IFAC Conference on Advances in PID Control
PID'12
Brescia (Italy), March 28-30, 2012

Joint Confidence Region for the Tuning Parameters of the PID Controller

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Abstract: The aim of this paper is to estimate the joint confidence region for the tuning parameters of the PID controller. Considerations on the statistical independence and the linear relationship between the tuning parameters for a more realistic scenario are taken into account. In order to capture the data necessary for the simulation, an appropriate structure was implemented, consisting of a process-model-based controller. By using the data resulting from the simulations, it was also possible to find the probability density function for each tuning parameter, as well the region of joint confidence of such parameters, which indicates a contraction of the region when compared with the standard level of significance. What is also shown is how to restore the joint confidence region by means of the Principal Components Technique.

Keywords: PID Controller, Tuning Parameters, Joint Confidence Region.

1. INTRODUCTION

Even after decades of research into and development of more efficient tuning methods and refined types of instrumentation and despite the process being better understood, good tuning remains a challenging problem.

Classical methods, such as that of Cohen-Coon, as well as the Ziegler-Nichols tuning technique, have been essential requirements to establish controller settings.

Where there has been poor understanding of the process in which linearized models are normally used and the nature of the process is neither stationary nor weakly stationary, a “Self-Adaptive Control System” has been systematically used, and associated with several other strategies, according to Skogestad (2003) and Aström (1995). Some statistical tools have also been cited by Ruel (2004) to identify if tuning parameters are appropriate. However, the heuristic approach based on the experience of technical personnel, associated with the trial and error method, has proved to be the best way to tune control parameters, at least from the practical standpoint. In spite of all the efforts, it is well-known that all these methods still leave much to be desired.

It is well-known there is a certain amount of inherent or natural variability or still background noise in any production process, regardless of how well designed or carefully maintained it is. However, it should be emphasized that one of the major difficulties about a product or process having a flawless quality characteristic lies in the variability arising from assignable causes or special.

Therefore the question which this paper focuses on is related to the difficulty of finding accurate deterministic values for the control parameters, and thus to provide a better practical answer for this problem. This is strongly related to determining the joint confidence region. This article also considers two intrinsically correlated matters, which play an essential role in such investigations: the ineffectiveness of the current methods and the stochastic nature of the process.

2. DETERMINISTIC OPTIMUM AND ROBUSTNESS

The conceptual understanding of deterministic is that if all input information in the model is specified, the model generates only one value for every output. In deterministic optimization, besides the objective function, the constraints are defined in a deterministic way. However, due to the complexity of the process, the incorporation of uncertainties into the system has been postulated by Padulo et al. (2008), as intrinsically necessary for the purposes of robustness.

It seems appropriate at this point to reflect on robustness, by indicating the problems which can be addressed by such an approach. In essence, robust methods enable the optimization of the deterministic response of a system to be driven about a mean value, thus maximizing the robust performance while minimizing the sensitivity to random parameters. In other words, the robust objective is obtained by minimizing the variance and expectation (mean) simultaneously.

A classical approach to a deterministic optimization problem can be formulated by the following treatment:

$$\text{Min } f(x) \text{ such that } g_i(x) \le 0, \text{ for I} = 1,2,3,\ldots,r \quad \text{and} \quad x^L \le x \le x^U$$

where L and U denote lower and upper bounds and x is the vector of design variables and parameters.

As to robust design, according to the work Paiva et al. (2010) and Padulo et al. (2008), such a formulation can be rewritten as follows:

$$Min \ \ F(\mu_f(x), \sigma_f(x))$$

$$\text{subject to} \quad G_i((\mu_g(x), \sigma_g(x)) \le 0$$

$$P\ (x_k^L \le x_k \le x_k^U) \le P_{bounds}$$

μ and σ denote the mean and standard deviation respectively of the probability distribution, which, without loss of generality, can be considered as a Gaussian distribution. To ensure the robustness, the design variables can be considered deterministic or stochastic, while the parameters r also included in vector x are characterized by their probability distribution. P_{bound}, describing an area within the distribution of probability and representing a universe of all possible results, means the probability with which the mean of the design variables and parameters belong to the original range, $x^L \leq x \leq x^U$, which correspond to the confidence limits associated with the level of significance (α). Since the probability distributions by definition can vary from $-\infty$ to $+\infty$, and as it is impracticable to work with the open interval, it is necessary to close the distributions, and set an interval, taking into account the type I error, or a level of significance (α), induced in such an approach.

