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Tuning Method of PI Controller with Desired Damping Coefficient for a First-order Lag Plus Deadtime System

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Abstract: This paper presents an entirely new tuning method of PI controller with a desired damping coefficient. We consider a plant model described by a first-order lag plus deadtime system in the process control. The ultimate sensitivity method presented by Ziegler and Nichols has been still widely used, but it has a disadvantage that gives an oscillatory response. In recent years a less oscillatory response has been judged to be more appropriate for process controls. In tuning PI controller it is often convenient to determine the damping coefficient to obtain the desired control performance. The deadtime element can be approximated by the Padé equation to determine the damping coefficient for the equivalent second-order lag system. In this paper the integral time is normally chosen equal to the time constant of a plant to degenerate the order of the closed-loop transfer function so that pole-zero is cancelled. As a result, the relations among the gain constant, the time constant, the deadtime of the plant and the proportional gain of the controller are clarified when the desired damping coefficient is provided.

Keywords: process control, PID control, damping coefficient.

1. INTRODUCTION

In most process controls, tuning PID controllers can be achieved on the assumption that a plant model is approximated by a first-order lag plus deadtime system. The ultimate sensitivity method developed experimentally by Ziegler and Nichols in 1942 has been by far the most common controller tuning method. However, since the decay ratio, which is called quarter damping ratio (i.e. the damping coefficient $\zeta = 0.22$), has been chosen traditionally, this leads to poor attenuation, so that the closed-loop response can be rapidly oscillatory due to huntings.

In typical specifications on a control system it is desirable to have a critically damped response with no overshoot. To do this, we need a simple, easy-to-use, intuitive tuning method that gives moderate damping effect. In this study a pair of conjugate complex poles to form an oscillatory mode for the closed-loop transfer function are primary concern for PI control used when the deadtime element of the plant can be approximated by the Padé equation. The PI controller tuning method with a desired damping coefficient for the closed-loop system is developed.

2. CONTROLLED PLANT AND OVERALL CONTROL SYSTEM

Consider a first-order lag plus deadtime system with the transfer function:

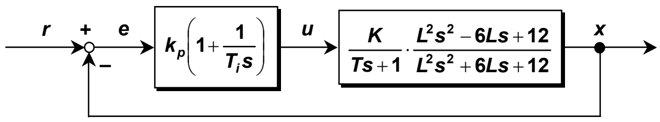


Fig. 1 PI control system

$$G_P(s) = \frac{K}{Ts+1}\cdot e^{-Ls}, \tag{1}$$

where K is the gain constant, T is the time constant and L is the deadtime of the controlled plant. Since the pure deadtime element cannot be described by a finite-order model, it can be practically approximated by the second-order Padé equation as:

$$G_P(s) = \frac{K}{Ts+1}\cdot\frac{L^2s^2-6Ls+12}{L^2s^2+6Ls+12} \ . \qquad (1)'$$

Consider the simple feedback loop, shown in Fig. 1, and composed of a plant and a PID controller. In Fig. 1, r is the setpoint, e is the control error, u is the control input, and x is the control output, k_p is the proportional gain and T_i is the integral time. For such a control system the transfer function from the setpoint r to the control output x is given by

$$G_{cl}(s) = \frac{\frac{k_p}{T_i}\left(\frac{T_i s+1}{s}\right)\frac{K}{Ts+1}\cdot\frac{L^2s^2-6Ls+12}{L^2s^2+6Ls+12}}{1+\frac{k_p}{T_i}\left(\frac{T_i s+1}{s}\right)\frac{K}{Ts+1}\cdot\frac{L^2s^2-6Ls+12}{L^2s^2+6Ls+12}}. \tag{2}$$

3. CHARACTERISTICS OF POLES FOR PI CONTROL SYSTEM

Expanding Eq. (2), $G_{cl}(s)$ can be expressed as a ratio of polynomials, and the orders of the numerator polynomial and the denominator one are 3 and 4, respectively. In pole assignment design it is attempted to assign a pair of conjugate complex poles of the closed-loop system equal to the dominant poles, and the PI parameters should be chosen so that the damping coefficient of the dominant poles leads to better response. In this current analysis it is desirable that the order of the denominator polynomial should be 3 to obtain satisfactory control performance.

