}} = \bar{T_\Sigma}$ and $\bar{\tau_{12}} = \bar{\tau_{22}}$ (or $\bar{\tau_{21}} = \bar{\tau_{22}}$) and $\bar{\tau}_{12} = 1$	$\frac{(1-\bar{K})}{\bar{s}+1}e^{-\bar{T}_{11}\bar{s}}$

Table 2. Summary of model order reduction for each case considered.

6. APPLICATION EXAMPLES

6.1 Vinante and Luyben (VL) Column

Consider the Vinante and Luyben distillation column (Vinante and Luyben (1972))

$$G(s) = \begin{bmatrix} \dfrac{-2.2e^{-s}}{7s+1} & \dfrac{1.3e^{-0.3s}}{7s+1} \\ \dfrac{-2.8e^{-1.8s}}{9.5s+1} & \dfrac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix} \tag{32}$$

The RETF dimensionless parameters are given in table 3.

The DRETF has no inverse response behavior because the discriminant is $\Delta = 0.07 > 0$, and applying Theorem 1 (condition (10)), we have

$$1.31 < \min\{1.46, 5.31\} \tag{33}$$

$\bar{K}$	$\bar{\tau_{12}}$	$\bar{\tau_{21}}$	$\bar{\tau_{22}}$	$\bar{T}_{11}$	$\bar{T}_{\Sigma}$
0.38	1	1.35	1.31	0.14	0.25

Table 3. Vinante and Luyben DRETF Dimensionless Numbers

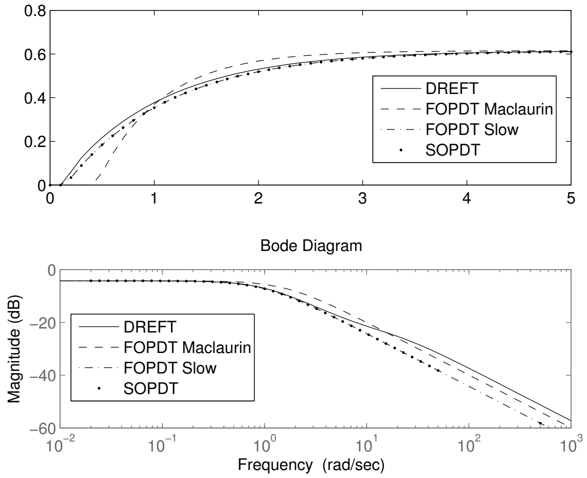

Fig. 2. Step response and Bode magnitude plot of reduced models of Vinante and Luyben DRETF.

which discards inverse response behavior.

Note that $\bar{\tau_{21}} \approx \bar{\tau_{22}}$, thus by the pole zero cancelation case, we can also discard non-minimum phase behavior using condition (22), that yields

$$0.38 < 1.33 \tag{34}$$

which is true and discards inverse response behavior of DRETF.

As inverse response behavior is discarded, two FOPDT models and one minimum phase SOPDT model are proposed. First, the 'FOPDT Maclaurin' approximation that requires no assumptions. Secondly, the 'FOPDT Slow' model obtained assuming that $\bar{K} << 1$, and given by (29). Thirdly, the 'SOPDT' model derived by assuming i) $\bar{\tau_{21}} = \bar{\tau_{22}}$, and ii) $\bar{T}_{11} = \bar{T}_{\Sigma}$. At the sight of dimensionless numbers in table 3, the first assumption is fully valid but the second one is just an approximation. In this case the model is given by SOPDT transfer function (31).

6.2 *Polymerization reactor*

Consider now the polymerization reactor (Garrido et al. (2010))

$$G(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.80e^{-0.4s}}{1.801s+1} \end{bmatrix} \tag{35}$$

The RETF dimensionless parameters are given in table 4. Note that $\bar{\tau_{12}} \approx \bar{\tau_{22}}$. Furthermore, we have that $\bar{T}_{11} = \bar{T}_{\Sigma}$, so we consider the time delay factorization and pole-zero cancelation case. Note also that in this case $\bar{K} < 0$, hence the results must be particularized to this case by realizing that no inverse response is possible because two first order transfer functions are being added in (23).

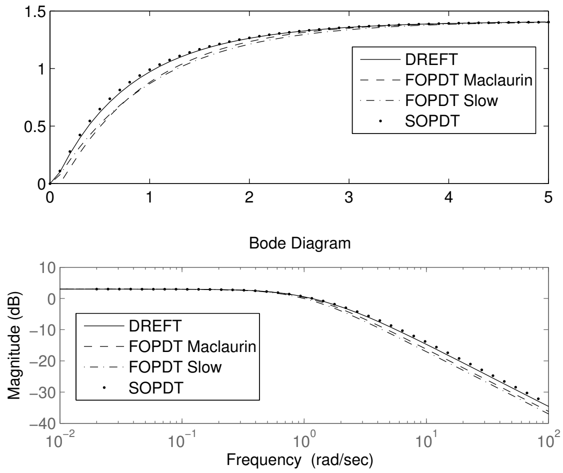

Fig. 3. Step response and Bode magnitude plot of reduced order models of Polymerization Reactor DRETF.

$\bar{K}$	$\bar{\tau_{12}}$	$\bar{\tau_{21}}$	$\bar{\tau_{22}}$	$\bar{T}_{11}$	$\bar{T}_{\Sigma}$
-0.41	0.39	0.47	0.39	0.0437	0.0437

Table 4. Polymerization Reactor DRETF Dimensionless Numbers

The same three models as previous examples are used and the results can be seen in figure 3. Note that in this case, the SOPDT model is equal to the DRETF.

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