Given that the robust objective and constraints are functions of the mean and the variance and both depend on the probability distribution of the variables and parameters, the universe of all possible results for the optimization process, established by a central tendency index and a dispersion index, should be carefully considered. In the case of the objective function being a function of many variables and parameters, the multivariate probability distribution with a joint probability density function should be taken into account.

Thus, this paper sets out to construct the joint confidence region for the PID parameters characterized by $P\ (x_k^L \leq x_k \leq x_k^U)$, which include the true values of the parameters with a determined degree of probability ($1-\alpha$).

It should be observed that a reduction of the confidence region has severe implications for the robustness desired which should be as wide as possible with regard to the design of robust controllers.

3. THE SYSTEM

As presented in Fig. 1, a jacketed vessel heater connected to a control structure including a supervisory system was considered for analysis.

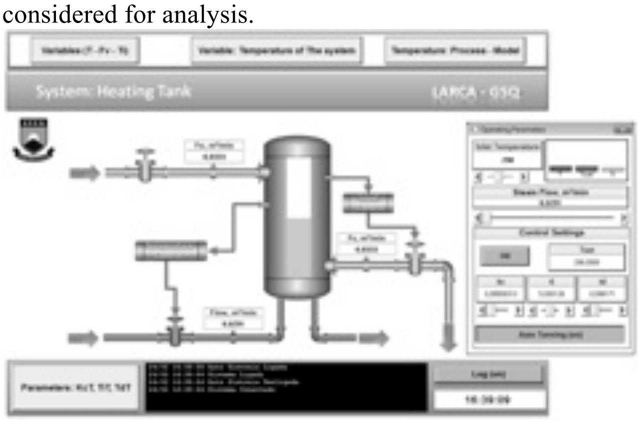

Fig. 1: System under study

Fig. 2 illustrates how information flows, as well as the block diagram that represents the identification procedure and at what point the tuning parameters are set. There is a list of symbols at the end of this paper in which the input and output signals and intermediaries, as well as the control parameters, are all clearly defined.

In order to obtain the best set of conditions for monitoring and control to be used in industrial plants, a supervisory system was built in the language of object-oriented programming, as shown previously in Fig. 1.

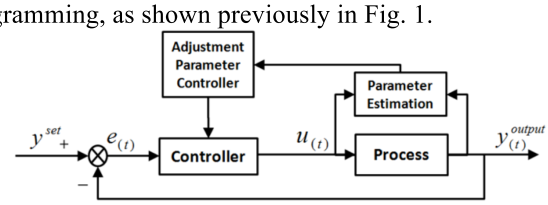

Fig. 2: Diagram of a generalized closed-loop structure with identification block

This enabled on-line information on the process variables and parameters to be extracted. Such information is needed to set up self-tuning control and is based on the work of Aström (1997). In order to simulate the operating conditions of an industrial plant, disturbances were introduced into the temperature of the inlet stream and into the stream itself.

4. THE CONTROL STRUCTURE

4.1 Building the Model

In order to provide a more realistic scenario, yet one in which the model differs in some way from the real process, this section describes the model and the process used for simulation purposes.

The dynamic modeling for the heating tank was derived from the conventional mass and energy balances, which results in the dynamic behavior for the liquid level and the temperature of the system under study. From this point on, such a strategy will represent, in fact, the process.

The discrete-time model to be used in the following procedures corresponds to a combination of the convolution model and the autoregressive model with exogenous inputs, which can be expressed by:

$$y_{(t)} + a_1 y_{(t-1)} + a_2 y_{(t-2)} + \ldots + a_n y_{(t-n)} = b_1 u_{(t-1)} + b_2 u_{(t-2)} + \ldots + b_m u_{(t-m)} + v_{(t)} \quad (1)$$

where a_i and b_i are the coefficients obtained by regression, and $\nu(t)$ denotes the combined effects of measurement noise, unmeasured disturbances and modelling errors. Since the order of the regressive model is described by n and m, the system may achieve better adjustment, relative to the mismatch between modelling and process, if large values are assumed for such parameters. However, the higher-order models can present some difficulties, namely: the distinction between the poles which correspond to structural modes and spurious poles, the computational efforts and memory requirements. Therefore, models of a lower order are always desired. How to estimate the order of such a model can be found in Moore et al. (2007).