3.1 Close-loop transfer function with integral time fixed

Considering the closed-loop transfer function given by Eq. (2) and letting the integral time T_i be the time constant T, the order of the denominator polynomial can be easily reduced through the pole-zero cancellation. As a result the closed-loop transfer function $G_{cl}'(s)$ is given by

$$G_{cl}'(s)=\frac{\frac{k_pK}{Ts}\cdot\frac{L^2s^2-6Ls+12}{L^2s^2+6Ls+12}}{1+\frac{k_pK}{Ts}\cdot\frac{L^2s^2-6Ls+12}{L^2s^2+6Ls+12}}$$

$$G_{cl}'(s)=\frac{k_pK/A\left\{(Ls)^2-6(Ls)+12\right\}}{(Ls)^3+\left\{6+k_pK/A\right\}(Ls)^2+\left\{12-6k_pK/A\right\}(Ls)+12k_pK/A} \tag{3}$$

where $A = T/L$ and $T_i = T$.

In this case we find that the closed-loop zeros of Eq. (3) are practically kept constant, and cannot be adjustable by the controller. On the contrary, it is useful that the closed-loop poles should be assigned in accordance with the required design criteria. Eq. (3) indicates major great advantage on the PI controller that the proportional gain k_p becomes the only one adjustable parameter because the integral time T_i is kept fixed. Therefore, the pole-assignment for the PI controller gives the simple problem solving the four parameters, namely k_p, K, T, and L.

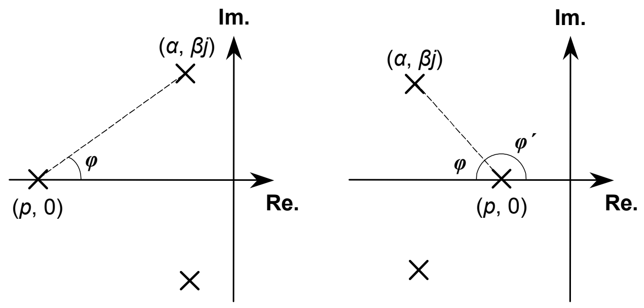
3.2 Relations of poles to damping coefficient

To get some insight into the relations between poles and damping coefficient, let us investigate the following simplified characteristic equation given in the denominator polynomial of Eq. (3):

$$L^3s^3+aL^2s^2+bLs+c=0 \tag{4}$$

where $a = 6 + k_pK/A$, $b = 12 - 6\,k_pK/A$, and $c = 12\,k_pK/A$.

The closed-loop poles are the roots of Eq. (4) and the pole-zero configuration may be varied significantly. Many simple feedback loops, however, will be given by a real pole s_1 and complex pair of poles s_2 and s_3 in case that $P^2 \geq 0$ (discussed later), which can be computed by



(a) $0^\circ \leq \varphi < 90^\circ$ (b) $90^\circ < \varphi \leq 180^\circ$

Fig. 2 Angle φ between a line connecting real pole and oscillatory poles, and real axis

$$\left.\begin{aligned}s_1&=\frac{1}{L}\left\{Q+R-\frac{a}{3}\right\}=p\\ s_{2,3}&=\frac{1}{L}\left\{\left(-\frac{Q+R}{2}-\frac{a}{3}\right)\pm j\left(\frac{\sqrt{3}(Q-R)}{2}\right)\right\}=\alpha\pm j\beta\end{aligned}\right\} \tag{5}$$

In this process the following interim parameters have been introduced:

$$M=\frac{3b-a^2}{9},N=\frac{2a^3-9ab+27c}{54},P=\sqrt{M^3+N^2}$$
$$Q=\sqrt[3]{P-N},R=\sqrt[3]{-(P+N)}.$$

According to Eq. (5) both real and oscillatory roots can be given by the function of $1/L$. Writing the real and imaginary parts of oscillatory roots as α and β we get the damping coefficient ζ through straightforward calculations as

$$\zeta=\frac{\alpha}{\sqrt{\alpha^2+\beta^2}}=\frac{\left(\frac{(Q+R)}{2}+\frac{a}{3}\right)}{\sqrt{\left(\frac{Q+R}{2}+\frac{a}{3}\right)^2+\left(\frac{\sqrt{3}(Q-R)}{2}\right)^2}}\ . \tag{6}$$

Notice that ζ can be determined by a, b, and c only, regardless of L. Since Q and R are the functions of a, b, and c, all of these parameters are the functions of k_pK/A, so that ζ can be easily obtained as the function of k_pK/A.

3.3 Effects of the real pole on the oscillatory poles

The damping coefficient can be determined by a set of the real pole and the oscillatory poles. Thus, if the damping coefficient of the closed-loop response is assumed to be equal to that of oscillatory poles, the real pole may be neglected by assigning far from the oscillatory poles of concern.

To examine the effects of the real pole on the oscillatory poles, we define the angle φ between the real axis and the connecting line of real pole and the oscillatory poles, as shown in Fig. 2.

From Eq. (5), the angle φ is given by,

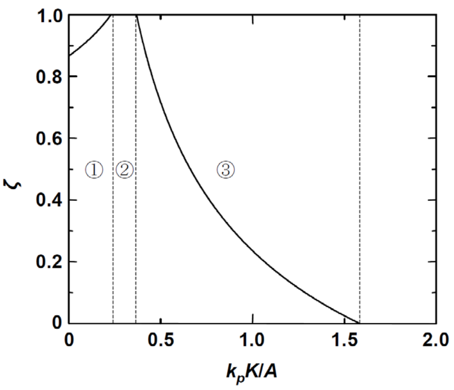


Fig. 3 Relation between ζ and k_pK/A

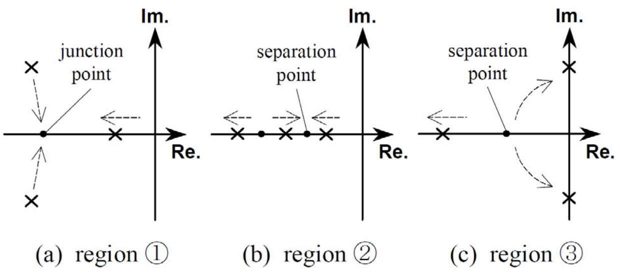


Fig. 4 Schematic diagram of pole assgignment

$$\varphi = \tan^{-1}\frac{\beta}{|p-\alpha|} = \tan^{-1}\left(-\frac{\sqrt{3}(Q-R)}{3(Q+R)}\right), \qquad (7)$$

where φ normally varies within the range from 0º to 90º (0º ≤ φ < 90º). In case that φ is gets over the range (90º < φ ≤ 180º), φ is given by,

$$\varphi' = 180° - \varphi\,. \qquad (8)$$

According to Eq. (7), (8) φ is the function of k_pK/A as well as ζ. In the previous paper 3), if φ is less than 50º (φ ≤ 50º) on third-order system consisted of real pole and oscillatory poles, it has been clarified that the effects of real pole can be neglected. Thus, when specifying ζ of oscillatory poles, it is necessary to examine if the condition for φ of less than 50º is satisfied. As a result, the value of φ affects the response (over damped or oscillation) of the control system.

4. PI PARAMETERS FOR DESIRED DAMPING COEFFICIENT

4.1 k_p for desired coefficient ζ

To derive k_p for the desired damping coefficient ζ, Eq. (6) can be varied with k_pK/A. The relation between ζ and k_pK/A is shown as in Fig. 3. It can be seen that there exist three regions as k_pK/A increases. The first region means the region where the oscillatory poles join the real axis at the junction point as k_pK/A increases. The Second region means the region where all three poles become real. The third region means the region where the oscillatory poles moves from the separation point to the stability limit at the imaginary axis. The behaviors for these three regions are depicted as shown in Fig. 4.

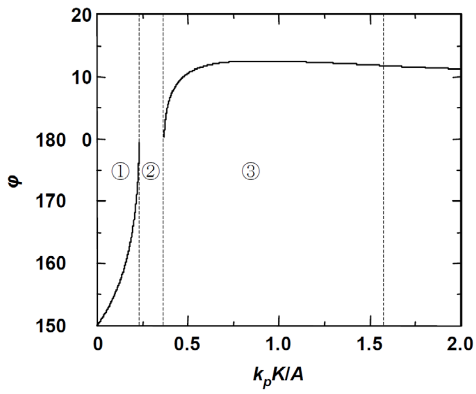


Fig. 5 Relation between φ and k_pK/A

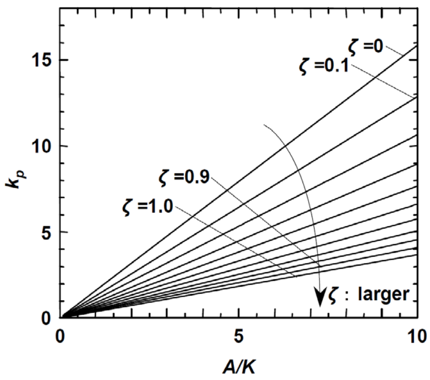


Fig. 6 k_p versus A/K in case that ζ is specified

Table 1 Relation between ζ and γ

ζ	γ	ζ	γ
1.0	0.368	0.5	0.661
0.9	0.407	0.4	0.765
0.8	0.454	0.3	0.896
0.7	0.510	0.2	1.06
0.6	0.578	0.1	1.29