Taking into account the backward shift operator and expressing this in a vectorial structure, the resulting in:

$$y_{(t)} = \theta^T \varphi_{(t)} + v_{(t)} \quad (2)$$

where θ, φ are the matrices of the coefficients and of the variables respectively.

Given that the classical recursive least squares method (RLSM), demonstrated by Ljung (1983) were used, the model parameters can be obtained by:

$$\hat{\theta}_{(N)} = \left[\sum_{t=1}^{N} \varphi_{(t)}\varphi_{(t)}^{T}\right]^{-1} \sum_{t=1}^{N} \varphi_{(t)}.y_{(t)} \tag{3}$$

Since the difference between the process and model was minimized, then, the parameters obtained result in the best predictions for the output variable in the sense of minimum variance.

4.2 The PID Control Law and Confidence Region

It is easy to show that the classical PID control law can be written into the discrete-time model as:

$$\Delta m_c = k_c(e_{(t)} - e_{(t-1)}) + \frac{k_c}{\tau_i}\left(\frac{(e_{(t)} + e_{(t-1)})}{2}\right)\Delta t + k_c\tau_d \frac{(e_{(t)} - 2e_{(t-1)} + e_{(t-2)})}{\Delta t} \tag{4}$$

In (4) can be rewritten, resulting in (5), where η , x_1, x_2 and x3 are denoted by Δmc, *(e(t) – e(t-1)), [(e(t) + e(t-1)) /2*] and *[(e(t) –2e(t-1) + e(t-2))/ Δt]* respectively.

$$\eta = k_c x_1 + \frac{k_c}{\tau_i} x_2 + k_c \tau_d x_3 \tag{5}$$

In order to establish the joint confidence of the parameters of the controller, a slight modification to (5) must be made, with the aim of fostering the most favourable conditions for such an approach. Hence, the model to be used for analysing the reliability of the tuning parameters should be expressed as per (6). This is written as a function of the expected value of the variables x_i, where the parameters β_0, β_1 and β_2 denote k_c, k_c/τ_i and $k_c\tau_d$, respectively. With such modifications, the estimates of β_0, β_1 and β_2, can be found without solving coupled sets of equations, which are statistically independent. Furthermore, using the least square method, the estimated parameters are unbiased, as they have the minimum variance.

$$\eta_i = \beta_0(x_{1i} - \bar{x}_1) + \beta_1(x_{2i} - \bar{x}_2) + \beta_2(x_{3i} - \bar{x}_3) \tag{6}$$

Thus, the estimates (b_j) of the parameters β_i can be calculated from the following:

$$b_j = \frac{\sum_{i=1}^{n} p_i \bar{y}_i (x_{(1+j)i} - \bar{x}_{(1+j)})}{\sum_{i=1}^{n} p_i (x_{(1+j)i} - \bar{x}_{(1+j)})^2}, \quad j = 0,1,2 \tag{7}$$

Consequently, since the data can be directly obtained from the systems, then the estimate of the output variable η can be easily found.
Bearing in mind the basic formalism for generating the joint confidence region, it must also be observed that the sum of squares for the deviation between the expected values β_0, β_1 and β_2 and their estimates b_0, b_1 and b_2 are distributed in accordance with the $\sigma^2_y\chi^2$ distribution with 1 degree of freedom, where χ^2 denotes the probability for the chi square distribution. Since all the deviations can be considered as statistically independent, their sum is also distributed as per $\sigma^2_y\chi^2$ but with a degree of freedom equal to 3, resulting in the following:

$$(b_0 - \beta_0)^2 \sum_{i=1}^{n} p_i (x_{1i} - \bar{x}_1)^2 + (b_1 - \beta_1)^2 \sum_{i=1}^{n} p_i (x_{2i} - \bar{x}_2)^2 + (b_2 - \beta_2)^2 \sum_{i=1}^{n} p_i (x_{3i} - \bar{x}_3)^2 = \sigma^2_{\bar{Y}_i}\chi^2 \tag{8}$$

It is quite straightforward from elementary statistics that a variance ratio can be formed by means of the F probability distribution expressed by the following relationship:

$$(b_0 - \beta_0)^2 \sum_{i=1}^{n} p_i (x_{1i} - \bar{x}_1)^2 + (b_1 - \beta_1)^2 \sum_{i=1}^{n} p_i (x_{2i} - \bar{x}_2)^2 + (b_2 - \beta_2)^2 \sum_{i=1}^{n} p_i (x_{3i} - \bar{x}_3)^2 = 2 s^2_{\bar{Y}_i} F_{1-\alpha} \tag{9}$$

where $v = (3, \sum p_i - 3)$ denotes the degree of freedom and *(1 – α)* the critical level of confidence.

When (9) is re-ordered, it can be clearly observed that it represents an ellipse in parameter space (coordinates: β_0, β_1 and β_2) for a given percent of confidence region, that is, (1 – α)100.

$$\frac{(\beta_0 - b_0)^2}{\frac{F_{1-\alpha,(v1,v2)}.2.s^2_{\bar{y}i}}{\sum_{i=1}^{n} p_i (x_{1i} - \bar{x}_1)^2}} + \frac{(\beta_1 - b_1)^2}{\frac{F_{1-\alpha,(v1,v2)}.2.s^2_{\bar{y}i}}{\sum_{i=1}^{n} p_i (x_{2i} - \bar{x}_2)^2}} + \frac{(\beta_2 - b_2)^2}{\frac{F_{1-\alpha,(v1,v2)}.2.s^2_{\bar{y}i}}{\sum_{i=1}^{n} p_i (x_{3i} - \bar{x}_3)^2}} = 1 \tag{10}$$

A pooled estimate of residual variances was used to obtain the variance of y_i ($\sigma^2_{\bar{y}_i}$).
Equations described in detail have been omitted throughout because of space constraints.

4.3 Tuning Procedure

It should be noted that classical auto-tuning is always implemented with the relay feedback connected to the process, what it can give rise to some disadvantages because the process is uncontrolled during the tuning time. To overcome this difficulty, a slight modification was made to the strategy, as shown in Fig. 3.

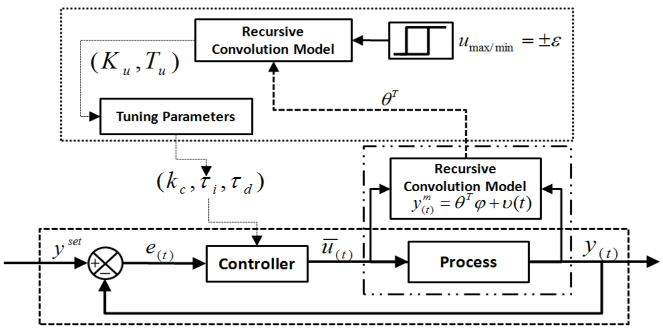

Fig. 3: Block diagram of generalized control structure

Two basic differences between classical auto-tuning and the strategy now developed can be noted, namely: classical auto-tuning is based on frequency response methods while the approach proposed operates in the time domain besides making effective use of a model for the process. In this

methodology, the convolution model presented in Section 4.1 was identified and used as the basic component in auto-tuning the relay. It can also be verified that the auto-tuning strategy may be run with the ongoing process and control system.
The implementation of the auto-tuning procedure was carried out by means of generating a stimulus introduced in the relay ($\overline{u}$), which fluctuates between ± *ε*, chosen suitably, for generating a controlled oscillation in the output variable of the model, $y_{(t)}^{m}$, with constant amplitude. By doing so, the ultimate gain and ultimate period can be determined in each sampling instance and the parameters of the controller are estimated using the classical tuning procedure.
By using the conventional rules reported by Aström (1997), the tuning parameters for the PID controller can be established.

5. RESULTS AND DISCUSSION

Despite the method of fitting being very good, the model to be used as expressed by (1) depends on the values of n and m in order to choose the appropriate functional form, taking into account the one that best describes the process. After conducting a few simulations, such values can be found with minimum computational efforts, as illustrated in Fig. 4.