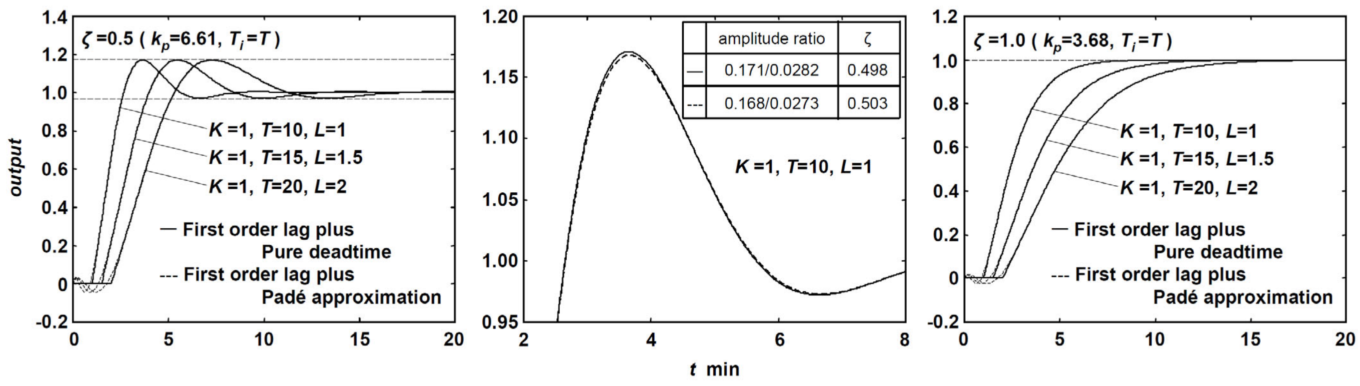
Next let us consider how Eq. (7) and (8) are varied with k_pK/A. The relation of φ to k_pK/A can be obtained as shown in Fig. 5. When looking at Fig. 5 corresponding to Fig. 3, φ is less than 50º in the third region and the damping coefficient ζ of oscillatory poles can be desired as the damping coefficient ζ of a step response in this region. Therefore, k_p for the desired damping coefficient ζ can be calculated by using the region ③ (0.368 ≤ k_pK/A < 1.58). In region ③ in Fig. 3, k_pK/A can be determined uniquely when ζ is desired. Thus, Eq. (9) can be obtained by replacing k_pK/A by γ

$$k_pK/A = \gamma(\zeta) \qquad (9)$$

where γ is the function of ζ. γ for desired ζ is shown in Table 1. From Eq. (10), k_p for the desired damping coefficient ζ can be given by