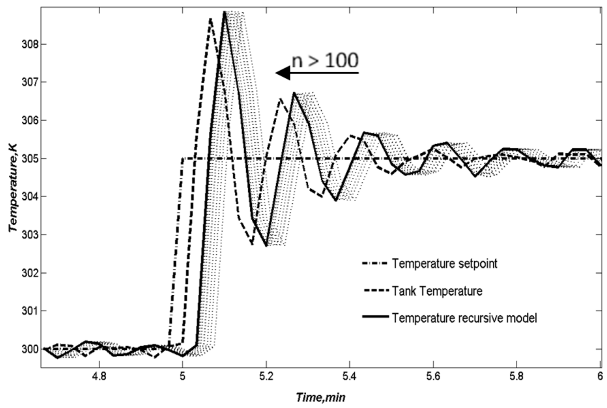

Fig.4: The behaviour of the model for n and m higher than 5.

It can e verified that for high values, the functional form representing the model becomes sufficiently close to the process. In the case of $n = m = 100$, the result proves to be satisfactory. For convenience, it is assumed n and m have the same value. For the ongoing analysis and since the aim of the study is not to develop an auto-tuning method, it should be highlighted that the over-specification of the model order can be reasonable and of interest, given that the mismatch between the model and the process can be thus minimized, and thereby it will be possible to reduce the additional interference of the model on such a procedure. In practice, the values obtained for the analysis of interest are provided by the system without the need of such an auto-tuning system, as introduced in this paper.
Since the model parameters were calculated according to (3), then the system can run in order to verify the dynamic behaviour of the output variable, and be compared with that of the process when submitted to a disturbance of 10% in the value of set point.

With the aim of adjusting the parameters of the PID controller on-line, a configuration for the control system was established, consisting of a relay, a convolution model and an algorithm for tuning parameters, as per Fig. 3. The stimulus generated by the relay is based on adequate amplitude and is stabilized, in the form of a square wave of ± *ε,* resulting in an output sinusoidal wave with constant amplitude. The relationship between the input waves and output present in Fig. 5 enables the final period and ultimate gain to be determined, as per the following in (11).

$$Ku = \frac{4\varepsilon}{\pi a} \qquad (11)$$

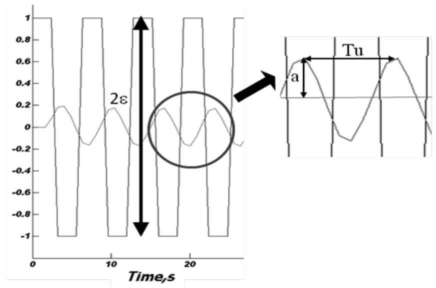

Fig. 5: Stimulus generated by the relay and response of the output variable

Since the system was running, the PID controller was automatically tuned, using the tuning rules in accordance with Aström (1997). Thus, a set of parameters in each sampling period was found resulting in the closed loop response with the auto-tuning as depicted in Fig. 6.1. and Fig.6.2 shows the corresponding close loop response without the auto-tuning procedure. The results indicate a very good agreement between the process and the model for both devices in spite of the presence of large disturbances. It can also be noted that the system operating with the auto-tuning yields a superior performance.

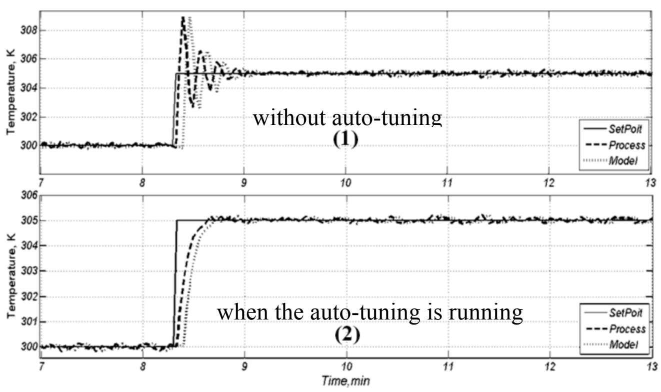

Fig. 6: The behavior of the output temperature for the model and process.

With the auto-tuning device implemented on-line and having in mind the joint confidence region drawn for the tuning parameters, the values of such parameters were recorded and distributed according to a probability function, as shown in Fig. 7.
It can be also verified that the tuning parameters follow a probability distribution which can be considered at least approximately a normal N(*μ*, σ^2). Furthermore, with regard

to the sampling variability and considering the degree of linear relationship between the tuning parameters, the data collected can be statistically analysed, resulting in the covariance matrix or in the correlation matrix given in Table 1.

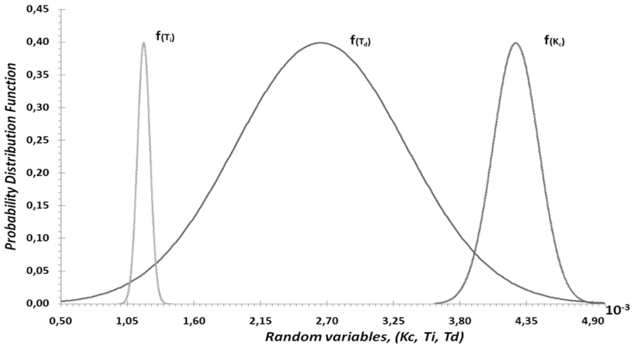

Fig. 7: The probability density function for the tuning parameters of the PID controller.

Table 1: The correlation coefficients ρ

Parameter	β_0	β_1	β_2
β_0	$\rho(\beta_0\beta_0) = 1$	$\rho(\beta_0\beta_1) = 0.137$	$\rho(\beta_0\beta_2) = 0.946$
β_1	$\rho(\beta_1\beta_0) = 0.137$	$\rho(\beta_1\beta_1) = 1$	$\rho(\beta_1\beta_2) = 0.128$
β_2	$\rho(\beta_2\beta_0) = 0.946$	$\rho(\beta_2\beta_1) = 0.128$	$\rho(\beta_2\beta_2) = 1$

Strictly speaking, real data always present some degree of correlation, as can be observed from Table 1. Consequently, the parameters are not statistically independent. However, for an ideal condition where the tuning parameters β_0, β_1 and β_2 can be considered statistically independent, the joint probability may be written as the product of the corresponding individual probabilities. Since it is normal practice to consider it acceptable to have a significance level of α=5% which corresponds to a 95% confidence interval for the individual parameters, then the joint probability given by the $P(\beta_0,\beta_1,\beta_2)$ results is equal to 0.857. This indicates a severe reduction in the joint confidence region when compared with a significance level of 5%.
Otherwise, if an approach is considered that is a little more realistic and in which the tuning parameters cannot be considered statistically independent, the joint probability is given by:

$$P(\beta_0\beta_1\beta_2) = P(\beta_0)P(\beta_1/\beta_0)P(\beta_2/\beta_0\beta_1) \tag{12}$$

However, due to the correlation coefficients $\rho(\beta_1\beta_0)$ and $\rho(\beta_1\beta_2)$ being very small, which indicates a very weak linear association between the parameters, (12) can be rewritten as:

$$P(\beta_0\beta_1\beta_2) = P(\beta_0)P(\beta_1)P(\beta_2) \tag{13}$$

even considering that the use of the correlation coefficient ρ is a conceptually weak measure of the conditional probability. The result yields $P(\beta_0,\beta_1,\beta_2)$=0.85.
From the practical standpoint, the last two considerations imply an 85% joint confidence region. It should be emphasized that for a real case, such a reduction is even more drastic.

When the values obtained for the joint confidence regions are introduced into (10), the following ellipsoid can be drawn, as shown in Fig. 8.

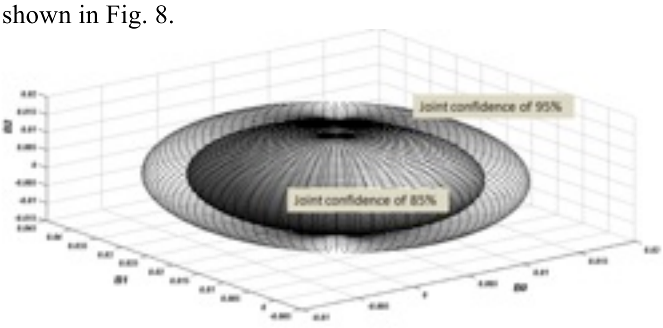

Fig. 8: Joint confidence region for the significance level α= 5% and 15%.