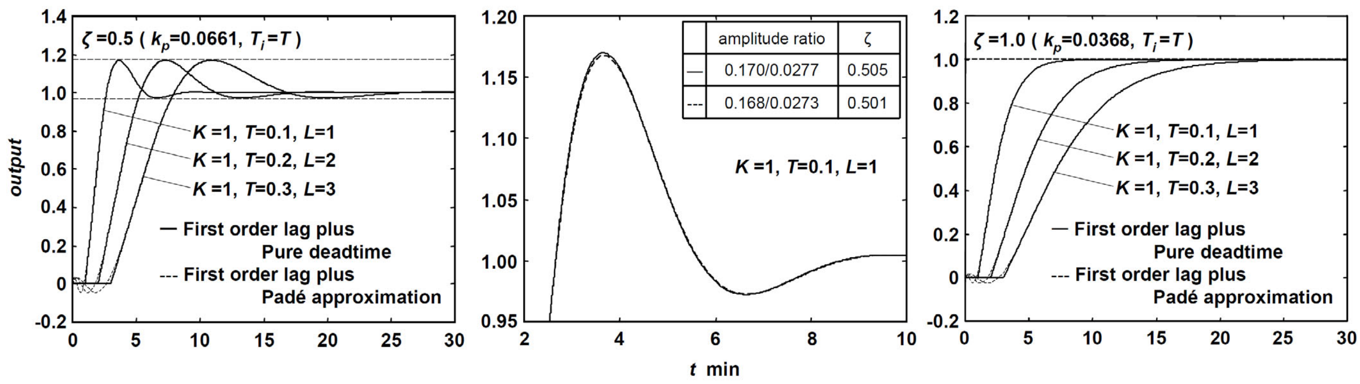
$$k_p = \gamma(\zeta)A/K \qquad (10)$$

From Eq. (10) and the results shown in Table 1, k_p for the desired damping coefficient ζ can be found to be a completely linear function of A/K, as shown in Fig. 6.



(a) ζ=0.5 (b) enlarged figure of Fig. 7(a) (c) ζ=1.0

Fig. 7 Step responses in case that ζ is specified ($T >> L$)



(a) ζ=0.5 (b) enlarged figure of Fig. 8(a) (c) ζ=1.0

Fig. 8 Step responses in case that ζ is specified ($T << L$)

4.2 Validation for the pure deadtime system

In section 4.1, we proposed the PI tuning method based on the damping coefficient ζ of oscillatory poles. So that it is response of ζ = 0.5 is required for this plant, the proportional gain k_p = 6.61 can be obtained from Fig. 6. Then the integral time T_i = 10 can be found as $T_i = T$. The step responses for the desired ζ = 0.5 and 1.0 are shown in Fig. 7(a), (c) respectively. Because the damping coefficient of step responses for the desired ζ = 1.0 cannot be obtained from the amplitude ratio, the damping coefficient is assumed to be ζ = 0.5.

An enlarged figure of the step responses desired as ζ = 0.5 is shown in Fig. 7(b). In this figure, the dashed line is the step responses for first order lag plus deadtime system approximated by the second-order Padé equation. In the calculation of the damping coefficient of the step responses from amplitude ratio, it is found that the step response corresponds approximately to the damping coefficient desired by oscillatory poles.

In this figure, the solid lines show the step responses for first-order lag plus deadtime system with pure deadtime, and it can be found that the step response (solid line) almost corresponds to the step response (dashed line). Thus, it can be concluded that this parameter method is also effective for pure deadtime.

As additional consideration, the plant which the deadtime is larger than the time constant, K = 1, A = 0.1 ($T << L$) is considered. For this plant, the step responses for the desired ζ = 0.5 and 1.0 are shown in Fig. 8(a), (c) respectively, and enlarged figure of Fig. 8 (a) is shown in Fig. 8(b). In case that $T << L$, the step response for the desired damping coefficient can be obtained successfully as well as $T >> L$.

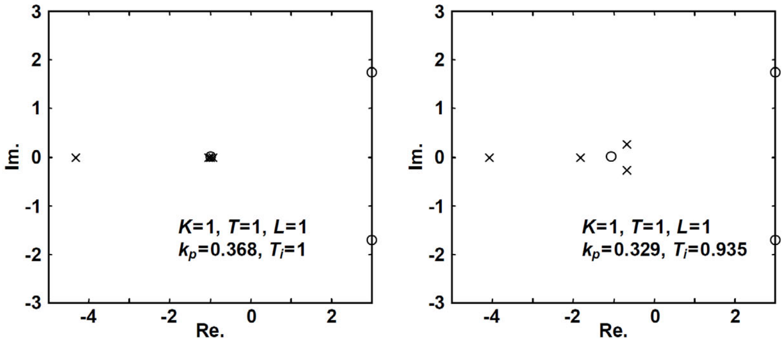
4.3 Comparison with PMM (Partial Model Matching)

Table 2 shows PI parameters obtained by the proposed method and PMM (Partial Model Matching method, proposed by Kitamori, 1985) for ζ =1.0.

As you can see Table 2, both PI parameters are about the same. This result is significantly interesting.

Since the zero of the controller compensates the pole of the plant in the proposed method, it can be considered that the pole-zero cancellation also occurs in the PMM. In the future, we will discuss about the other damping coefficients.

Fig. 9 shows pole-zero maps for the closed-loop system (the characteristics of the plant is A = 1.) using PI controller. In Fig. 9(a), the PI parameters are decided by using the proposed method, and the PI parameters are determined by using the PMM in Fig. 9(b). Fig. 10 shows the results of the step response for both control systems.



(a) Proposed method (b) Partial model matching method

Fig. 9 Pole-zero map (A = 1)

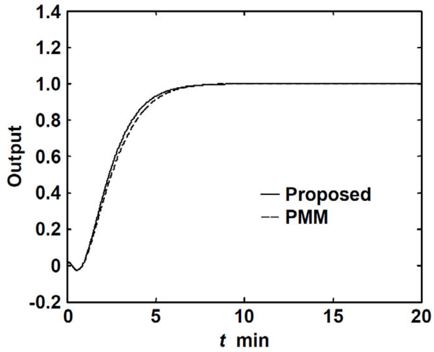
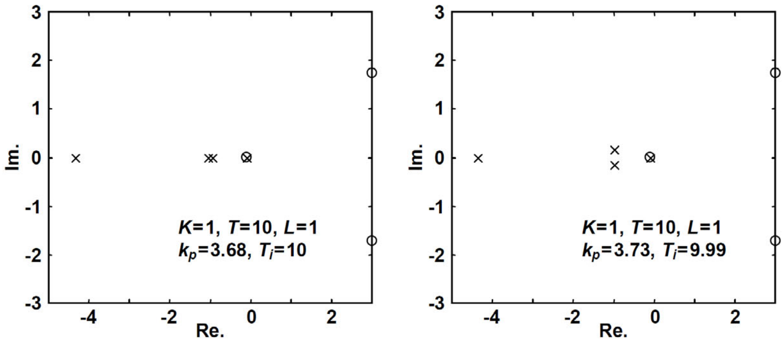


Fig. 10 Step responses (A = 1)



(a) Proposed method (b) Partial model matching method

Fig. 11 Pole-zero map (A = 10)

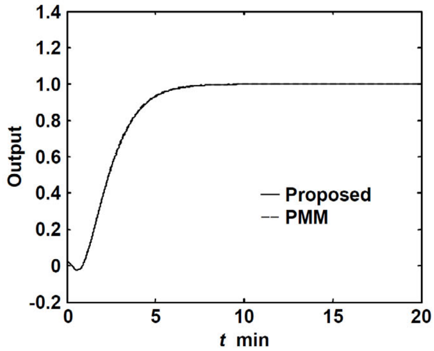


Fig. 12 Step responses (A = 10)

Table 2: Comparison of PI parameters

Plant			Proposed		PMM	
T	L	T/L	k_p	T_i	k_p	T_i
1	1	1	0.368	1	0.329	0.935
2	2	1	0.368	2	0.329	1.87
3	3	1	0.368	3	0.329	2.80
5	1	5	1.84	5	1.85	4.99
10	2	5	1.84	10	1.85	9.98
15	3	5	1.84	15	1.85	15.0
10	1	10	3.68	10	3.73	9.99
20	2	10	3.68	20	3.73	20.0
30	3	10	3.68	30	3.73	30.0

(K=1) (ζ=1.0)

It can be seen from Fig. 9 that the control system with the PMM cannot achieve pole-zero cancellation for the plant (A = 1) that the deadtime L is the same as the time constant T. However, in this situation, the non-oscillation response can be obtained. And, both responses are the almost same by comparing the response using the proposed method with that using the PMM.

In addition, Fig. 11 shows pole-zero maps for the closed-loop system (the characteristics of the plant is A = 10.) using PI controller. In Fig. 11(a), the PI parameters are decided by using the proposed method, and the PI parameters are determined by using the PMM in Fig. 11(b). Fig. 12 shows the results of the step response for both control systems. In this case, the both characteristics (the pole-zero map and the step response) are the about perfectly same. Moreover, although the both response between Fig. 10 and Fig. 12 are the same, the speed of the response depends on the value of the deadtime L.

5. CONCLUSIONS

We proposed PI controller tuning method for first-order lag plus deadtime system. On the assumption that the integral time T_i is chosen equal to the time constant T of a plant to degenerate the order of the closed-loop transfer function, we obtained the following conclusions.

1) The step response desired by ζ of oscillatory poles can be approximated the equivalent response of second-order system since real pole can be almost neglected.
2) A proportional gain k_p for the desired damping coefficient is an exactly linear function of A/K ($A = T/L$).
3) We demonstrated an applicability of the tuning method of PI controller for the given damping coefficient.

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