5.1 Restoring the joint confidence region

As shown in the previous section, the joint confidence region for the system mentioned presented a considerable reduction, at the most favorable estimate, of 15%. This means that the robust region within the constraints at the expected probabilistic level was reduced, which may adversely affect the robustness of control design. Thus, it is essential to restore the robust region at the significance level acceptable.
The basic tool with sufficient ability to deal with this problem relies on Principal Components Analysis (PCA). PCA is related to the variance-covariance structure by means of diagonalizing the covariance matrix, thus making it possible to transform the related variables into a set of uncorrelated variables denoted as Principal Components, which are linear combinations of the original variables. The essence of the method is to reduce the dimension of the system, in which the maximal variance lies in the first principal component. This is highly meaningful due to the possibility of ignoring the other principal components in view of their insignificant contributions to the total variability. Such a technique has been substantially described in work of Jackson (1991) and Johnson (1992).
By using the covariance matrix S obtained from the data previously mentioned, such a matrix can be reduced to a diagonal matrix D by premultiplying and postmultiplying by an orthonormal matrix U, by the relationship U'SU=D, where U and D are given by:

$$U = \begin{bmatrix} 0.7076 & 0.1276 & 0.6950 \\ -0.0071 & -0.9822 & 0.1875 \\ -0.7066 & 0.1376 & 0.6941 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 0.0544 & 0 & 0 \\ 0 & 0.9643 & 0 \\ 0 & 0 & 1.9813 \end{bmatrix}$$

The columns vectors of U are the eigenvectors of the covariance matrix, being used to transform the correlated variables into the new uncorrelated variables (z) called the principal components (PC's), expressed by $z = U'(\beta - \overline{\beta})$ and given by:

$$z_1 = 0.7076(\beta_0 - \overline{\beta}_0) - 0.0071(\beta_1 - \overline{\beta}_1) - 0.7066(\beta_2 - \overline{\beta}_2)$$
$$z_2 = 0.1276(\beta_0 - \overline{\beta}_0) - 0.9822(\beta_1 - \overline{\beta}_1) + 0.1376(\beta_2 - \overline{\beta}_2)$$
$$z_3 = 0.6950(\beta_0 - \overline{\beta}_0) + 0.1875(\beta_1 - \overline{\beta}_1) + 0.6941(\beta_2 - \overline{\beta}_2)$$

The elements of diagonal of D are the eigenvalues of S, which reveal the individual contribution of each principal component for the total variability. In other words, for the case considered, the first principal component corresponding to z_3 contributes to 66% of total variability while the second principal component given by z_2 explains 32% of the total variance. The third PC, z_1, has an insignificant contribution. Therefore, the total variability can be well explained by the first two principal components, thereby reducing the dimensionality of the system to 2 new variables, z_3 and z_2, which are, in fact, uncorrelated. It must be observed that based on the properties of PC, the original variables may be stated as a function of the principal components by inversion of z, that is, $\beta = \overline{\beta} + Uz$.

Thus, when a significance level of α=5% which corresponds to a 95% confidence interval for the individual parameters is considered acceptable, then the joint probability for both uncorrelated PCs is equal to 0.9, related to α=10%, which indicates a satisfactory recovery of the joint confidence region, as shown in Fig.9.

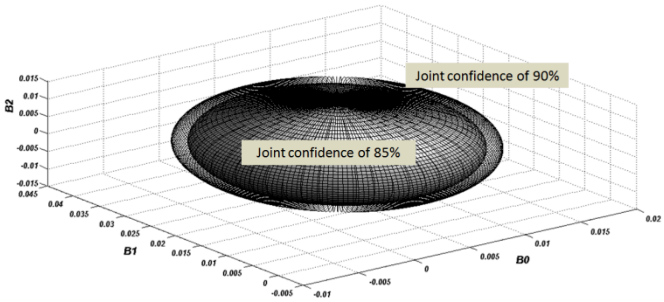

Fig. 9: Joint confidence region for the significance level α=10% and 15%

6. CONCLUSIONS

Due to the need to capture the data from simulations, a structure consisting of a block for estimating the recursive parameter and automatic tuning, connected to the process, was developed, in which the parameters of the process model are updated on-line. This allowed the on-line generation of information to be used in determining the joint confidence region.

With the aim of providing a more realistic scenario for the purposes of simulation, a convolution model was used and fitted while the process corresponding to a stirred tank heater was modeled based on the first principle.

Although not the primary goal, a strategy for auto-tuning device was also proposed in a way that allows a continuous operation of the process to be always governed by the control system. The results of the simulation, shown in Fig.6, indicate that the performance was satisfactory for the purposes of this paper.

Based on a statistical procedure that takes the sum of squares for deviation between the parameters into account, the joint confidence region was established for the values of significance level α, when the parameters are considered either statistically independent or to be cases in which statistical dependence should be considered.

Fig.9 illustrates the joint confidence region for the parameters of the PID controller, which allows the conclusion to be drawn that even for an ideal case study, the confidence region presents a significant reduction to 85%, which cannot be tolerated. When a more realistic approach is carried out, in which the parameters show strong statistical dependence, then, clearly, an even more severe reduction of the joint confidence region can be determined.

Such a reduction has a severe impact on the size of robust range in the design of control structure. However, by using the methodology of Principal Components, it was also shown how it is possible to minimize the loss of robustness by means of reducing the dimensionality of the system, by transforming correlated variables into new uncorrelated principal components. Thus, with the generation and appropriate choice of uncorrelated variables, which contribute strongly to the total variability, the robustness can be kept at an acceptable level of significance of 10%. Additional improvements in the control structure can enable a still more meaningful recovery of robustness.

LIST OF SYMBOLS

a_n, b_m coefficients of the dynamics recursive model

b_i, coefficients' estimate of the regression coefficient of the model; $e(t)$, deviation variable; F(1- α);(ν_1,ν_2), F-distribution test ; $s^2_{\hat{y}_i}$, $s^2_{\hat{b}_i}$ variance of output variable and variance of $\hat{b}_i$ t_{n-2};α/2, t-student distribution; u(t), process input variable; $u_{max}(t)$, $u_{min}(t)$, maximum and minimum value of variable input delay; x_j deviation variable and average deviation of the variable; $y(t)$, $y_m(t)$, process output variable and of the model; $\hat{y}_i$ estimated process output with parameters $\hat{\beta}_j$ of the model; τ_i, τ_d, integral and derivative time parameter

REFERENCES

Aström, K. J. and Hagglund, T. (1995). *PID controllers: Theory, design and tuning.* 2nd ed. Inst. Society of America.

Aström, K. and Wittenmark, B., (1997). *Computer Controlled Systems*, 3rd ed., Pr.-Hall.

Jackson, J. E., (1991). *A User's Guide to Principal Components*, John Wiley and Sons.

Johnson, R. A. and Wichern, D. W., (1992). *Applied multivariate statistical analysis*.Prent-Hall,NJ, USA, 6 ed.

Ljung, L. and Söderström, T. L. (1983). *Theory and Practice of Recursive Identification*, Cambridge, MIT Press.

Moore, S.M., Lai, J.C.S. and Shankar, K., (2007). ARMAX modal parameter identification in the presence of unmeasured excitation—I: Theoretical background, *Mechanical Systems and Signal Processing* 21,pp. 1616–1641.

Padulo, M., Forth, S. A. and Guenov, M. D., (2008). *Robust Aircraft Conceptual Design using Automatic Differentiation in Matlab. in Advances in Automatic Differentiation,* C. H. Bischof, H. M. B¨ucker, P. Hovland, U. Naumann, and J. Utke, eds., Lect. Notes Comput. Sci. Eng. 64, Springer, Berlin.

Paiva, M. R., Carvalho, A., Crawford, C., (2010). A Robust and Reliability Based Design Optimization Framework for Wing Design. *2nd International Conference on Engineering Optimization*. Sptember 6-9, Lisboa/Pt.

Ruel, M., (2004) Tools to troubleshoot process. Instrumentation, Systems and Automation Society. *ISA EXPO 2004*, Houston, TX.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller Tuning. *J. Process Control*, 13, 291–309.